

CHAPTER TWO

KINEMATICS OF PARTICLES

2/4 $\int v dv = \int a ds$; $\int_{30/3.6}^{200/3.6} v dv = a \int_0^{600} ds$

$200 \text{ km/h} = 200/3.6 \text{ m/s}$

$a(600) = \frac{1}{2(3.6)^2} (30^2 - 200^2)$

$a = -2.51 \text{ m/s}^2$

2/5 $v^2 = v_1^2 + 2as$; $v^2 = 4^2 + 2 \frac{9.81}{6} (5)$

$v^2 = 32.4$, $v = 5.69 \text{ m/s}$

2/6 $a = -9.81 \text{ m/s}^2$

$v^2 = v_1^2 + 2as$; $0 = (200)^2 + 2(-9.81)h$

$h = 2040 \text{ m}$

$v = v_1 + at$; $-200 = 200 + (-9.81)t$, $t = 40.8 \text{ s}$

2/7 $\Delta s = v_1(t_2 - t_1) + \frac{1}{2}a(t_2^2 - t_1^2)$

30 m/s

$= 30(3-1) + \frac{1}{2}(-9.81)(3^2 - 1^2) = 20.8 \text{ m}$

Particle rises 20.8 m

$v = v_1 + at$; $v = 30 - 9.81(4) = -9.24 \text{ m/s}$

2/8 $v dv = a ds$

$a = v \frac{dv}{ds} = 15 \frac{55-40}{15}$

$= 15 \text{ m/s}^2$

2/9 $\Delta v = \int a dt = \int_0^5 (-5g) dt = -4(5)(9.81)$

$v = -196.2 \text{ m/s}$

$v_{12} - 100 = -196.2$, $v_{12} = -96.2 \text{ m/s}$

2/10 $v dv = a ds$; $v = a \frac{dv}{ds} = 30 / (-\frac{25}{50})$

$= 60 \text{ m/s}$

2/11 $S_1 = \int v dt = \int_0^2 (300 - 75t^2) dt = 300t - 25t^3 \Big|_0^2$

$= 400 \text{ mm}$

$S_2 = \int_2^3 (300 - 75t^2) dt$

$= 300t - 25t^3 \Big|_2^3 = -175 \text{ mm}$

Distance $D = 400 + 175 = 575 \text{ mm}$

Displacement $s = 400 - 175 = 225 \text{ mm}$

2/12 $v dv = a ds$ gives $a = \frac{1}{2} \frac{d(v^2)}{ds}$ constant

$a = \frac{1}{2} \frac{-900}{200} = -2.25 \text{ m/s}^2$

$v = v_1 + at$; $0 = 30 - 2.25t$, $t = 13.33 \text{ s}$

$v_{10.33} = 30 - 2.25(10.33) = 6.75 \text{ m/s}$

$\Delta s = v_{av} \Delta t = \frac{6.75 + 0}{2} (3) = 10.12 \text{ m}$

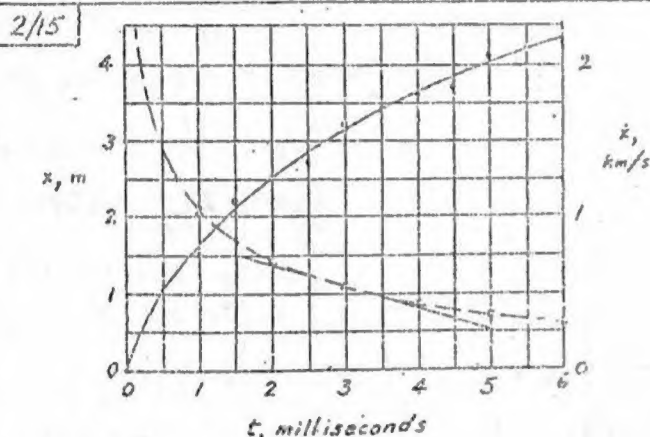
2/13 $a = -Kv$; $v dv = a ds$; $v dv = -Kv ds$

$\int_{v_0}^0 dv = -K \int_{s_0}^s ds$; $v_0 = KS$, $S = v_0/K$

$dv = a dt$; $\int_{v_0}^{v_0/2} \frac{dv}{-Kv} = \int_0^t dt$; $t = \frac{\ln 2}{K} = \frac{0.693}{K}$

2/14 $a = \frac{dv}{dt} = -cv^n$, $\int_{v_1}^v \frac{v^n}{v^n} dv = -c \int_0^t dt$

$\frac{v^{1-n}}{1-n} \Big|_{v_1}^v = -ct$, $v = [v_1^{1-n} + c(n-1)t]^{1/(1-n)}$



at $t = 3 \text{ ms}$, $v \approx 550 \text{ m/s}$

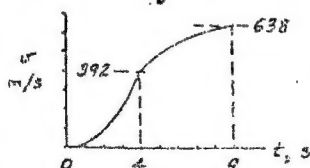
$a \approx -\frac{0.5(10^3)}{3.6(10^{-3})} = -140 \text{ km/s}^2$

2/34 $v dv = a ds$; $\frac{v dv}{-Kv^2} = ds$; $\int_1^{v_2} \frac{dv}{v^2} = -K \int_0^x dx$
 $\ln \frac{v_2}{v_1} = -Kx$, $K = \frac{1}{x} \ln \frac{v_2}{v_1} = \frac{1}{400} \ln \frac{160}{30} = 4.18(10^{-3}) \text{ m}^{-1}$
 $\frac{dv}{dt} = -Kv^2$, $\int_{v_1}^{v_2} \frac{dv}{v^2} = -Kt$, $t = \frac{1}{K} \left(\frac{1}{v_2} - \frac{1}{v_1} \right)$
 $t = \frac{10^3}{4.18} \left(\frac{1}{30} - \frac{1}{160} \right) 3.6 = 23.3 \text{ s}$

2/35 $v dv = a ds$; $a = \frac{T - 4.50v^2}{m}$
 $\int_0^s \frac{m v dv}{T - 4.50v^2} = \int_0^s ds$; $s = \frac{m}{9.00} \ln \frac{T}{T - 4.50v^2}$
 $s = \frac{16000}{9.00} \ln \frac{250}{250 - 4.50(14 \frac{1.852}{3.6})^2}$
 $= 1778 \ln 15.08 = 1778(2.714) = 4820 \text{ m or } 4.82 \text{ km}$
 $s = \frac{4.82}{1.852} = 2.60 \text{ mi (nautical)}$

2/36 $a = \frac{d^2x}{dt^2} = Kt - k^2x$, $\frac{d^2x}{dt^2} + k^2x = Kt$
 $x = x_c + x_p = A \sin kt + B \cos kt + \frac{K}{k^2} t$
 $\dot{x} = A k \cos kt - B k \sin kt + \frac{K}{k^2} = 0$ for $t=0$
 $0 = A k - 0 + \frac{K}{k^2}$, $A = -K/k^3$
 $x=0$ when $t=0$, so $0 = 0 + B + 0$, $B=0$
 Thus $x = \frac{K}{k^3} (kt - \sin kt)$

2/37 $0-4 \text{ s}$; $a = 5g t = \frac{dv}{dt}$; $\int_0^v dv = \int_0^t 5g t dt$
 $v = \frac{5}{2} g t^2$; $v_4 = \frac{5}{2} g (4)^2 = 40g = 392 \text{ m/s}$
 $4-9 \text{ s}$; $a = 10g - 2gt'$ where t' counted from $t=4 \text{ s}$
 $\frac{dv}{dt'} = 2g(5-t')$, $\int_{40g}^v dv = 2g \int_0^{t'} (5-t') dt'$
 $v = 40g + 2g(5t' - \frac{t'^2}{2})$
 $0-4 \text{ s}$; $s = \int v dt = \frac{5}{2} g \int_0^4 t^2 dt = \frac{5}{2} (9.81) \frac{4^3}{3} = 523 \text{ m}$
 $4-9 \text{ s}$; $4s = \int_0^5 g(40 + 10t' - t'^2) dt' = g(40t' + 5t'^2 - \frac{t'^3}{3})_0^5$
 $= 2780 \text{ m}$
 Total $s = 3.30 \text{ km}$

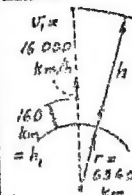


2/38 $\frac{dv}{dt} = ke^{-bt} - cv - g$; $\frac{dv}{dt} + cv = ke^{-bt} - g$
 In the standard form for the solution of first order linear equation, $e^{-\int c dt} = e^{-ct}$
 $\oint v = Ae^{-ct} + e^{-ct} \int (ke^{-bt} - g) e^{ct} dt$
 $= Ae^{-ct} + \frac{k}{c-b} e^{-bt} - \frac{g}{c}$; $v=0$ when $t=0$
 $A = \frac{g}{c} - \frac{k}{c-b}$
 Thus $v = \frac{g}{c} (e^{-ct} - 1) + \frac{k}{c-b} (e^{-bt} - e^{-ct})$

2/39 $a = g_0 \frac{r^2}{(D-s)^2}$ where $g_0 = 3.73 \text{ m/s}^2$ at surface
 $\int_{v_1}^v v dv = \int_0^s a ds$
 $\frac{1}{2} (v^2 - v_1^2) = \int_0^s g_0 \frac{r^2}{(D-s)^2} ds$
 $= g_0 r^2 \left(\frac{1}{D-s} - \frac{1}{D} \right)$
 $v^2 = v_1^2 + 2g_0 r^2 \frac{s}{(D-s)D}$

So for $s = 10000 \text{ km}$,
 $v^2 = (20000)^2 + 2(3.73)(3200) \frac{10000}{(3200)(13200)}$ (3600/10000)
 $= 400(10^6) + 234(10^6) = 634(10^6)$
 $v = 25200 \text{ km/h}$ at impact

2/40 $+s = \text{away from earth}$; $a = -g = -g_0 \frac{r^2}{(s+r)^2}$
 $\int_{v_1}^v v dv = \int_{h_1}^h -g_0 \frac{r^2}{(s+r)^2} ds$
 $\frac{v^2}{2} = -g_0 r^2 \left[\frac{1}{s+r} \right]_{h_1}^h = g_0 r^2 \left[\frac{1}{h_1+r} - \frac{1}{r+h} \right]$
 $\frac{1}{r+h} = \frac{1}{r+h_1} - \frac{v^2}{2g_0 r^2} = \frac{1}{6360+160} - \frac{(16000)^2}{2(9.833)}$
 $= 1.534(10^{-4}) - 0.2483(10^{-4}) = 1.285$



So $6360 + h = \frac{10^4}{1.285} = 7780 \text{ km}$, $h = 1420$

2/41 $\int_{v_1}^v dv = \int_{t_1}^t a dt$; $v = v_1 + \frac{eE_0}{m} \int_{t_1}^t \sin \omega t dt$
 $= v_1 + \frac{eE_0}{m\omega} \left(\frac{1}{\sqrt{2}} - \cos \omega t \right)$

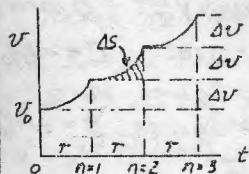
For $t = t_2 = \frac{3\pi}{4\omega}$, $v = v_2 = v_1 + \frac{eE_0}{m\omega} \sqrt{2}$

$\Delta l = v_2(t_2 - t_1) = \left(v_1 + \frac{eE_0 \sqrt{2}}{m\omega} \right) \left(\frac{3\pi}{2\omega} \right)$

$s = \int_{t_1}^{t_2} v dt = \left(v_1 + \frac{eE_0}{m\omega \sqrt{2}} \right) \left(\frac{2\pi}{4\omega} - \frac{\pi}{4\omega} \right) - \frac{eE_0}{m\omega} \int_{\pi/4\omega}^{3\pi/4\omega} \cos \omega t dt$

$s = \frac{\pi}{2\omega} \left(v_1 + \frac{eE_0}{m\omega \sqrt{2}} \right) + 0$

2/42 $\dot{v} = \frac{eE}{m} = \frac{eE_0}{m\tau} t$, $\Delta v = \int_0^{\tau} \frac{eE_0}{m\tau} t dt = \frac{eE_0 \tau}{2m}$
 (for each interval)



so $v_n = v_0 + \frac{neE_0 \tau}{2m}$

Due to Δv , $\Delta s = \int v dt = \int_0^{\tau} \frac{eE_0 t^2}{2m\tau} dt = \frac{eE_0 \tau^2}{6m}$

For arithmetic sum $S = a + (a+d) + (a+2d) + \dots$

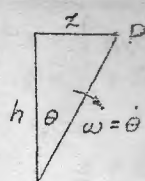
$S = n \frac{a+l}{2}$, $l = \text{last term}$
 $= a + (n-1)d$

$d = \Delta v(\tau) = \frac{eE_0 \tau^2}{2m}$, $S = na + \frac{n(n-1)d}{2}$

$S = D = n \left(v_0 \tau + \frac{eE_0 \tau^2}{6m} \right) + \frac{n(n-1)}{2} \frac{eE_0 \tau^2}{2m}$

$D = nv_0 \tau + \frac{n(3n-1)}{12} \frac{eE_0 \tau^2}{m}$

2/45 $x = h \tan \theta$, $\dot{x} = h \sec^2 \theta \dot{\theta}$



$a = \ddot{x} = 2h \sec^2 \theta \tan \theta \dot{\theta}^2 + 0$

$a = 2h\omega^2 \sec^2 \theta \tan \theta$

2/46 $N = \int \omega dt = \text{area under curve}$
 $= 5.4 \text{ rev}$

2/47 $\alpha = \frac{d\omega}{dt}$, $\int_{\omega_0}^0 \frac{d\omega}{-(b+c\omega^2)} = \int_0^t dt$
 $\frac{-1}{\sqrt{bc}} \tan^{-1} \frac{\omega \sqrt{bc}}{b} \Big|_{\omega_0}^0 = t$, $t = \frac{1}{\sqrt{bc}} \tan^{-1} \left(\omega_0 \sqrt{\frac{c}{b}} \right)$

2/48 $\omega = 20(1 + 2t/3 - t^2/3)$

$\alpha = \dot{\omega} = 40/3 - 40t/3$

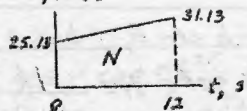
For $t = 3 \text{ s}$, $\alpha = 40(1-3)/3 = -26.7 \text{ rad/s}^2$

$N = \frac{\theta}{2\pi} = \frac{1}{2\pi} \int \omega dt = \frac{1}{2\pi} \int_0^3 20 \left(1 + \frac{2t}{3} - \frac{t^2}{3} \right) dt$

$= \frac{10}{\pi} \left[t + \frac{t^2}{3} - \frac{t^3}{9} \right]_0^3 = \frac{10}{\pi} \frac{5}{9} = 1.768 \text{ rev}$

2/49 $\alpha = \frac{30}{60} = \frac{1}{2} \text{ rad/s}^2$, constant

$\omega = \omega_0 + \alpha t$, $\omega_{12} = 4(2\pi) + \frac{1}{2}(12) = 31.13 \text{ rad/s}$



$N = \int \omega dt = \frac{25.13 + 31.13}{2} \cdot 12$
 $= 337.6 \text{ rad}$

$N = \frac{337.6}{2\pi} = 53.7 \text{ rev}$

2/50 From $\omega d\omega = \alpha d\theta$, $\alpha = \frac{1}{2} \frac{d(\omega^2)}{d\theta}$

so $\alpha = \frac{1}{2} \frac{1600 - 900}{100(2\pi)} = 0.557 \text{ rad/s}^2$

$\Delta \omega = \int \alpha dt$, $\sqrt{1600} - \sqrt{900} = 0.557 t$

$t = \frac{40 - 30}{0.557} = 17.95 \text{ s}$

2/51 $\omega = \omega_0 + k\theta = d\theta/dt$

$\int_0^{\theta} \frac{d\theta}{\omega_0 + k\theta} = \int_0^t dt$, $t = \frac{1}{k} \ln \frac{\omega_0 + k\theta}{\omega_0}$

$\frac{\omega_0 + k\theta}{\omega_0} = e^{kt}$, $\theta = \frac{\omega_0}{k} (e^{kt} - 1)$

$\omega = \dot{\theta} = \omega_0 e^{kt}$; $\alpha = \dot{\omega} = \omega_0 k e^{kt}$

2/52 $\alpha = \frac{d\omega}{dt} = \frac{1}{I} (M - k\omega^2)$

$\int_0^{\omega} \frac{d\omega}{M - k\omega^2} = \frac{1}{I} \int_0^t dt$, $\frac{1}{\sqrt{MK}} \tanh^{-1} \frac{\sqrt{MK} \omega}{M} \Big|_0^{\omega} = t$

$\frac{\sqrt{MK} \omega}{M} = \tanh \frac{\sqrt{MK} t}{I}$, $\omega = \sqrt{\frac{M}{K}} \tanh \frac{\sqrt{MK} t}{I}$

2/53 $\Delta \omega = \int \alpha dt; -300 - \omega_1 = -150(6)$

ω_1 rev/min (CW)



$\omega = 600 \text{ rev/min CW}$

$\Delta \theta = \int \omega dt$

$\theta_1 = \frac{1}{2} \left(\frac{-600}{-150} \right) \frac{600}{60} = 20 \text{ rev CW}$

$\theta_2 = \frac{1}{2} (2) \frac{300}{60} = 5 \text{ rev CCW}$

$N = \theta_1 + \theta_2 = 25 \text{ rev (CW + CCW)}$

2/54 During deceleration $\omega = 120 - 6\theta \text{ rev/min}$

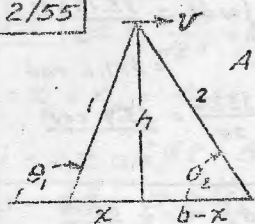
$\omega d\omega = \alpha d\theta, \omega \frac{d\omega}{d\theta} = \alpha \text{ but } d\omega = -6 d\theta$

so $\omega(-6) = \alpha = \frac{d\omega}{d\theta}$. Thus $\int_{120}^{60} \frac{d\omega}{\omega} = -6 \int_0^t dt$

$\ln \frac{60}{120} = -6t, t = \frac{1}{6} \ln 2 = \frac{0.6931}{6} = 0.1155 \text{ min}$

or $t = 0.1155(60) = 6.93 \text{ s}$

2/55



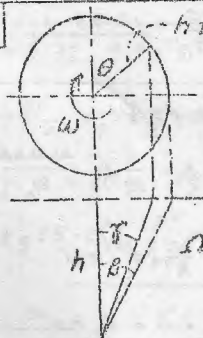
$x = -h \cot \theta_1, b-x = h \cot \theta_2$

Add & get $h = \frac{b}{\cot \theta_2 - \cot \theta_1}$

$v = \dot{x} = h \csc^2 \theta_1 \dot{\theta}_1, \omega_1 = \dot{\theta}_1$

$v = \frac{b \omega_1 \csc^2 \theta_1}{\cot \theta_2 - \cot \theta_1}$

2/56



$-h \tan \beta; h \tan \gamma = h \tan \beta \sin \theta$

$\sec^2 \gamma \dot{\gamma} = \tan \beta \cos \theta \dot{\theta}$

$\tan \beta \tan \gamma \cos \theta = \frac{\sqrt{\tan^2 \beta - \tan^2 \gamma}}{\tan \beta}$

$\dot{\omega} = \dot{\theta}$

$\Omega = \dot{\gamma} = \omega \cos^2 \gamma \sqrt{\tan^2 \beta - \tan^2 \gamma}$

2/57 $\alpha = \ddot{\theta} = -K\theta$, General solution diff. eq. $\ddot{\theta} + K\theta = 0$ is

$\theta = A \sin bt + B \cos bt$ where A, B are constants. Substitute & get

$-Ab^2 \sin bt - Bb^2 \cos bt + K A \sin bt + K B \cos bt = 0$ which requires $A(K - b^2) = 0, b = \sqrt{K}$.

$\theta = \theta_0, \dot{\theta} = 0, t = 0$ so $\theta = A\sqrt{K}, A = 0$

$\theta_{\max} = \theta_0$ when $t = 0$, so $\theta_0 = B$ & $\theta = \theta_0 \cos \sqrt{K}t$

$\omega = \dot{\theta} = -\theta_0 \sqrt{K} \sin \sqrt{K}t, |\omega|_{\max} = \theta_0 \sqrt{K}$

2/58 $\alpha = \ddot{\theta} = \frac{M_0}{I} + \frac{k}{I} \theta, \frac{d^2 \theta}{dt^2} - \frac{k}{I} \theta = \frac{M_0}{I}$

Solution is $\theta = A \sinh \sqrt{\frac{k}{I}} t + B \cosh \sqrt{\frac{k}{I}} t$

Substitute into D.E. - gives $C = -M_0/k$

Also $\dot{\theta} = 0$ when $t = 0$ gives $A = 0$

& $\theta = 0$ when $t = 0$ gives $B = -M_0/k$

Thus $\theta = \frac{M_0}{K} (\cosh \sqrt{\frac{k}{I}} t - 1)$

2/59

$y = \sqrt{b^2 + h^2} \tan \theta$
 $v = \dot{y} = \sqrt{b^2 + h^2} \sec^2 \theta \dot{\theta}$ where $\dot{\theta} = \omega$
 $\underline{\omega} = -\underline{i} \omega \cos \beta - \underline{k} \omega \sin \beta$
 $= \frac{-v \cos^2 \theta}{\sqrt{b^2 + h^2}} (\underline{i} h + \underline{k} b) \frac{1}{\sqrt{b^2 + h^2}}$
 $\underline{\omega} = \frac{-v \cos^2 \theta}{b^2 + h^2} (\underline{i} h + \underline{k} b)$

[Note: $\omega_z \neq -v/b$ & $\omega_x \neq -v/h$]

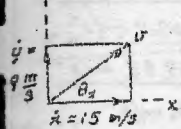
2/63 $x = 2t^2 + 3t, \dot{x} = 4t + 3, \ddot{x} = 4 \text{ m/s}^2$
 $y = t^3/3 - 8, \dot{y} = t^2, \ddot{y} = 2t \text{ m/s}^2$

When $t = 5 \text{ s}, \dot{x} = 4(5) + 3 = 15 \text{ m/s}$

$\dot{y} = (5)^2 = 9 \text{ m/s}$

$\ddot{x} = 4 \text{ m/s}^2$

$\ddot{y} = 2(5) = 6 \text{ m/s}^2$

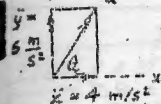


$v = \sqrt{(15)^2 + 9^2} = 17.49 \text{ m/s}$

with $\theta_x = \tan^{-1} \frac{9}{15} = 31.0^\circ$

$a = \sqrt{4^2 + 6^2} = 7.21 \text{ m/s}^2$

with $\theta_x = \tan^{-1} \frac{6}{4} = 56.3^\circ$



2/64 Angle θ for maximum range and hence minimum u is 45° .

From Sample Prob. 2/60 range is

$2s = \frac{u^2}{g} \sin 2\theta, \text{ so } 10(10^3) = \frac{u^2}{9.81} (1)$

$u = \sqrt{9.81 (10^4)} = 313 \text{ m/s}$

2/65 Real coordinates $X = 0.25x, Y = 1.25y, T = 0.04t$

so $x = 4X, y = 0.8Y, t = 25T$

$4X = 50 + 0.1(25T)^2, X = 12.5 + 15.62T^2$

$0.8Y = 50 + 0.0015(25T)^3 - 0.04(25T)^2/2,$

$Y = 62.5 + 29.3T^3/3 - 31.25T^2/2$

For actual motion $\dot{x}_x = dX/dT = 31.25T$

$\dot{y}_y = dY/dT = 29.3T^2 - 31.25T$

$a_x = 31.25 \text{ m/s}^2$

$a_y = 58.6T - 31.25$

$T = 2 \text{ s}, \dot{x}_x = 62.5 \text{ m/s}, \dot{y}_y = 54.7 \frac{\text{m}}{\text{s}}$

$a_x = 31.25 \text{ m/s}^2, a_y = 85.9 \frac{\text{m}}{\text{s}^2}$

$a = \sqrt{31.25^2 + 85.9^2} = 91.4 \frac{\text{m}}{\text{s}^2}$

2/66 $\underline{r} = \underline{i} = (2t^3 - 3t)\underline{i} + \frac{t^3}{3}\underline{j}$

$\underline{a} = \underline{\ddot{r}} = (4t - 3)\underline{i} + t^2\underline{j}$

$t = 3 \text{ s}, \underline{v} = (18 - 9)\underline{i} + 9\underline{j} = 9\underline{i} + 9\underline{j} \text{ m/s}$

$\underline{a} = (12 - 3)\underline{i} + 9\underline{j} = 9\underline{i} + 9\underline{j} \text{ m/s}^2$

$\underline{v} = 9\sqrt{2} \text{ m/s}$

$\underline{a} = 9\sqrt{2} \text{ m/s}^2$

\underline{v} & \underline{a} have same direction, so at $t = 3 \text{ s}$ particle is at inflection point on curve.

2/67 $y = 1 + x^2/10, \dot{y} = \frac{dy}{dx} \dot{x} = \frac{x}{5} \dot{x}$

But $x = 2t^3/3, \text{ so } \dot{y} = \frac{2t^3}{15} \dot{t} = \frac{4}{15} t^5$

$\ddot{y} = \frac{4}{3} t^4 = \frac{4}{3} 2^4 = \frac{64}{3} \text{ m/s}^2$

$\dot{x} = 2t^2, \ddot{x} = 4t = 4(2) = 8 \text{ m/s}^2 \text{ for } t = 2 \text{ s}$

$a = \sqrt{(64/3)^2 + 8^2} = 22.8 \text{ m/s}^2$

2/68 For y-motion; $a_y = g = 9.81 \text{ m/s}^2$
 $v_0 = 200 \text{ km/h}$

$y = \frac{1}{2} g t^2$

For x-motion; $a_x = 0, x = v_0 t$

so $y = \frac{g}{2} \frac{x^2}{v_0^2}$ & $x = v_0 \sqrt{\frac{2y}{g}}$

$x = \frac{200}{3.6} \sqrt{\frac{2(150)}{9.81}} = 307 \text{ m}$

$\theta = \tan^{-1} \frac{150}{307} = 26.0^\circ$

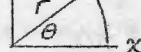
2/69 $x = r \cos \frac{\alpha t^2}{2}, \dot{x} = -r\alpha t \sin \frac{\alpha t^2}{2}$

$\ddot{x} = -r\alpha \sin \frac{\alpha t^2}{2} - r\alpha^2 t^2 \cos \frac{\alpha t^2}{2}$

$y = r \sin \frac{\alpha t^2}{2}, \dot{y} = r\alpha t \cos \frac{\alpha t^2}{2}$

$\ddot{y} = r\alpha \cos \frac{\alpha t^2}{2} - r\alpha^2 t^2 \sin \frac{\alpha t^2}{2}$

$\theta = \frac{1}{2} \alpha t^2, \alpha = \text{ang. or accel. of radial line}$



(a) $t = 0, a_x = \ddot{x} = 0; a_y = \ddot{y} = r\alpha$

(b) $\alpha t^2/2 = \pi, a_x = \ddot{x} = 2\pi r\alpha, a_y = \ddot{y} = -r\alpha$

2/70 $\dot{y} = 8t, y = 4t^2 + C; y = 2 \text{ m when } t = 0,$

so $C = 2 \text{ m}$

$y = 4t^2 + 2$

$\ddot{x} = 4t, \dot{x} = 2t^2 + C_2; \dot{x} = 0 \text{ when } t = 0, \text{ so } C_2 = 0$

$x = 2t^3/3 + C_3; x = 0 \text{ when } t = 0, \text{ so } C_3 = 0$

Eliminate t & get $y = 4\sqrt[3]{\frac{9x^2}{4}} + 2$

or $(y - 2)^3 = 144 x^2$

When $x = 18 \text{ m}, \frac{2t^3}{3} = 18, t^3 = 27, t = 3 \text{ s}$

$\dot{x} = 2(9) = 18 \text{ m/s}$

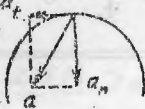
$\dot{y} = 8(3) = 24 \text{ m/s}$

$v = \sqrt{18^2 + 24^2} = 30 \text{ m/s}$

2/71 $a = a_n = r\omega^2 = R \cos \gamma \omega^2$
 $= \frac{12.742}{2} (10^6) \cos 40^\circ (0.729)^2 (10^{-4})^2$
 $= 0.0259 \text{ m/s}^2$



2/72 $a_n = \sqrt{a^2 - a_t^2} = \sqrt{100^2 - 60^2}$
 $= 80 \text{ m/s}^2$
 $a_n = v^2/r, r = \frac{v^2}{a_n} = \frac{10^2}{80} = 1.25 \text{ m}$



2/73 $a_n = g = 9.81 \text{ m/s}^2$
 $a_n = v^2/\rho, \rho = \frac{v^2}{a_n} = \frac{620^2}{9.81} = 39,200 \text{ m}$
 or 39.2 km

2/74 $a_n = v\dot{\theta} = g, \dot{\theta} = \frac{9.81}{720 \times 10^3 / 3600} = 0.0491 \text{ rad/s}$
 or $\dot{\theta} = 0.0491 \frac{180}{\pi} = 2.81 \text{ deg/s}$

2/75 For A; $\omega^2 = \omega_1^2 + 2\alpha\theta, \alpha = \frac{120^2 - 20^2}{2(5)} \frac{2\pi}{(60)^2}$
 $= 2.44 \text{ rad/s}^2$

For B; $v_B = v_A = r_A \omega_A = 0.1 \frac{50(2\pi)}{60} = 0.514 \text{ m/s}$
 $a_n = v^2/r = 0.514^2 / 0.15 = 0.658 \text{ m/s}^2$
 $a_{tB} = a_{tA} = r \alpha = 0.1(2.44) = 0.244 \text{ m/s}^2$
 $a_p = \sqrt{a_n^2 + a_t^2} = \sqrt{0.658^2 + 0.244^2} = 0.702 \text{ m/s}^2$



Given: $a_{At} = 1 \text{ m/s}^2$
 $a_{Bn} = 0.6 \text{ m/s}^2$
 $a_t = r\alpha; \alpha = 1/0.2 = 5 \text{ rad/s}^2$
 $a_{Bt} = 0.15(5) = 0.75 \text{ m/s}^2$
 $a_n = r\omega^2; \omega = \sqrt{0.6/0.15} = 2 \text{ rad/s}$
 $v = r\omega, v_A = 0.2(2) = 0.4 \text{ m/s}$
 $a_B = \sqrt{0.6^2 + 0.75^2} = 0.960 \text{ m/s}^2$

$r_A = 0.2 \text{ m}, r_B = 0.15 \text{ m}$
 $a_B = \sqrt{0.6^2 + 0.75^2} = 0.960 \text{ m/s}^2$

2/77 $r = r_1; v = \dot{r} = r\dot{\theta};$ but $\dot{r} = \dot{\theta}e_\theta$
 so $v = r\dot{\theta}e_\theta; a = \ddot{r} = \dot{v} = r\ddot{\theta}e_\theta + r\dot{\theta}(-\dot{\theta}e_r)$
 But $\ddot{\theta} = 0$, so $a = -r\dot{\theta}^2 e_r$

2/78 $|\Delta v| = 2v \sin \frac{\Delta\theta}{2} = 20 \sin \frac{\Delta\theta}{2}$
 $\Delta t = \frac{\Delta s}{v} = \frac{r\Delta\theta}{v} = \frac{2\Delta\theta}{10} = \frac{\Delta\theta}{5}$

$(a_n)_{AV} = \frac{|\Delta v|}{\Delta t} = 100 \frac{\sin \Delta\theta/2}{\Delta\theta}$

$30^\circ, \Delta\theta = 0.5236 \text{ rad}, (a_n)_{AV} = 100 \frac{0.2588}{0.5236} = 49.4 \text{ m/s}^2$

$15^\circ, \Delta\theta = 0.2618 \text{ rad}, (a_n)_{AV} = 100 \frac{0.2305}{0.2618} = 49.9 \text{ m/s}^2$

$5^\circ, \Delta\theta = 0.0873 \text{ rad}, (a_n)_{AV} = 100 \frac{0.0436}{0.0873} = 49.98 \text{ m/s}^2$

Instantaneous $a_n = v^2/r = 100^2/2 = 50 \text{ m/s}^2$

2/79 $d\mathbf{r}_1 = |r_1| d\theta \mathbf{e}_1$
 $\dot{r}_1 = \frac{d\mathbf{r}_1}{dt} = \frac{d\theta}{dt} \mathbf{e}_1 = \dot{\theta} \mathbf{e}_1$
 $d\mathbf{e}_1 = |\mathbf{e}_1| d\theta (-\mathbf{e}_2)$
 $\dot{\mathbf{e}}_1 = \frac{d\mathbf{e}_1}{dt} = \frac{d\theta}{dt} (-\mathbf{e}_2) = -\dot{\theta} \mathbf{e}_2$

2/80 $a_r = \ddot{r} - r\dot{\theta}^2; a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
 $\ddot{r} = 0, \dot{r} = -0.3 \text{ m/s}, \ddot{\theta} = -280 \frac{2\pi}{60} = -29.3 \text{ rad/s}^2$
 $\dot{\theta} = 80 \frac{2\pi}{60} = 8.38 \text{ rad/s}$
 $a = a_r \mathbf{e}_1 + a_\theta \mathbf{e}_2$
 so $a = (0 - 0.25(8.38)^2) \mathbf{e}_1 + [0.25(-29.3) + 2(-0.3)(8.38)] \mathbf{e}_2$
 $= -17.55 \mathbf{e}_1 - 12.36 \mathbf{e}_2 \text{ m/s}^2$

2/81 $a_r = \ddot{r} - r\dot{\theta}^2; 0 = \ddot{r} - 0.25(4)^2, \ddot{r} = 4 \text{ m/s}^2$
 $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}; 0 = 0.25(16) + 2\dot{r}(4), \dot{r} = -0.5 \text{ m/s}$

2/82 For $\theta = 0, a_r = \ddot{r} - r\dot{\theta}^2$ where $\dot{\theta} = v/r$
 $a_n = v^2/\rho$
 But $c = -a_n$, so $\ddot{r} - v^2/r = -v^2/\rho$
 $\ddot{r} = v^2(\frac{1}{r} - \frac{1}{\rho}) = -v^2(\frac{1}{\rho} - \frac{1}{r})$

2/83 $r = 2t^2, \dot{r} = 4t, \ddot{r} = 4 \text{ m/s}^2$
 $\theta = 0.4 \sin \frac{\pi t}{5}, \dot{\theta} = \frac{4\pi}{50} \cos \frac{\pi t}{5}, \ddot{\theta} = -\frac{4\pi^2}{90} \sin \frac{\pi t}{5}$
 For $t = 2 \text{ s}, r = 8 \text{ m}, \dot{r} = 8 \text{ m/s}, \ddot{r} = 4 \text{ m/s}^2$
 $\dot{\theta} = \frac{4\pi}{50} \left(\cos \frac{2\pi}{5} \right) = -0.209 \text{ rad/s}, \ddot{\theta} = -\frac{4\pi^2}{90} \left(\sin \frac{2\pi}{5} \right) = -0.380 \text{ rad/s}^2$
 $v_r = \dot{r} = 8 \text{ m/s}, v_\theta = r\dot{\theta} = 8(-0.209) = -1.676 \text{ m/s}$
 $\mathbf{v} = 8\mathbf{r}_1 - 1.676\mathbf{\theta}_1, \text{ m/s}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 4 - 8(-0.209)^2 = 3.65 \text{ m/s}^2$
 $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 8(-0.380) + 2(8)(-0.209) = -6.39 \text{ m/s}^2$
 $\mathbf{a} = 3.65\mathbf{r}_1 - 6.39\mathbf{\theta}_1, \text{ m/s}^2$


2/84 $\dot{r} \neq \mathbf{v}, \dot{\theta} \neq \mathbf{v}, \ddot{r} \neq \mathbf{a}, \ddot{\theta} \neq \mathbf{a}$ since scalar \neq vector
 $\dot{r} \neq \mathbf{v}$ since $\mathbf{v} \neq (v_r = \dot{r})$
 $\dot{\theta} \neq \mathbf{a}$ since $\dot{\theta}$ is only one part of magnitude of \mathbf{a}
 $\ddot{r} \neq \mathbf{a}$ since $\mathbf{v} = \dot{r}$ also contains a θ -component
 $\ddot{\theta} \neq \mathbf{a}$ since $\mathbf{a} = \ddot{r}$ contains another r -component and also the θ -component
 $\ddot{r} \neq r\ddot{\theta}$ since $\dot{r} = \mathbf{v}$ also contains an r -component


2/85 $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{1}{r} \frac{d}{dt}(r^2\dot{\theta})$
 From graph, at $t = 7 \text{ s}, \frac{d}{dt}(r^2\dot{\theta}) \approx \frac{24-12}{10-4} = 2 \text{ m}^2/\text{s}^2$
 For $r = 0.5 \text{ m}, a_\theta = \frac{1}{0.5}(2) = 4 \text{ m/s}^2$

2/86 $r = b - c \cos \theta, \dot{r} = cw \sin \theta, \ddot{r} = cw^2 \cos \theta$
 $a_r = \ddot{r} - r\dot{\theta}^2 = cw^2 \cos \theta - (b - c \cos \theta)\omega^2 = (2c \cos \theta - b)\omega^2$
 $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2cw^2 \sin \theta$
 $a = \sqrt{a_r^2 + a_\theta^2} = \omega^2 \sqrt{4c^2 - 4bcc \cos \theta + b^2}$

2/87 $a_n = g = v^2/p, p = \frac{v^2}{g} = \frac{(720)^2}{9.81} = 52800 \text{ m}$
 $= 52.8 \text{ km}$

2/88 Horizontal velocity constant
 at $72 \text{ km/h} = 20 \text{ m/s}$
 At top of trajectory $a_n = \frac{v^2}{p} = g$
 $p = \frac{v^2}{g} = \frac{(20)^2}{9.81} = 40.8 \text{ m}$

2/89  $dA = \frac{1}{2} r d\theta (r) = \frac{1}{2} r^2 d\theta$
 $\dot{A} = \frac{1}{2} r^2 \dot{\theta} = \text{constant since}$
 $a_\theta = \frac{1}{r} \frac{d}{dt}(r^2\dot{\theta}) = 0$

2/90  $a_n = g \sin 30^\circ = 7.32(0.5) = 3.66 \text{ m/s}^2$
 or $a_n = \frac{3.66}{1000} (3600)^2 = 47400 \text{ km/h}^2$
 $a_n = \frac{v^2}{p}, p = \frac{v^2}{a_n} = \frac{(40000)^2}{47400} = 33700 \text{ km}$
 $\dot{v} = a_t = 7.62 - 7.32 \frac{\sqrt{3}}{2} = 1.28 \text{ m/s}^2$

2/91 $x = 10y^2, \dot{x} = 20y\dot{y}$; but $\dot{x} = 0.1 \text{ m/s}$ const.,
 so $y\dot{y} = 1/200, y^2 + y\dot{y} = 0, \dot{y} = -\frac{\dot{y}}{y} \neq \ddot{y} = 0$
 $\dot{y} = \frac{1}{200y} = \frac{1}{200} \frac{1}{\sqrt{x}}$; For $x = 0.1 \text{ m}, \dot{y} = \frac{1}{200} \frac{1}{\sqrt{0.1}} = -0.0250 \text{ j m/s}^2$

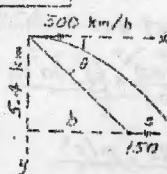
For $x = 0.1 \text{ m}, \dot{y} = \frac{1}{200} \frac{1}{\sqrt{0.1}} = 0.05 \text{ m/s}$
 so $\mathbf{v} = (20 \frac{1}{\sqrt{0.1}} \mathbf{i} + \mathbf{j}) 0.05 = 0.05(2\mathbf{i} + \mathbf{j}) \text{ m/s}$

2/92 $\overline{OA} = r = 2b \cos \theta, \dot{r} = -2bK \sin \theta, \ddot{r} = -2bK^2 \cos \theta$
 where $K = \dot{\theta} = \text{const.}$
 $a_r = \ddot{r} - r\dot{\theta}^2 = -2bK^2 \cos \theta - 2bK^2 \cos \theta = -4bK^2 \cos \theta$
 $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 - 4bK^2 \sin \theta$
 $a = \sqrt{a_r^2 + a_\theta^2} = 4K^2 b$ independent of θ

2/93 From Sample Prob. 2/60,
 $h = \frac{u^2 \sin^2 \theta}{2g}, R = 2s = \frac{u^2 \sin 2\theta}{g}$
 Eliminate θ ; $\sin \theta = \frac{\sqrt{2gh}}{u}$ so $\cos \theta = \sqrt{1 - \frac{2gh}{u^2}}$
 Thus $R = \frac{2u^2}{g} \sin \theta \cos \theta = \frac{2u^2 \sqrt{2gh}}{g} \sqrt{1 - \frac{2gh}{u^2}}$
 Simplify & get $R = 4h \sqrt{\frac{u^2}{2gh} - 1}$

2/94 $r = r_0 e^{n\theta}, \dot{r} = r_0 n \dot{\theta} e^{n\theta} = r_0 n K e^{n\theta}$
 $\ddot{r} = r_0 n^2 K^2 e^{n\theta} = r n^2 K^2$
 $a_r = \ddot{r} - r\dot{\theta}^2 = r n^2 K^2 - r K^2 = (n^2 - 1) r K^2$
 $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2r n K^2$
 $a = \sqrt{a_r^2 + a_\theta^2} = r K^2 \sqrt{(n^2 - 1)^2 + 4n^2} = R K^2 (n^2 + 1), r = R$

2/95

Time of drop, $h = \frac{1}{2}gt^2$

$$t = \sqrt{2(5400)/9.81} = 33.2 \text{ s}$$

$$s = ut = \frac{150}{3.6} 33.2 = 1382 \text{ m}$$

$$b + s = ut = \frac{500}{3.6} 33.2 = 4610 \text{ m}$$

$$\theta = \tan^{-1} \frac{5400}{4610 - 1382} = 59.2^\circ$$

2/96

$$y = ut \cos 45^\circ + \frac{1}{2}gt^2; u = \frac{1000}{3.6} = 278 \text{ m/s}$$

$$1800 = 278(0.707)t + \frac{1}{2}9.81t^2$$

solve & get $t = 7.69 \text{ s}$ (or $t = -47.7 \text{ s}$)

$$s = ut \sin 45^\circ = 278(7.69)(0.707)$$

$$= 1510 \text{ m}$$

$$\tan\left(\frac{\pi}{4} - \delta\right) = \frac{1510}{1800} = 0.8389$$

$$\frac{\pi}{4} - \delta = 39.99^\circ, \delta = 45^\circ - 39.99^\circ = 5.01^\circ$$

2/97

$$a_n = g = \frac{v^2}{\rho} \text{ where } g = g_0 \left(\frac{R}{R+h} \right)^2$$

$$\rho = \frac{1}{g_0} \left(\frac{R+h}{R} \right)^2 v^2 = \frac{1}{9.821(10^{-3})} \left(\frac{18702}{12743/2 + 400} \right)^2 \frac{32000}{3600} = 9090 \text{ km}$$

2/98

$$a_n = \frac{v^2}{\rho} = \frac{v^2}{R+h} \text{ where } R = 1738 \text{ km}$$

$$g_0 = 1.62 \text{ m/s}^2$$

$$a_n = g = g_0 \left(\frac{R}{R+h} \right)^2 = 1.62 \left(\frac{1738}{1738+160} \right)^2 = 1.359 \text{ m/s}^2$$

$$v^2 = (1738+160)1.359(3600)^2(10^{-3}) = 33.42(10^6) (\text{km/h})^2$$

$$v = 5781 \text{ km/h}$$

2/99



$$a_y = g, v_y = gt$$

$$a_x = 2v_y \omega \cos \theta = 2gt \omega \cos \theta$$

$$\dot{x} = \int a_x dt = gt^2 \omega \cos \theta$$

$$x = \int \dot{x} dt = \frac{1}{3}gt^3 \omega \cos \theta = \delta$$

But time of fall is given by $h = \frac{1}{2}gt^2$

$$\text{so } \delta = \left(\frac{2h}{g} \right)^{3/2} \frac{g\omega}{3} \cos \theta, \delta = \frac{2\sqrt{2}}{3} \omega h \sqrt{\frac{h}{g}} \cos \theta$$

2/100

Eq. of trajectory (Sample Prob. 2/60)

$$y = x \tan \theta - \frac{gx^2}{2u^2} \sec^2 \theta$$

$$= x \tan \theta - \frac{gx^2}{2u^2} (1 + \tan^2 \theta)$$

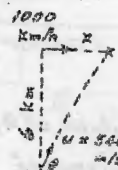
$$1500 = 5000 \tan \theta$$

$$- \frac{9.81(5000)^2}{2(400)^2} (1 + \tan^2 \theta)$$

$$\text{or } \tan^2 \theta - 6.524 \tan \theta + 2.957 = 0$$

$$\text{Solution gives } \theta = 26.1^\circ \text{ or } \theta = 80.6^\circ$$

2/101

Aircraft: $x = \frac{1000}{3.6} t$ 

$$\text{Shell: } x = 540 t \cos \theta$$

$$6000 = 540 t \sin \theta - \frac{9.81}{2} t^2$$

From the two equations,

$$\cos \theta = \frac{1000}{3.6(540)} = 0.5144, \theta = 59.0^\circ$$

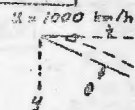
$$\sin \theta = 0.8575$$

$$6000 = 540(0.8575)t - \frac{9.81}{2} t^2$$

$$\text{or } t^2 - 94.41t + 1223 = 0, t = 15.50 \text{ s (} t = 78.9 \text{ s)}$$

2/102

$$\ddot{x} = 0.5g, \dot{x} = u + 0.5gt, x = ut + 0.25gt^2$$



$$\ddot{y} = g, \dot{y} = gt, y = \frac{1}{2}gt^2$$

$$\text{so } t = \sqrt{2h/g}, s = u\sqrt{\frac{2h}{g}} + \frac{h}{2}$$

$$\theta = \tan^{-1} \frac{h}{u\sqrt{\frac{2h}{g}} + \frac{h}{2}} = \tan^{-1} \frac{1500}{\frac{1000\sqrt{3000}}{3.6} + \frac{1500}{2}} = \tan^{-1} 0.2675 = 14.98^\circ$$

2/103

Acceleration in all directions is zero, so

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0, \ddot{r} = r\dot{\theta}^2$$

$$r = h/\sin \theta, r = 8 \frac{2}{\sqrt{3}} = 9.24 \text{ km}$$

$$\ddot{r} = 9.24(0.025)^2 = 0.00577 \text{ km/s}^2$$

$$= 5.77 \text{ m/s}^2$$

$$v = |\dot{r}|/\sin \theta = \frac{h\dot{\theta}}{\sin^2 \theta}$$

$$v = \frac{8000(0.025)}{(\sqrt{3}/2)^2} = 267 \text{ m/s} = 960 \text{ km/h}$$

2/104

$$a_t = -ks; \int_{v_0}^v v dv = \int_0^s -ks ds, v_0^2 = v^2 + ks^2$$

$$144 = 36 + k(324), k = \frac{1}{3} \text{ s}^{-2}; a_t = -\frac{1}{3}18 = -6 \text{ m/s}^2$$

$$a_n = \sqrt{a^2 - a_t^2} = \sqrt{10^2 - 6^2} = 8 \text{ m/s}^2 = v^2/\rho, \rho = \frac{36}{8} = 4.5 \text{ m}$$

2/105 $v = 2 + 0.3t^2$, $a_t = \dot{v} = 0.6t = 0.6(2) = 1.2 \text{ m/s}^2$
for $t = 2 \text{ s}$

$a^2 = a_t^2 + a_n^2$, $a_n^2 = 3.2^2 - 1.2^2 = 4.32$, $a_n = 2.078 \text{ m/s}^2$

$v_{t=2} = 2 + 0.3(2)^2 = 3.2 \text{ m/s}$

$a_n = v^2/\rho$, $\rho = v^2/a_n = 3.2^2/2.078 = 4.93 \text{ m}$

2/106 $a_n = v^2/\rho = 6^2/2 = 18 \text{ m/s}^2$; $a_t = \sqrt{a^2 - a_n^2} = \sqrt{50^2 - 18^2}$

$a_t = \rho\ddot{\theta} + \dot{\rho}\dot{\theta}$, $\dot{\theta} = v/\rho = 6/2 = 3 \text{ rad/s}$ $\dot{\rho} = 24 \text{ m/s}$

$24 = 2(3) + \dot{\rho}(3)$, $\dot{\rho} = 6 \text{ m/s}$

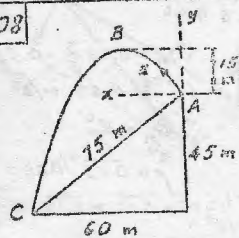
2/107 $r = K\theta$, $\dot{r} = K\dot{\theta}$, $\ddot{r} = K\ddot{\theta} = K\alpha$, $\dot{\theta}^2 = 2 \int \alpha d\theta = \alpha\theta$
 $\alpha = 3\pi/4$

$a_r = \ddot{r} - r\dot{\theta}^2 = K\alpha - \frac{3}{4}K\pi^2\alpha = K\alpha(1 - 3\pi^2/4)$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{3K\pi\alpha}{4} + 2K\pi\alpha = K\alpha(11\pi/4)$

$a = \sqrt{a_r^2 + a_\theta^2} = K\alpha\sqrt{1 + \frac{97\pi^2}{16} + \frac{9}{16}\pi^4} = 10.76 K\alpha$

2/108



y-dir, $v^2 = v_1^2 + 2as$
 $s = v_1 t + \frac{1}{2}at^2$

$0 = v_y^2 - 2g(15)$ A to B

$v_y = \sqrt{30g} = 17.16 \text{ m/s}$

$-45 = 17.16t - \frac{1}{2}9.81t^2$

$t = 5.25 \text{ s}$ (or $t = -1.749 \text{ s}$)

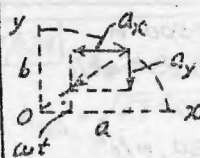
x-dir, $60 = v_x(5.25)$, $v_x = 11.44 \text{ m/s}$

$u = \sqrt{11.44^2 + 17.16^2} = 20.6 \text{ m/s}$

2/110 $x = a \cos \omega t$, $\dot{x} = -a\omega \sin \omega t$, $\ddot{x} = -a\omega^2 \cos \omega t$
 $y = b \sin \omega t$, $\dot{y} = b\omega \cos \omega t$, $\ddot{y} = -b\omega^2 \sin \omega t$
 $\cos^2 \omega t + \sin^2 \omega t = 1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$, an ellipse

$|a_x|_{\max} = |\ddot{x}|_{\max} = a\omega^2$ when $t = \frac{n\pi}{\omega}$, $n = 0, 1, 2, \dots$

$|a_y|_{\max} = |\ddot{y}|_{\max} = b\omega^2$ when $t = \frac{n\pi}{2\omega}$, $n = 1, 3, 5, \dots$

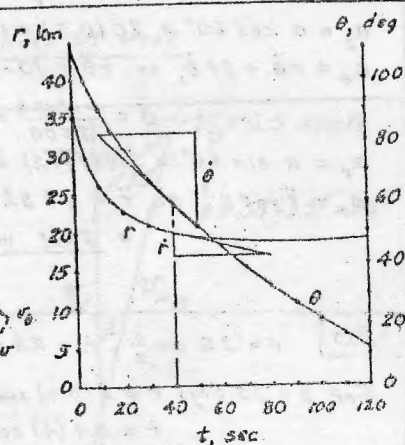
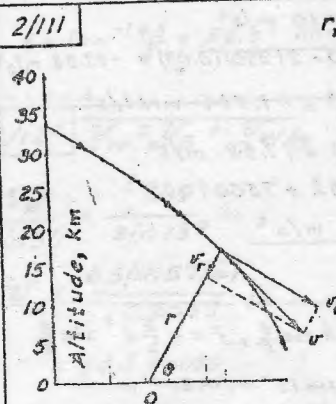


$\frac{a_y}{a_x} = \frac{-b\omega^2 \sin \omega t}{-a\omega^2 \cos \omega t} = \frac{b}{a} \tan \omega t$

$\frac{y}{x} = \frac{b \sin \omega t}{a \cos \omega t} = \frac{b}{a} \tan \omega t$

Therefore accel. is directed toward O

2/111



$t = 40 \text{ s}$, $r = 20 \text{ km}$

$\dot{r} = -77.5 \text{ m/s}$

$\dot{\theta} = -0.014 \text{ rad/s}$

$v = \sqrt{\dot{r}^2 + (r\dot{\theta})^2} = \sqrt{(77.5)^2 + (20 \times 10^3)^2(0.014)^2} = 284 \text{ m/s}$

or $v = 1020 \text{ km/h}$

2/109 $r = r_0 + b_0 \sin 2\pi n t$, $\dot{r} = 2\pi b_0 n \cos 2\pi n t$

$\ddot{r} = -4\pi^2 b_0 n^2 \sin 2\pi n t$

$\dot{\theta} = \omega$, $\ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2$, $a_r = -4\pi^2 b_0 n^2 \sin 2\pi n t - (r_0 + b_0 \sin 2\pi n t)\omega^2$
 $= -(4\pi^2 n^2 + \omega^2)b_0 \sin 2\pi n t - r_0 \omega^2$

$|a_r|_{\max} = (4\pi^2 n^2 + \omega^2)b_0 + r_0 \omega^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$, $a_\theta = 0 + 4\pi b_0 n \omega \cos 2\pi n t$

$|a_\theta|_{\max} = 4\pi b_0 n \omega$

2/112 Accel. directed from A to N is

$v = 17970 \text{ km/h}$

$a = g = g_0 \left(\frac{R}{R+h} \right)^2$

$= 9.821 \left(\frac{6370}{16000} \right)^2 = 1.557 \text{ m/s}^2$

$a_n = 1.557 \cos \theta = 1.557 (0.866)$

$= 1.348 \text{ m/s}^2$

$a_n = \frac{v^2}{\rho}$, $\rho = \frac{(17970)^2}{1.348 (10^{-3})(3600)^2} = 18480 \text{ km}$

2/113

$\dot{y} = 2 \text{ m/s}$ constant so $\ddot{y} = 0$
 $\&$ accel. can have no vertical component
 $v = \dot{y} / \cos 30^\circ = 2 / 0.866 = 2.31 \text{ m/s}$
 $a_n = \frac{v^2}{r} = \frac{2.31^2}{0.2} = 26.7 \text{ m/s}^2$
 $a_t = 26.7 \tan 30^\circ = 15.40 \text{ m/s}^2$
 $a_t = r\ddot{\theta}$, $\ddot{\theta} = 15.40 / 0.2 = 77.0 \text{ rad/s}^2$

2/114

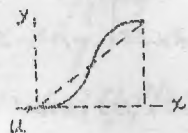
Given $r = 7500 \text{ m}$
 $\dot{\theta} = 0.03 \text{ rad/s}$
 $\theta = 60^\circ$
 $a = 20 \text{ m/s}^2$
 $v_\theta = r\dot{\theta} = 7500(0.03) = 225 \text{ m/s}$
 $v = 225 / \cos 60^\circ = 450 \text{ m/s}$
 $v_r = \dot{r} = 225 \tan 60^\circ = 390 \text{ m/s}$
 $a_\theta = a \cos 60^\circ = 20(0.5) = 10 \text{ m/s}^2$
 $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$, so $r\ddot{\theta} = 10 - 2(390)(0.03) = -13.38 \text{ m/s}^2$
 $\ddot{\theta} = \frac{-13.38}{7500} = -1.784 \text{ mrad/s}^2$
 $a_r = a \sin 60^\circ = 20(\sqrt{3}/2) = 17.32 \text{ m/s}^2$
 $a_r = \ddot{r} - r\dot{\theta}^2$, so $\ddot{r} = 17.32 + 7500(0.03)^2$
 $= 24.1 \text{ m/s}^2$

2/115

$r = 2R \sin \frac{\theta}{2}$, $\dot{r} = R\dot{\theta} \cos \frac{\theta}{2}$, $\ddot{r} = -\frac{R}{2}\ddot{\theta} \sin \frac{\theta}{2}$
 For $\theta = 30^\circ$, $r = 2(0.4) \sin 15^\circ = 0.207 \text{ m}$
 $\dot{r} = 0.4(4) \cos 15^\circ = 1.545 \text{ m/s}$
 $\ddot{r} = -\frac{0.4}{2}(4)^2 \sin 15^\circ = -0.828 \text{ m/s}^2$
 $a_r = \ddot{r} - r\dot{\theta}^2 = -0.828 - 0.207(4)^2 = -4.14 \text{ m/s}^2$
 $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.545)(4) = 12.36 \text{ m/s}^2$
 $a = \sqrt{4.14^2 + 12.36^2} = 13.04 \text{ m/s}^2$

2/116

$\ddot{y} = \frac{eE_0}{m} \sin pt$, $\dot{y} = -\frac{eE_0}{mp} \cos pt + K$
 $\dot{y} = 0$ when $t = 0$, so $K = +\frac{eE_0}{mp}$
 $\&$ $\dot{y} = \frac{eE_0}{mp} (1 - \cos pt)$
 $y = \frac{eE_0}{mp} (t - \frac{1}{p} \sin pt) + K$, $r = 0$
 $x = ut$
 For one complete cycle, $t = 2\pi/p$, and
 $x = 2\pi u/p$ $\&$ $y = \frac{eE_0}{mp^2} 2\pi$



2/117

$\ddot{y} = -g$, $\dot{y} = u \sin 45^\circ - gt$, $y = ut \sin 45^\circ - \frac{1}{2}gt^2$
 $\ddot{x} = 0$, $\dot{x} = u \cos 45^\circ$, $x = ut \cos 45^\circ$
 $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = 1 - \frac{gt}{u} \sqrt{2} = 1 - \frac{2gx}{u^2}$, $x = \frac{u^2}{2g} (1 - \frac{dy}{dx})$
 $\frac{dy}{dx} = \tan 20^\circ = 0.3640$, $x = \frac{(15)^2(10^4)}{(3600)^2(2)(9)(10^{-3})} (1 - 0.364) = 613 \text{ m}$
 $t = \frac{x}{u \cos 45^\circ} = \frac{613}{15000 \sqrt{2}} = 0.0578 \text{ h or } 3 \text{ min } 28 \text{ sec}$
 $h = y = \frac{15(10^3)(0.0578)}{\sqrt{2}} - \frac{9(10^{-3})}{2} (0.0578)^2 (3600)^2 = 418 \text{ km}$

2/118

Eg. of path: $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$
 Let $m = \tan \theta$; $\frac{1}{\cos^2 \theta} = 1 + m^2$
 Thus, $y = mx - \frac{gx^2}{2u^2} (1 + m^2)$
 or $m^2 - \left(\frac{2u^2}{gx}\right)m + \left(1 + \frac{2u^2 y}{gx^2}\right) = 0$
 Roots are equal if discriminant = 0,
 Thus $\frac{4u^4}{g^2 x^2} - 4\left(1 + \frac{2u^2 y}{gx^2}\right) = 0$, $y = \frac{u^4}{2g} - \frac{gx^2}{2u^2}$
 a vertical parabola

2/119

$v = \sqrt{0.15^2 + 0.2^2} = 0.25 \text{ m/s}$ (direction of path)
 $v_A = 0.2 \text{ m/s}$, $v_B = 0.15 \text{ m/s}$
 $a_B = 0.5 \text{ m/s}^2$
 $a_n = 0.15 \frac{v}{r} = 0.50 \frac{v}{r}$
 $= 0.05 \text{ m/s}^2$
 $a_t = \frac{v^2}{r}$
 $r = \frac{0.25^2}{0.05} = 1.25 \text{ m}$
 $a_t = -0.75 \frac{v}{r} - 0.50 \frac{v}{r} = -0.90 \text{ m/s}^2$
 $a_t = \frac{d}{dt}(p\omega) = \dot{p} \frac{\omega}{p} + p\dot{\omega}$, \dot{p} cannot be found until $\dot{\omega}$ is known.

2/120

$a_r = \ddot{r} - r\dot{\theta}^2 = 0$, $\frac{d^2 r}{dt^2} - K^2 r = 0$
 Try $r = A \cosh Kt + B \sinh Kt$
 Substitute & get
 $AK^2 \cosh Kt + BK^2 \sinh Kt - AK^2 \cosh Kt - BK^2 \sinh Kt = 0$
 Thus r is the solution.
 When $r = r_0$, $t = 0$; Thus $r_0 = A + 0$, $A = r_0$
 " $\dot{r} = 0$, $\dot{r} = 0$; $r_0 K \sinh Kt = BK \cosh Kt$
 $0 + BK = 0$, $B = 0$
 Thus equation of path is
 $r = r_0 \cosh Kt$

1/21

$$(r \sin \theta)^2 + (r \cos \theta - e)^2 = b^2$$

$$r^2 - 2re \cos \theta + e^2 = b^2$$

$$r \ddot{\theta} - \dot{r} \dot{\theta} \cos \theta + \dot{r} e \omega \sin \theta = 0$$

$$\text{For } \theta = \pi/2, \dot{r} = -e\omega$$

$$r^2 + r \ddot{r} - \dot{r} e \cos \theta + r e \omega \sin \theta + r e \omega \sin \theta + r e \omega^2 \cos \theta = 0$$

$$\text{For } \theta = \pi/2, r^2 + r \ddot{r} + 2re\omega = 0, \ddot{r} = \frac{e^2 \omega^2}{\sqrt{b^2 - e^2}}$$

$$v_r = \dot{r} = -e\omega, v_\theta = r\dot{\theta} = \sqrt{b^2 - e^2} \omega$$

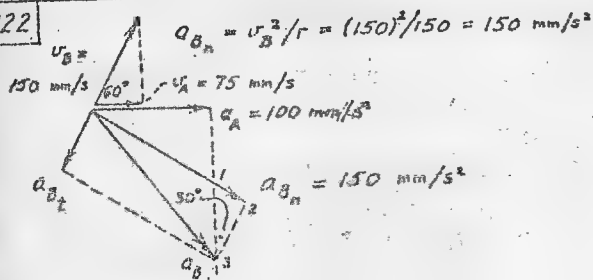
$$v = \sqrt{v_r^2 + v_\theta^2} = \omega \sqrt{b^2 - e^2 + e^2} = b\omega$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{e^2 \omega^2}{\sqrt{b^2 - e^2}} - \sqrt{b^2 - e^2} \omega^2 = \frac{2e^2 - b^2}{\sqrt{b^2 - e^2}} \omega^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-e\omega)\omega = -2e\omega^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \frac{b^2 \omega^2}{\sqrt{b^2 - e^2}}$$

2/122

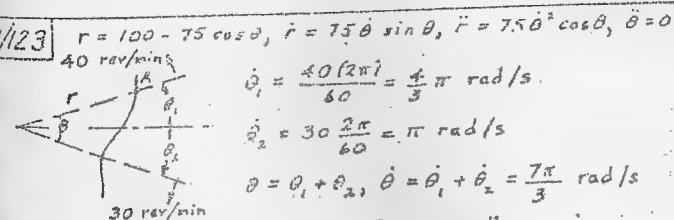


$$a_{Bn} = v_B^2 / r = (150)^2 / 150 = 150 \text{ mm/s}^2$$

$$a_{Bt} = \sqrt{2^2 - 1^2} \sin 30^\circ = (150 - 100/0.866) / \sqrt{3} = 59.8 \text{ mm/s}^2$$

$$\alpha = a_{Bt} / 150 = \frac{59.8}{150} = 0.399 \text{ rad/s}, \ddot{\theta} = -0.399 \text{ rad/s}^2$$

2/123



$$\text{For } \theta = 30^\circ, r = 100 - 75 \cos \theta = 35.0 \text{ mm}, \dot{r} = 75 \frac{7\pi}{3} \frac{1}{2} = 275 \text{ mm/s}$$

$$\ddot{r} = 75 \left(\frac{7\pi}{3} \right)^2 \frac{1}{2} = 3490 \text{ mm/s}^2$$

$$a_r = 3490 - 35.0 \left(\frac{4\pi}{3} \right)^2 = 2875 \text{ mm/s}^2$$

$$a_\theta = 35.0 (0) + 2(275) \left(\frac{4\pi}{3} \right) = 2303 \text{ mm/s}^2$$

$$a = \sqrt{2875^2 + 2303^2} = 3680 \text{ mm/s}^2 = 3.68 \text{ m/s}^2$$

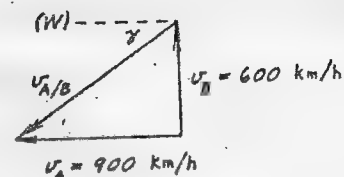
2/127

$$v_A = v_B + v_{A/B}$$

$$v_{A/B} = \sqrt{600^2 + 900^2} = 1082 \text{ km/h}$$

$$\gamma = \tan^{-1} \frac{600}{900} = 33.7^\circ$$

South of west



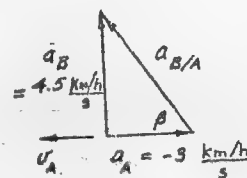
2/128

$$a_B = a_A + a_{B/A}$$

$$a_{B/A} = \sqrt{4.5^2 + 3^2} = 5.41 \text{ km/h}$$

$$\text{or } a_{B/A} = \frac{5.41}{3.6} = 1.502 \text{ m/s}^2$$

$$\beta = \tan^{-1} \frac{4.5}{3} = 56.3^\circ \text{ north of west}$$

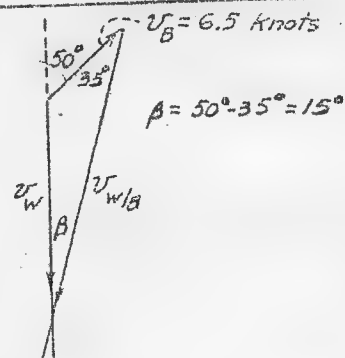


2/129

$$v_W = v_B + v_{W/B}$$

$$\frac{v_W}{\sin 35^\circ} = \frac{6.5}{\sin 15^\circ}$$

$$v_W = \frac{6.5(0.5736)}{0.2598} = 14.4 \text{ knots}$$



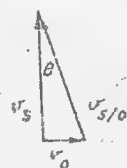
2/130

Let S = satellite, O = observer

$$v_S = v_O + v_{S/O}; v_O = 6378 (0.729)(10^{-4}) (3600) = 1674 \text{ km/h (East)}$$

$$v_S = 27940 \text{ km/h (North)}$$

$$\theta = \tan^{-1} \frac{1674}{27940} = 3.43^\circ$$

Satellite appears to travel 3.43° west of north

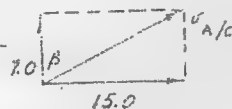
2/131

$$v_{A/B} = v_{A/C} + v_{C/B}; v_{A/C} = v_{A/E} - v_{C/B}$$

$$v_{A/C} = (16.8i - 3.6j) - (-8.2i - 10.6j) = 15.0i + 7.0j$$

$$v_{A/C} = \sqrt{15.0^2 + 7.0^2} = 16.55 \text{ knots}$$

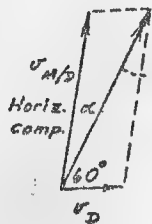
$$\beta = \tan^{-1} \frac{15.0}{7.0} = 65.0^\circ$$



2/132 Let v_D = velocity of destroyer = $30(1.852)$
= 55.6 km/h

or $v_D = \frac{55.6}{3.6} = 15.43$ m/s

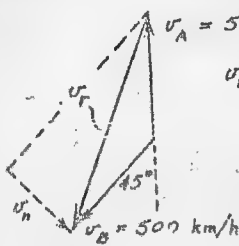
$(v_{M/D})_{\text{Horiz.}} = 75 \cos 30^\circ = 65.0$ m/s



$\sin \alpha = \frac{v_D}{v_{M/D}} \sin 60^\circ = \frac{15.43}{65.0} \frac{\sqrt{3}}{2} = 0.2058$

$\alpha = 11.88^\circ$

2/133 $v_B = v_A + v_r$ where $v_r = v_{B/A}$



$v_r^2 = 500^2 + 500^2 + 2(500)(500)\cos 45^\circ$
= $854\,000$ (km/h)²

$v_r = 924$ km/h

$v_r = 924$ km/h

$v_n = 500 \cos 45^\circ = 354$ km/h

Let h = possible constant difference in altitude

k = unit vector in vertical direction

Since $\frac{d}{dt}(kh) = 0$, vertical separation has no influence on relative-velocity equations.

2/134 $a_A = a_B + a_{A/B}$



$|a_A| = \frac{45}{3.6} = 12.5$ m/s²

$|a_B| = \frac{30}{3.6} = 8.33$ m/s²

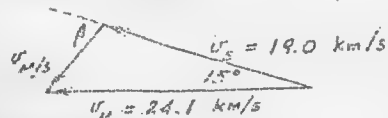
$a_{A/B} = 12.5 \underline{i} - 2.33 \left(\frac{\underline{i}}{\sqrt{2}} + \frac{\underline{j}}{\sqrt{2}} \right)$
= $10.39 \underline{i} - 5.87 \underline{j}$ m/s²

or $a_{A/B} = 19.31(0.952 \underline{i} - 0.305 \underline{j})$ m/s²

2/135 Mars will appear to be approaching

Spacecraft head on when $v_{M/S}$ is along line of sight M-S.

$v_M = v_S + v_{M/S}$



$v_{M/S}^2 = 19.0^2 + 24.1^2 - 2(19.0)(24.1) \cos 15^\circ = 57.2$ (km/s)²

$v_{M/S} = 7.56$ km/s, $\frac{24.1}{\sin(\pi - \beta)} = \frac{7.56}{\sin 15^\circ}$

$\sin(\pi - \beta) = \sin \beta = 0.3246, \beta = 55.6^\circ$

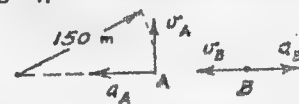
2/136 $a_A = a_B = 0$ so $a_{A/B} = 0$

In polar coordinates relative to B,

$(a_{A/B})_r = \ddot{r} - r\dot{\theta}^2 = 0$ so $\ddot{r} = r\dot{\theta}^2$

$(a_{A/B})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$ so $\ddot{\theta} = -2\dot{r}\dot{\theta}/r$

2/137 $v_A = 54/3.6 = 15$ m/s, $v_B = 81/3.6 = 22.5$ m/s

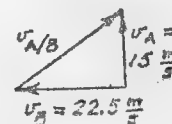


$a_A = \frac{v_A^2}{\rho} = \frac{15^2}{150} = 1.5$ m/s²

$a_B = 3$ m/s²

$v_A = v_B + v_{A/B}$

$a_A = a_B + a_{A/B}$



$a_A = 1.5$ m/s², $a_B = 3$ m/s²

$v_{A/B} = \sqrt{22.5^2 + 15^2} = 27.0$ m/s

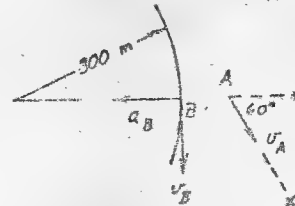
$a_{A/B} = 4.5$ m/s²

in dir. shown

in dir. shown

2/138 $a_B = v_B^2/r = (90/3.6)^2/300 = 2.08$ m/s²

$a_A = a_B + a_{A/B}$, $a_A = 0$, so $a_{A/B} = -2.08$ m/s²

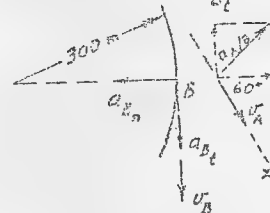


$(a_{A/B})_x = 2.08 \cos 60^\circ$

= 1.042 m/s²

2/139 $a_B = 2.08$ m/s² (From 2/138)

$a_{B_1} = 12/3.6 = 3.33$ m/s²



$a_{A/B} = a_{B_1} - a_B$

$a_{A/B} = 3.33 - 2.08$

$(a_{A/B})_x = 2.08 \cos 60^\circ - 3.33$

= -1.845 m/s²

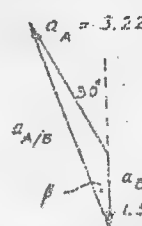
2/140 $a_A = a_B + a_{A/B}$, $a_A = v_A^2/\rho = (150/3.6)^2/600 = 3$ m/s²

$a_{A/B} = \sqrt{(3.22)^2 + (1.5)^2} = 3.56$ m/s²

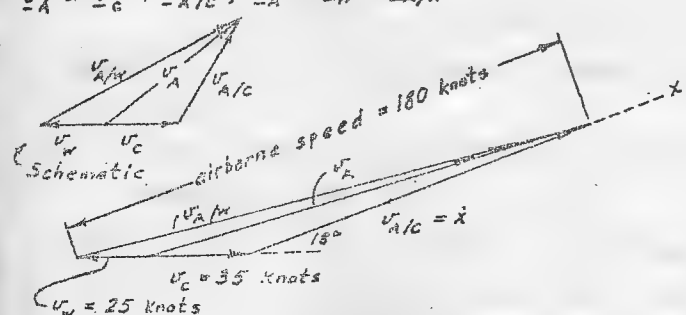
= 4.58 m/s²

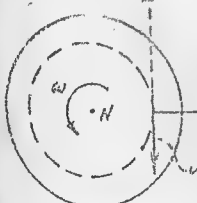
$\beta = \tan^{-1} \frac{1.608}{4.28} = \tan^{-1} 0.376$

= 20.6° west of north

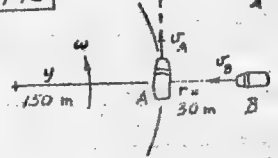


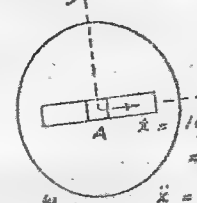
2/141 $\vec{v}_B = \vec{v}_A + (\vec{v}_{B/A})_r + (\vec{v}_{B/A})_\theta$
 $\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = (1500 - 1000)/3.6 = 138.9 \frac{m}{s}$
 $(\vec{v}_{B/A})_r = \vec{v}_{B/A} \cos \theta = \dot{r}$
 so $\dot{r} = 138.9(0.866) = 120.3 \text{ m/s}$
 $(\vec{v}_{B/A})_\theta = -\vec{v}_{B/A} \sin \theta = r\dot{\theta}$
 so $\dot{\theta} = \frac{-138.9(0.5)}{(18-12)(10^3)/0.5} = -5.79 \text{ mrad/s}$
 $\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_r + (\vec{a}_{B/A})_\theta$
 $(\vec{a}_{B/A})_\theta = 0.6 \text{ m/s}^2$
 $(\vec{a}_{B/A})_r = \ddot{r} - r\dot{\theta}^2; -1.039 = \ddot{r} - 12(10^3)(-5.79)^2(10^{-3})^2$
 $\ddot{r} = -0.637 \text{ m/s}^2$
 $(\vec{a}_{B/A})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}; 0.600 = 12(10^3)\ddot{\theta} + 2(120.3)(-5.79)(10^{-3})$
 $\ddot{\theta} = 0.1660 \text{ mrad/s}^2$

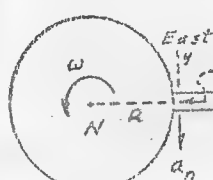
2/142 A = aircraft, C = carrier, W = wind
 $\vec{v}_A = \vec{v}_C + \vec{v}_{A/C}; \vec{v}_A = \vec{v}_W + \vec{v}_{A/W}$

 $v_C = 35 \text{ knots}$
 $v_W = 25 \text{ knots}$
 At takeoff $v_{A/C} = 180 - (25+35) \cos 18^\circ = 122.9 \text{ knots}$
 or $\dot{x} = 122.9(1.852)/3.6 = 63.2 \text{ m/s}$
 $\int_0^x \dot{x} dx = \int_0^x \dot{x} dx, \frac{1}{2} \dot{x}^2 = \dot{x} x \text{ for } \ddot{x} \text{ constant}$
 so $\ddot{x} = \dot{x}^2 / (2x) = \frac{(63.2)^2}{2(150)} = 13.33 \text{ m/s}^2$

2/143 Coriolis component does not depend on latitude for E-W rel. velocity.

 $|2\omega \times \vec{v}_{rel}| = a_c = 2(0.729)(10^{-4})(2500)/3.6$
 $= 0.1012 \text{ m/s}^2$
 away from earth normal to polar axis

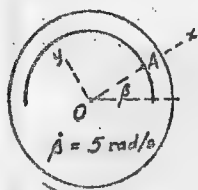
2/144 Eq. 23, $\vec{a}_A = \dot{\omega} \times \vec{r} + \omega \times (\omega \times \vec{r}) + 2\omega \times \vec{v}_{rel} + \vec{a}_{rel}$
 $\dot{\omega} \times \vec{r} = k\ddot{\theta} \times r\hat{i} = r\ddot{\theta}\hat{j}$
 $\omega \times (\omega \times \vec{r}) = k\dot{\theta} \times (k\dot{\theta} \times r\hat{i}) = -r\dot{\theta}^2\hat{i}$
 $2\omega \times \vec{v}_{rel} = 2k\dot{\theta} \times \dot{r}\hat{i} = 2\dot{r}\dot{\theta}\hat{j}$
 $\vec{a}_{rel} = \ddot{r}\hat{i}$
 So $(a_A)_{\text{for } x} = \ddot{r} - r\dot{\theta}^2, (a_A)_{\text{for } y} = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
 which agree with Eq. 18 for $a_r \neq a_\theta$

2/145 $\vec{v}_A = 54/3.6 = 15 \text{ m/s}, \vec{v}_B = \frac{81}{3.6} = 22.5 \text{ m/s}$

 x-y axes have angular velocity of
 $\omega = \frac{v_A}{r} = \frac{15}{150} = 0.10 \text{ rad/s}$
 $\vec{v}_B = \vec{v}_A + \omega \times \vec{r} + \vec{v}_{rel}; \omega \times \vec{r} = 0.10(30)\hat{i} = 3\hat{i} \text{ m/s}$
 $22.5\hat{j} = 15\hat{i} + 3\hat{i} + \vec{v}_{rel}, \vec{v}_{rel} = -18\hat{i} + 22.5\hat{j} \text{ m/s}; \text{ No } \vec{v}_{rel} \text{ differs from } -\vec{v}_{A/B} \text{ by term } \omega \times \vec{r}$

2/146 For A at origin, $\vec{r} = 0, \omega \times \vec{r} = 0, \omega \times (\omega \times \vec{r}) = 0, \dot{\omega} \times \vec{r} = 0$

 $\vec{a}_A = 2\omega \times \vec{v}_{rel} + \vec{a}_{rel}$
 $= 2(12)(100)\hat{j} + 0 \text{ mm/s}^2$
 $\vec{a}_A = 2.40\hat{j} \text{ m/s}^2$
 $\omega = 12 \text{ rad/s}$
 $\dot{\omega} = 15 \text{ rad/s}^2$

2/147 For constant accel. g, $\vec{v}_{rel}^2 = 2gh$
 $v_{rel} = \sqrt{2(9.781)(25)} = 22.1 \text{ m/s}$
 take origin B at particle so $\vec{r} = 0$

 $\vec{a}_A = \vec{a}_B + 2\omega \times \vec{v}_{rel} + \vec{a}_{rel}$
 $\vec{a}_{rel} = -ig + iR\omega^2 - 2\omega \sqrt{2gh}\hat{j}$
 Rel. accel. normal to pipe is
 $a_n = 2\omega \sqrt{2gh} \text{ west} = 2(0.729)(10^{-4})(22.1)$
 $= 3.22 \text{ mm/s}^2 \text{ west}$

2/148 By Eqs. 18; Let $\dot{\theta}$ = absolute angular velocity of OP



$\omega = 10 \text{ rad/s}$
const.

$$\begin{aligned}\dot{\theta} &= \omega + \dot{\beta} = 10 + 5 = 15 \text{ rad/s} \\ \ddot{\theta} &= 0 \\ a_r &= \ddot{r} - r\dot{\theta}^2 = 0 - 0.15(15)^2 = -33.8 \text{ m/s}^2 \\ a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0\end{aligned}$$

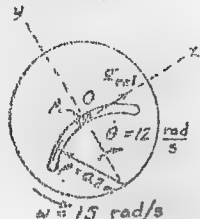
By Eqs. 23; x-y axes attached to disk with x-axis passing through position of A.

$$\begin{aligned}a_A &= \dot{\omega} \times r + \omega \times (\omega \times r) + 2\omega \times v_{rel} + a_{rel} \\ \dot{\omega} \times r &= 0; \omega \times (\omega \times r) = -0.15(10)^2 \hat{i} = -15 \hat{i} \text{ m/s}^2 \\ 2\omega \times v_{rel} &= 2(10 \hat{k}) \times (0.15)(5) \hat{j} = -15 \hat{i} \text{ m/s}^2 \\ a_{rel} &= -0.15 \dot{\beta}^2 \hat{i} = -3.75 \hat{i} \text{ m/s}^2 \\ \text{Thus } a &= a_r = -15 - 15 - 3.75 = -33.8 \text{ m/s}^2\end{aligned}$$

2/149 $a_p = \dot{\omega} \times r + \omega \times (\omega \times r) + 2\omega \times v_{rel} + a_{rel}$

$$\begin{aligned}r &= 100\sqrt{2} \text{ mm} \\ |\dot{\omega} \times r| &= 10(0.1\sqrt{2}) = 1.414 \text{ m/s}^2 \\ |\omega \times (\omega \times r)| &= 0.1\sqrt{2}(5)^2 = 3.54 \text{ m/s}^2 \\ |2\omega \times v_{rel}| &= 2(5)(0.15) = 1.50 \text{ m/s}^2 \\ |a_{rel}| &= \ddot{x} = 0.5 \text{ m/s}^2 \\ a_x &= 1.414/\sqrt{2} - 3.54/\sqrt{2} + 0.50 = -1.00 \text{ m/s}^2 \\ a_y &= 1.414/\sqrt{2} + 3.54/\sqrt{2} - 1.50 = 2.00 \text{ m/s}^2\end{aligned}$$

2/150 $a_A = 2\omega \times v_{rel} + a_{rel}; r = \vec{OA} = 0, a_O = 0$



$$\begin{aligned}v_{rel} &= r\dot{\theta} \hat{i} = 0.2(12) \hat{i} = 2.4 \hat{i} \text{ m/s} \\ 2\omega \times v_{rel} &= 2(15)(2.4) \hat{j} = 72 \hat{j} \text{ m/s}^2 \\ a_{rel} &= -r\dot{\theta}^2 \hat{j} \text{ since } \ddot{\theta} = 0 \\ &= -0.2(12)^2 \hat{j} = -28.8 \hat{j} \text{ m/s}^2 \\ a_A &= 72 \hat{j} - 28.8 \hat{j} = 43.2 \hat{j} \text{ m/s}^2\end{aligned}$$

2/151 x-y axes attached to B have the angular velocity $\omega = 10 \frac{\pi}{180} = 0.1745 \text{ rad/min}$

$$\begin{aligned}v_A &= 12 \text{ knots} \\ r &= 2 \text{ n.mi} \\ v_B &= 10 \text{ knots} \\ v_A &= v_B + \omega \times r + v_{rel} \\ \omega \times r &= 0.1745(60)(2) \hat{j} = 20.94 \hat{j} \text{ knots} \\ v_{rel} &= -12 \hat{j} - 10 \hat{i} - 20.94 \hat{j} = -10 \hat{i} - 32.94 \hat{j} \text{ knots} \\ |v_{rel}| &= \sqrt{10^2 + 32.94^2} = 34.4 \text{ knots}\end{aligned}$$

$$\beta = \tan^{-1} \frac{10}{32.94} = 16.87^\circ$$



2/152 x-y axes attached to B with angular velocity $\omega = 0.1745 \text{ rad/min}$ (Prob. 2/151)

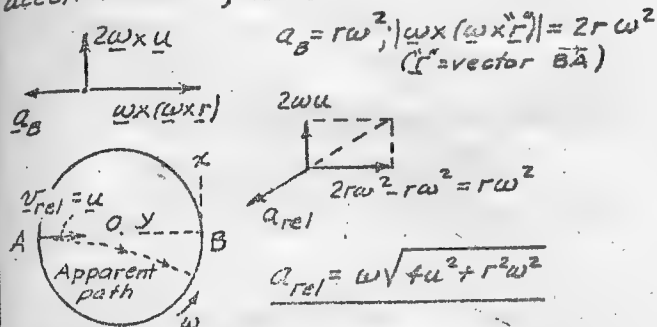
$$\begin{aligned}v_A &= 12 \text{ knots} \\ r &= 2 \text{ n.mi} \\ v_B &= 10 \text{ knots} \\ v_{rel} &= -10 \hat{i} - 32.94 \hat{j} \text{ knots} \\ a_A &= a_B + \dot{\omega} \times r + \omega \times (\omega \times r) + 2\omega \times v_{rel} + a_{rel} \\ a_B &= 0, \dot{\omega} = 0 \\ \omega \times (\omega \times r) &= \omega \hat{k} \times (\omega \hat{k} \times r \hat{i}) = -2(0.1745[60])^2 \hat{i} = -219.2 \hat{i} \text{ n.mi/h}^2 \\ 2\omega \times v_{rel} &= 2\omega \hat{k} \times v_{rel} = 2(0.1745)(60) \hat{k} \times (-10 \hat{i} - 32.94 \hat{j}) \\ &= -209.4 \hat{j} + 689.8 \hat{i} \text{ n.mi/h}^2 \\ a_B &= v_B \omega \hat{j} = 10(0.1745)(60) \hat{j} = 104.7 \hat{j} \text{ n.mi/h}^2 \\ \text{So } a_{rel} &= 0 - 104.7 \hat{j} + 219.2 \hat{i} + 209.4 \hat{j} - 689.8 \hat{i} \\ &= -471 \hat{i} + 105 \hat{j} \text{ n.mi/h}^2\end{aligned}$$

2/153 $v_A = v_B + \omega \times r + v_{rel}; \omega = \frac{v_B}{r} = \frac{50/26}{150} \hat{k}$

$$\begin{aligned}v_A &= 13.89 \text{ m/s} \\ r &= 150 \text{ m} \\ v_B &= 13.89 \text{ m/s} \\ \omega \times r &= 0.0926(150)(-\hat{i}) = -13.89 \hat{i} \text{ m/s} \\ v_{rel} &= -13.89 \hat{i} - 13.89 \hat{j} + 13.89 \hat{i} = -13.89 \hat{j} \text{ m/s} \\ a_A &= a_B + \dot{\omega} \times r + \omega \times (\omega \times r) + 2\omega \times v_{rel} + a_{rel} \\ a_B &= 0; a_{rel} = \frac{v_B^2}{r} = \frac{(13.89)^2}{150} \hat{j} = 1.286 \hat{j} \text{ m/s}^2 \\ \dot{\omega} \times r &= 0; \omega \times (\omega \times r) = -150(0.0926)^2 \hat{j} = -1.286 \hat{j} \text{ m/s}^2 \\ 2\omega \times v_{rel} &= 2(0.0926) \hat{k} \times (-13.89 \hat{j}) = -2.57 \hat{j} \text{ m/s}^2 \\ \text{Thus } a &= 1.286 \hat{j} + 0 - 1.286 \hat{j} - 2.57 \hat{j} + a_{rel} \\ a_{rel} &= 2.57 \hat{j} \text{ m/s}^2; a_{rel} = 2.57 \text{ m/s}^2 \text{ from B to A}\end{aligned}$$

2/154 (Accel. of ball) $_{xy} = \underline{a} = \underline{a}_B + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

Once ball leaves A its horiz. accel. is zero, $a = 0$



(Analysis is simplified slightly by attaching origin of axes to A.)

2/156 Let A be the point (1, 2, 3, or 4) in question

Let reference coordinates be fixed to OM with origin coincident with each point in turn. Thus $\underline{r} = 0$, $\underline{\omega} \times \underline{r} = 0$, $\underline{\omega} \times (\underline{\omega} \times \underline{r}) = 0$. Let P be point fixed to rotating reference frame & coincident with A.

$\underline{a}_P = r\omega^2$; $\underline{a}_P = 1.5(2)^2 \underline{i} = 6 \underline{i} \text{ m/s}^2$

$\underline{a}_{P_1} = (0.9 \underline{i} - 0.6 \underline{j})(2)^2 \text{ m/s}^2$

$\underline{a}_{P_2} = 0.3(2)^2 = 1.2 \underline{i} \text{ m/s}^2$

$\underline{a}_{P_4} = (0.9 \underline{i} + 0.6 \underline{j})(2)^2 \text{ m/s}^2$

$\underline{v}_{rel} = r\dot{\theta} = 0.6(3) = 1.8 \text{ m/s}$

$\underline{a}_c = 2\underline{\omega} \times \underline{v}_{rel}$; $\underline{a}_c = -2(2)(1.8)\underline{i} = -7.2 \underline{i} \text{ m/s}^2$

$\underline{a}_{c_1} = 7.2 \underline{j} \text{ m/s}^2$; $\underline{a}_{c_2} = 7.2 \underline{i} \text{ m/s}^2$; $\underline{a}_{c_4} = -7.2 \underline{j} \text{ m/s}^2$

$\underline{a}_{rel} = r\ddot{\theta}$; $\underline{a}_{rel} = 0.6(3)^2 \underline{i} = 5.4 \underline{i}$; $\underline{a}_{rel_1} = -5.4 \underline{j}$; $\underline{a}_{rel_2} = -5.4 \underline{i}$

$\underline{a}_A = \underline{a}_P + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$ $\underline{a}_{rel_4} = 5.4 \underline{j} \text{ m/s}^2$

$\underline{a}_1 = 4.2 \underline{i} \text{ m/s}^2$; $\underline{a}_2 = 3.6 \underline{i} - 0.6 \underline{j} \text{ m/s}^2$

$\underline{a}_3 = 3.0 \underline{i} \text{ m/s}^2$; $\underline{a}_4 = 3.6 \underline{i} + 0.6 \underline{j} \text{ m/s}^2$

2/155 $x = 50 \sin 4\pi t$, $\dot{x} = 200\pi \cos 4\pi t$, $\ddot{x} = -800\pi^2 \sin 4\pi t$

$\theta = 0.2 \sin 8\pi t$, $\dot{\theta} = 1.6\pi \cos 8\pi t$, $\ddot{\theta} = -12.8\pi^2 \sin 8\pi t$

(a) For $x=0$ and $\dot{x}>0$; $t=0$, $\underline{v}_{rel} = \dot{x} = 200\pi \text{ mm/s}$

$\underline{a}_A = 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$ $\underline{a}_{rel} = \ddot{x} = 0$

$= 2(1.6\pi)(200\pi)\underline{j} + 0$ $\omega = \dot{\theta} = 1.6\pi \text{ rad/s}$

$= 6320\pi \text{ mm/s}^2 = 6.32 \text{ m/s}^2$ $\dot{\omega} = \ddot{\theta} = 0$

(b) For $x = +50 \text{ mm}$, $\sin 4\pi t = 1$, $\cos 4\pi t = 0$, $t = \frac{1}{8} \text{ s}$

$\theta = 0$ $\underline{v}_{rel} = \dot{x} = 0$, $\ddot{x} = -800\pi^2 \text{ mm/s}^2$

$\omega = \dot{\theta} = -1.6\pi \text{ rad/s}$

$\dot{\omega} = \ddot{\theta} = 0$

$\underline{a}_A = \underline{\omega} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{\omega} \times \underline{r} = 0$, $\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -50(1.6\pi)^2 \underline{i} = -128\pi^2 \underline{i} \text{ mm/s}^2$

$2\underline{\omega} \times \underline{v}_{rel} = 0$; $\underline{a}_{rel} = \ddot{x} = -800\pi^2 \underline{i} \text{ mm/s}^2$

$\underline{a}_A = -128\pi^2 \underline{i} - 800\pi^2 \underline{i} \text{ mm/s}^2 = -9.16 \underline{i} \text{ m/s}^2$

2/157 Let P = point on disk coincident with A

$\underline{a}_A = \underline{a}_P + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_P = r\omega^2 = 0.15 \frac{2}{\sqrt{3}} (3)^2 = \frac{2.7}{\sqrt{3}} \text{ m/s}^2$

$x = 0.15 \tan \theta$, $\dot{x} = 0.15 \dot{\theta} \sec^2 \theta$

$\dot{x} = 0.15(2)\left(\frac{2}{\sqrt{3}}\right)^2 = 0.4 \text{ m/s}$

$2\underline{\omega} \times \underline{v}_{rel} = 2\left(\frac{3}{\sqrt{3}}\right) \times (0.4 \underline{i}) = 2.4 \underline{j} \text{ m/s}^2$

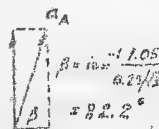
$\underline{a}_{rel} = \ddot{x} = 0.30 \dot{\theta}^2 \sec^2 \theta \tan \theta$, $\ddot{\theta} = 0$

$\ddot{x} = 0.30(2)^2 \left(\frac{2}{\sqrt{3}}\right)^2 \left(\frac{1}{\sqrt{3}}\right) = \frac{1.6}{\sqrt{3}} \text{ m/s}^2$

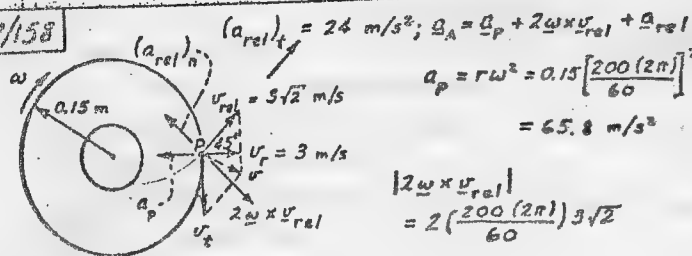
$\underline{a}_A = \frac{2.7}{\sqrt{3}} \left(\frac{-1}{2}\right) + \frac{2.7}{\sqrt{3}} \left(\frac{-1}{2}\right) + 2.4 \underline{j} + \frac{1.6}{\sqrt{3}} \underline{i}$

$= \frac{0.25}{\sqrt{3}} \underline{i} + 1.05 \underline{j} \text{ m/s}^2$

$\underline{a}_A = \sqrt{\frac{0.0625}{3} + 1.05^2} = 1.060 \text{ m/s}^2$



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$$(a_{rel})_n = \frac{v_{rel}^2}{\rho} = \frac{(3\sqrt{2})^2}{0.2} = 90 \text{ m/s}^2$$

Let \underline{n}_1 = unit vector in n-dir., \underline{t}_1 = unit vector in t-dir.

$$\underline{a} = \frac{65.8}{\sqrt{2}} \underline{n}_1 - \frac{65.8}{\sqrt{2}} \underline{t}_1 - 177.7 \underline{n}_1 + 24 \underline{t}_1 + 90 \underline{n}_1$$

$$= -41.2 \underline{n}_1 - 22.5 \underline{t}_1 \text{ m/s}^2$$

$$a = \sqrt{41.2^2 + 22.5^2} = 46.9 \text{ m/s}^2$$

2/159 Refer to Prob. figure & Fig 15

n-components of $a_{A/p}$ are due to $\omega ds + \dot{s}(d\theta + d\tau)$
Divide by dt in the limit & get

$\omega \dot{s} + \dot{s} \dot{\theta} + \dot{s} \dot{\tau}$; But $\dot{s} = v_{rel}$, $\dot{\theta} = \omega$, angular vel of path

$$\dot{\tau} = \dot{s}/\rho$$

So n-components becomes

$$2\omega v_{rel} + v_{rel}^2/\rho \text{ or } 2\omega v_{rel} + (a_{rel})_n$$

t-component of $a_{A/p}$ is due to $d\dot{s}$ & is

$$\ddot{s} = (a_{rel})_t$$

Thus $\underline{a}_A = \underline{a}_p + (\underline{a}_{A/p})_n + (\underline{a}_{A/p})_t$

$$= \underline{a}_p + 2\omega \times v_{rel} + \underline{a}_{rel} \text{ which is Eq. 2.2b}$$

Coriolis accel. comes from

ωds term & $\dot{s} d\theta$ term, both due to rotation of the path

2/162 Vectors are perpendicular if

$$\underline{l}_1 \cdot \underline{l}_2 + m_1 m_2 + n_1 n_2 = 0$$

$\frac{3}{v} \frac{a}{a} + \frac{2}{v} \frac{a}{a} - \frac{9}{v} \frac{a}{a} = 0$ so v & a are perpendicular. In the osculating plane this means that the acceleration is normal to the curve & hence the tangential acceleration is zero.

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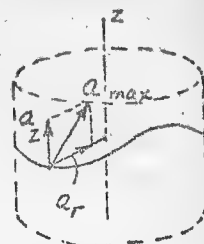
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - r\omega^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$$

$$a_z = \frac{d^2}{dt^2}(z_0 \sin 2\pi n t) = -4n^2 \pi^2 z_0 \sin 2\pi n t$$

$$a = \sqrt{(r\omega^2)^2 + (4n^2 \pi^2 z_0 \sin 2\pi n t)^2}$$

$$a_{max} = \sqrt{r^2 \omega^4 + 16n^4 \pi^4 z_0^2}$$



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$$d\underline{R}_1 = d\phi \underline{\phi}_1 + d\theta \cos \phi \underline{\theta}_1, \underline{\dot{R}}_1 = \dot{\phi} \underline{\phi}_1 + \dot{\theta} \cos \phi \underline{\theta}_1$$

$$d\underline{\theta}_1 = -d\theta \cos \phi \underline{R}_1 + d\theta \sin \phi \underline{\phi}_1, \underline{\dot{\theta}}_1 = -\dot{\theta} \cos \phi \underline{R}_1 + \dot{\theta} \sin \phi \underline{\phi}_1$$

$$d\underline{\phi}_1 = -d\phi \underline{R}_1 - d\theta \sin \phi \underline{\theta}_1, \underline{\dot{\phi}}_1 = -\dot{\phi} \underline{R}_1 - \dot{\theta} \sin \phi \underline{\theta}_1$$

$$2/165 \underline{R} = R \underline{R}_1, \underline{\dot{R}} = \dot{R} \underline{R}_1 + R(\dot{\phi} \underline{\phi}_1 + \dot{\theta} \cos \phi \underline{\theta}_1)$$

$$\underline{\ddot{R}} = \ddot{R} \underline{R}_1 + \dot{R}(\dot{\phi} \underline{\phi}_1 + \dot{\theta} \cos \phi \underline{\theta}_1) + R(\ddot{\phi} \underline{\phi}_1 + \ddot{\theta} \cos \phi \underline{\theta}_1 - \dot{\phi} \dot{\theta} \sin \phi \underline{\theta}_1 + \dot{\theta} \dot{\phi} \cos \phi \underline{R}_1 - \dot{\theta}^2 \sin \phi \underline{\phi}_1 + \dot{\theta} \cos \phi (-\dot{\theta} \cos \phi \underline{R}_1 + \dot{\theta} \sin \phi \underline{\phi}_1))$$

Collecting terms and combining give

$$\underline{\ddot{R}} = (\ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2 \cos^2 \phi) \underline{R}_1 + \left(\frac{\cos \phi}{R} \frac{d}{dt}(R^2 \dot{\theta}) - 2R\dot{\theta} \dot{\phi} \sin \phi \right) \underline{\theta}_1 + \left(\frac{1}{R} \frac{d}{dt}(R^2 \dot{\phi}) + R\dot{\theta}^2 \sin \phi \cos \phi \right) \underline{\phi}_1$$

2/166

Distance along arc = $ds = r d\theta / \cos \theta$

$$v dv = a ds = g \sin \theta \frac{r d\theta}{\cos \theta}, v^2 = 2gr \tan \theta, v = \sqrt{2gh \tan \theta}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - \frac{(r\dot{\theta})^2}{r} \text{ but } r\dot{\theta} = v \cos \theta$$

$$\text{so } a_r = -\frac{v^2 \cos^2 \theta}{r} = -\frac{2gh \cos^2 \theta}{r} = -2g(2.7 \tan \theta) \cos^2 \theta = -27g \sin \theta$$

2/167 $R = 24 \text{ m const.}, \dot{\theta} = \omega = \frac{2(2\pi)}{60} = \frac{\pi}{15} \text{ rad/s}, \ddot{\theta} = 0$

$\beta = 30^\circ, \varphi = \pi/2 - \beta, \dot{\varphi} = -\dot{\beta} = -0.10 \text{ rad/s}, \ddot{\varphi} = -\ddot{\beta} = 0$

$v_R = \dot{R} = 0, v_\theta = R\dot{\theta} \cos \varphi = 24 \left(\frac{\pi}{15}\right) \frac{1}{2} = 2.51 \text{ m/s}$

$v_\varphi = R\dot{\varphi} = 24(-0.10) = -2.4 \text{ m/s}$

$v = \sqrt{2.51^2 + 2.4^2} = 3.48 \text{ m/s}$

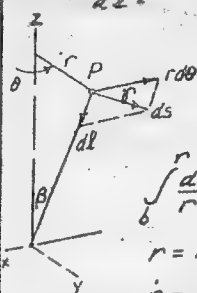
$a_R = \ddot{R} - R\dot{\varphi}^2 - R\dot{\theta}^2 \cos^2 \varphi = 0 - 24(-0.10)^2 - 24\left(\frac{\pi}{15}\right)^2 \left(\frac{1}{2}\right)^2$
 $= -0.503 \text{ m/s}^2$

$a_\theta = \frac{\cos \varphi}{R} \frac{d}{dt}(R^2 \dot{\theta}) - 2R\dot{\theta}\dot{\varphi} \sin \varphi = 0 - 2(24)\left(\frac{\pi}{15}\right)(-0.10)\frac{\sqrt{3}}{2}$
 $= 0.871 \text{ m/s}^2$

$a_\varphi = \frac{1}{R} \frac{d}{dt}(R^2 \dot{\varphi}) + R\dot{\theta}^2 \sin \varphi \cos \varphi = 0 + 24\left(\frac{\pi}{15}\right)^2 \frac{1}{2} \frac{1}{2}$
 $= 0.456 \text{ m/s}^2$

$a = \sqrt{(-0.503)^2 + (0.871)^2 + (0.456)^2} = 1.104 \text{ m/s}^2$

2/170 $ds = \text{differential distance along curve}$
 $dL = \text{ " " " in direction of cone element}$



$r = \frac{b}{h} z, \tan \beta = b/h$

$dL = r d\theta \tan \varphi$

$dr = -dL \sin \beta = -r d\theta \tan \varphi \sin \beta$

$\int \frac{dr}{r} = -\tan \varphi \sin \beta \int d\theta$

$r = be^{-K\theta}$ where $K = \tan \varphi \sin \beta$

$\dot{r} = -bK\dot{\theta}e^{-K\theta}, \ddot{r} = bK^2\dot{\theta}^2e^{-K\theta}, \ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = b\dot{\theta}^2e^{-K\theta}(K^2 - 1)$

or $a_r = b\dot{\theta}^2e^{-\theta \tan \varphi \sin \beta} (\tan^2 \varphi \sin^2 \beta - 1)$

where $\beta = \tan^{-1} \frac{b}{h}$

2/168 $R = 200 + 50 \sin 4\pi t, \dot{R} = 200\pi \cos 4\pi t$

$\ddot{R} = -800\pi^2 \sin 4\pi t$

$\dot{\theta} = \omega = 4\pi \text{ rad/s}, \ddot{\theta} = 0, \varphi = \pi/2 - \beta, \dot{\varphi} = \ddot{\varphi} = 0$

For \dot{R} maximum, $\cos 4\pi t = 1$ & $\sin 4\pi t = 0$

so $a_R = \ddot{R} - R\dot{\varphi}^2 - R\dot{\theta}^2 \cos^2 \varphi$
 $= 0 - 200(0) - 200(4\pi)^2 \left(\frac{1}{2}\right)^2 = -800\pi^2 \text{ mm/s}^2$

$a_\theta = \frac{\cos \varphi}{R} \frac{d}{dt}(R^2 \dot{\theta}) - 2R\dot{\theta}\dot{\varphi} \sin \varphi$
 $= 2\dot{R}\dot{\theta} \cos \varphi - 2R\dot{\theta}\dot{\varphi} \sin \varphi = 2(200\pi)(4\pi)\frac{1}{2} = 800\pi^2 \frac{\text{mm}}{\text{s}^2}$

$a_\varphi = \frac{1}{R} \frac{d}{dt}(R^2 \dot{\varphi}) + R\dot{\theta}^2 \sin \varphi \cos \varphi$
 $= 0 + 200(4\pi)^2 \frac{1}{2} \frac{1}{2} = 800\pi^2 \frac{\text{mm}}{\text{s}^2}$

$a = 800\pi^2 \sqrt{(-1)^2 + 1^2 + 1^2} = 800\pi^2 \sqrt{3} = 17660 \frac{\text{mm}}{\text{s}^2} = 17.66 \frac{\text{m}}{\text{s}^2}$

2/169 For $\ddot{R} = \text{max.}$ & directed toward O,

from solution to Prob. 2/168, $\sin 4\pi t = 1$,
 $\cos 4\pi t = 0$, & $R = 250 \text{ mm}, \dot{R} = 0, \ddot{R} = -800\pi^2 \text{ mm/s}^2$
 and with $\dot{\theta} = 4\pi \text{ rad/s}, \ddot{\theta} = 0, \varphi = 60^\circ, \dot{\varphi} = \ddot{\varphi} = 0$,
 acceleration components from solution to

Prob. 2/168 become

$a_R = -800\pi^2 - 0 - 250(4\pi)^2 \left(\frac{1}{2}\right)^2 = -1800\pi^2 \text{ mm/s}^2$

$a_\theta = 0 - 0 = 0$

$a_\varphi = 0 + 250(4\pi)^2 \frac{1}{2} \frac{1}{2} = 1000\pi^2 \frac{\text{mm}}{\text{s}^2}$

$a = 0.2\pi^2 \sqrt{(-9)^2 + (5\sqrt{3})^2} = 0.2\pi^2 \sqrt{156} = 24.7 \text{ m/s}^2$

2/171 $\theta = \theta_0 \cos pt, \dot{\theta} = -\theta_0 p \sin pt, \ddot{\theta} = -\theta_0 p^2 \cos pt$

$\phi = K, \dot{\phi} = 0, R = b, \dot{R} = \ddot{R} = 0$

From Eqs. 28

$a_R = 0 - bK^2 - b\theta_0^2 p^2 \sin^2 pt \cos^2 \phi$

$a_\theta = b \cos \phi (-\theta_0 p^2 \cos pt) - 2b(-\theta_0 p \sin pt)K \sin \phi$

$a_\phi = 0 + b(\theta_0 p \sin pt)^2 \sin \phi \cos \phi$

At A, $\cos pt = -1, \sin pt = 0$

$a_R = -bK^2, a_\theta = b p^2 \theta_0 \cos \phi, a_\phi = 0$

so $a = b \sqrt{K^2 + p^2 \theta_0^2 \cos^2 \phi}$

At B, $\cos pt = 0, \sin pt = 1, \phi = \pi/2$

$a_R = -bK^2, a_\theta = 2bpK\theta_0, a_\phi = 0$

so $a = bK \sqrt{K^2 + 4p^2 \theta_0^2}$

2/172 Use Eqs. 28 where $\dot{\varphi} = -\dot{\beta}, R = L, \dot{\theta} = \omega$

$a_R = 0 - 1.2 \left(-\frac{3}{2}\right)^2 - 1.2(2)^2 \frac{1}{2} = -5.10 \text{ m/s}^2$

$a_\theta = \frac{\sin \beta}{L} (2LL\omega + 0) + 2L\omega\dot{\beta} \cos \beta = 2\omega(L \sin \beta + L\dot{\beta} \cos \beta)$

$= 2(2) \left(0.9 \frac{1}{\sqrt{2}} + 1.2 \left(\frac{3}{2}\right) \frac{1}{\sqrt{2}}\right) = \frac{10.8}{\sqrt{2}} = 7.64 \text{ m/s}^2$

$a_\varphi = -2L\dot{\beta} + L\omega^2 \cos \beta \sin \beta = -2(0.9) \left(\frac{3}{2}\right) + 1.2(2)^2 \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}}$
 $= -2.7 + 2.4 = -0.3 \text{ m/s}^2$

2/173 Use Eqs. 28 where $\dot{\psi} = -\dot{\theta}$, $\dot{R} = L$,
 $\dot{\theta} = 4\pi(0.12) \cos 4\pi t = 0.48\pi \text{ rad/s}$, $\ddot{\theta} = -1.92\pi^2 \sin 4\pi t = 0$
 $a_R = 0 - 1.2(-\frac{3}{2})^2 - 1.2(0.48\pi)^2(\frac{\sqrt{3}}{2})^2 = -4.75 \text{ m/s}^2$
 $a_\theta = \cos \psi (2\dot{R}\dot{\theta} + R\ddot{\theta}) - 2R\dot{\theta}\dot{\psi} \sin \psi$
 $= \frac{\sqrt{3}}{2} 2(0.9)(0.48\pi) - 2(1.2)(0.48\pi)(-\frac{3}{2})\frac{1}{2} = 5.07 \text{ m/s}^2$
 $a_\psi = 2\dot{R}\dot{\psi} + R\ddot{\psi} + R\dot{\theta}^2 \sin \psi \cos \psi$, $\ddot{\psi} = 0$
 $= 2(0.9)(-\frac{3}{2}) + 0 + 1.2(0.48\pi)^2 \frac{1}{2} \frac{\sqrt{3}}{2} = -1.518 \text{ m/s}^2$
 $a = \sqrt{(-4.75)^2 + (5.07)^2 + (-1.518)^2} = 7.11 \text{ m/s}^2$

2/174 $\{s_{r\theta z}\} = [T_\theta] \{s_{xyz}\}$
 where $s_x = 7-2 = 5 \text{ m}$, $s_y = 4-2 = 2 \text{ m}$, $s_z = 6-3 = 3 \text{ m}$
 $\{s_{r\theta z}\} = \begin{Bmatrix} s_r \\ s_\theta \\ s_z \end{Bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 5 \\ 2 \\ 3 \end{Bmatrix}$ where $\sin \theta = \frac{2}{\sqrt{29}}$, $\cos \theta = \frac{5}{\sqrt{29}}$
 so $s_r = 5 \cos \theta + 2 \sin \theta = \frac{1}{\sqrt{29}}(25+4) = \sqrt{29} \text{ m}$
 $s_\theta = -5 \sin \theta + 2 \cos \theta = \frac{1}{\sqrt{29}}(-10+10) = 0$
 $s_z = 3 \text{ m}$

2/175 $\{v_{xyz}\} = [T_\theta]^{-1} [T_\psi] \{v_{R\theta\psi}\}$
 $= \begin{bmatrix} \cos \psi \cos \theta & -\sin \theta & -\sin \psi \cos \theta \\ \cos \psi \sin \theta & \cos \theta & -\sin \psi \sin \theta \\ \sin \psi & 0 & \cos \psi \end{bmatrix} \begin{Bmatrix} 3 \\ -4 \\ 2 \end{Bmatrix}$
 where $\sin \theta = \sqrt{3}/2$, $\cos \theta = 1/2$, $\sin \psi = 1/2$, $\cos \psi = \sqrt{3}/2$
 so $v_x = \frac{\sqrt{3}}{2} \frac{1}{2} (3) + \frac{\sqrt{3}}{2} (-4) - \frac{1}{2} \frac{1}{2} (2) = -4.26 \text{ m/s}$
 $v_y = \frac{\sqrt{3}}{2} \frac{1}{2} (3) + \frac{1}{2} (-4) - \frac{1}{2} \frac{\sqrt{3}}{2} (2) = -0.616 \text{ m/s}$
 $v_z = \frac{1}{2} (3) + 0 + \frac{\sqrt{3}}{2} (2) = 3.23 \text{ m/s}$

2/176 $\{v_{R\theta\phi}\} = [T_\theta] [T_\phi] \{v_{xyz}\}$
 $\{v_{R\theta\phi}\} = \begin{bmatrix} \cos \phi \cos \theta & \cos \phi \sin \theta & \sin \phi \\ -\sin \theta & \cos \theta & 0 \\ -\sin \phi \cos \theta & -\sin \phi \sin \theta & \cos \phi \end{bmatrix} \begin{Bmatrix} u \\ 0 \\ 0 \end{Bmatrix}$
 $\begin{Bmatrix} v_R \\ v_\theta \\ v_\phi \end{Bmatrix} = \begin{Bmatrix} u \cos \phi \cos \theta \\ -u \sin \theta \\ -u \sin \phi \cos \theta \end{Bmatrix}$ Ans

2/177 $\{v_{R\theta\phi}\} = [T_\theta] [T_\phi] \{v_{xyz}\}$
 $\{v_{R\theta\phi}\} = \begin{bmatrix} \cos \phi \cos \theta & \cos \phi \sin \theta & \sin \phi \\ -\sin \theta & \cos \theta & 0 \\ -\sin \phi \cos \theta & -\sin \phi \sin \theta & \cos \phi \end{bmatrix} \begin{Bmatrix} 0 \\ \dot{y} \\ \dot{z} \end{Bmatrix}$
 $v_R = R\dot{\theta} \cos \phi = 0 + \dot{y} \cos \theta$
 $v_\phi = R\dot{\phi} = -\dot{y} \sin \phi \sin \theta + \dot{z} \cos \phi$
 But $\dot{y} = u \cos \beta$, $\dot{z} = u \sin \beta - gt$
 so $\begin{cases} \dot{\theta} = \frac{\dot{y} \cos \theta}{R \cos \phi} = \frac{u \cos \beta \cos \theta}{R \cos \phi} \\ \dot{\phi} = \frac{-u \cos \beta \sin \phi \sin \theta + (u \sin \beta - gt) \cos \phi}{R} \end{cases}$
 where $R = \sqrt{b^2 + (ut \cos \beta)^2 + (ut \sin \beta - \frac{1}{2}gt^2)^2}$

2/178 $\{c_{xyz}\} = [T_\theta]^{-1} [T_\psi] \{c_{R\theta\psi}\}$
 $= \begin{bmatrix} \cos \psi \cos \theta & -\sin \theta & -\sin \psi \cos \theta \\ \cos \psi \sin \theta & \cos \theta & -\sin \psi \sin \theta \\ \sin \psi & 0 & \cos \psi \end{bmatrix} \{c_{R\theta\psi}\}$
 $\begin{Bmatrix} c_x \\ c_y \\ c_z \end{Bmatrix} = \begin{bmatrix} 1/2 & -1/\sqrt{2} & -1/2 \\ 1/2 & 1/\sqrt{2} & -1/2 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} \begin{Bmatrix} -5.10 \\ 7.64 \\ -0.3 \end{Bmatrix}$ $\begin{cases} \sin \psi = \cos \psi = 1/\sqrt{2} \\ \sin \theta = \cos \theta = 1/\sqrt{2} \end{cases}$
 $c_x = -5.10 \text{ m/s}^2$
 $c_y = 7.64 \text{ m/s}^2$
 $c_z = -0.3 \text{ m/s}^2$
 $a_x = \frac{1}{2}(-5.10) - \frac{1}{\sqrt{2}}(7.64) - \frac{1}{2}(-0.3) = -7.80 \text{ m/s}^2$
 $a_y = \frac{1}{2}(-5.10) + \frac{1}{\sqrt{2}}(7.64) - \frac{1}{2}(-0.3) = 3.00 \text{ m/s}^2$
 $a_z = \frac{1}{\sqrt{2}}(-5.10) + 0 + \frac{1}{\sqrt{2}}(-0.3) = -3.82 \text{ m/s}^2$

2/181 In x-y-z, $\Sigma a/c = \Sigma a_n + \Sigma a_t$, $a_n = r\omega^2$, $a_t = r\dot{\omega}$
 $\omega = 0.4t$, $\dot{\omega} = 0.4 \text{ rad/s}^2$, $\omega_{t=2} = 0.8 \text{ rad/s}$
 For $t = 2 \text{ s}$, $a_n = 0.9(0.8)^2 = 0.576 \text{ m/s}^2$
 $a_t = 0.9(0.4) = 0.360 \text{ m/s}^2$

$\Sigma a/c = \sqrt{0.576^2 + 0.360^2} = 0.675 \text{ m/s}^2$

$\Sigma a/c = \Sigma a_n + \Sigma a_t/c$ $\Sigma a_t/c = 9.81(0.1+0.05[2]) = 1.962 \text{ m/s}^2$

$\Sigma a/c = \sqrt{1.962^2 + 0.675^2} = 2.06 \text{ m/s}^2$

2/182 $\vec{v}_A = \vec{v}_E + \vec{v}_{A/B}$ $\vec{v}_{A/B} = 150\hat{i} - 35\hat{j} \text{ m/s}$
 $= 150\hat{i} - 64\hat{j} \text{ km/h}$

From Eq. 35, $|\vec{R}| = |\vec{v}_{A/B}|_R = (v_{A/B})_R \cos \theta + (v_{A/B})_\theta \cos \phi \sin \theta = 0$

where $\cos \theta = -\frac{6.6}{7.6}$, $\sin \theta = \frac{6.6}{7.6}$, $\cos \phi = \frac{7}{\sqrt{7^2 + 6.6^2}}$

$\dot{R} = 150(0.8575)(-0.6) - 64.8(0.8575)(\frac{7}{10}) = -12.1 \text{ km/h}$

2/183 $\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$; $\underline{a}_A = \underline{a}_B = 0$

$$\underline{a}_{A/B} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$$

But from Prob. 2/182

$$(\underline{v}_{A/B})_x = 150 \text{ km/h}$$

$$(\underline{v}_{A/B})_y = -64.8 \text{ km/h}$$

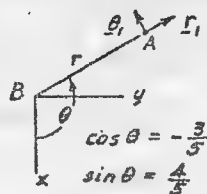
$$\text{so } \underline{v}_r = \dot{r} = (-\frac{3}{5})150 + (\frac{4}{5})(-64.8) = -141.9 \text{ km/h}$$

$$\underline{v}_\theta = r\dot{\theta} = (-\frac{4}{5})150 + (-\frac{3}{5})(-64.8) = -81.1 \text{ km/h}$$

Since $\dot{h} = 0$ and $\underline{a}_{A/B} = 0$, $\ddot{r} - r\dot{\theta}^2 = 0$ & $r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$$\ddot{r} = r\dot{\theta}^2 = (-81.1)^2 / 7.5 = 877 \text{ km/h}^2$$

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} = -\frac{2(-141.9)(-81.1)}{(7.5)^2} = -409 \text{ rad/h}^2$$



2/184 $\underline{v}_\theta = \frac{2\pi a}{T} (-\underline{i}) = -\frac{2\pi (149.6)(10^6)}{365.25(24)} \underline{i} = -107\,200 \underline{i} \text{ km/h}$

Satellite vel. rel. to O is $\underline{v}_r = (R+h)\frac{2\pi}{T} = \frac{6370+480}{1.57} 2\pi = 27\,400 \text{ km/h}$

At A, $\underline{v}_r = 27\,400 \underline{i}$; $\underline{v}_A = (27\,400 - 107\,200) \underline{i} = -79\,800 \underline{i} \text{ km/h}$

At B, $\underline{v}_r = 27\,400 \underline{j}$; $\underline{v}_B = -107\,200 \underline{i} + 27\,400 \underline{j} \text{ km/h}$

At C, $\underline{v}_r = -27\,400 \underline{i}$; $\underline{v}_C = (-27\,400 - 107\,200) \underline{i} = -134\,600 \underline{i} \text{ km/h}$

2/185 Point P has zero accel. if

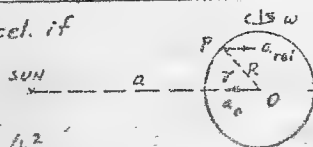
$$|\underline{a}_{rel}| = |\underline{a}_0|$$

$$\underline{a}_0 = \underline{v}_0^2 / a$$

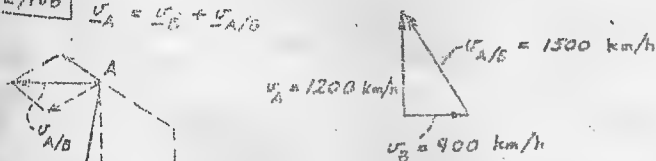
$$= \frac{(107\,200)^2}{149.6(10^6)} = 76.9 \text{ km/h}^2$$

$$\underline{a}_{rel} = R \cos \gamma \omega^2 = 6370 (0.723)^2 (15^\circ)^2 \cos \gamma (3600)^2 = 439 \cos \gamma \text{ km/h}^2$$

$$\cos \gamma = 76.9 / 439 = 0.1752, \gamma = 79.91^\circ$$



2/186 $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$



$$(\underline{v}_{A/B})_R = R = 0 \text{ when } \underline{v}_{A/B} \perp \underline{R}$$

$$\underline{v}_{A/B} = -1200 \underline{i} - 900 \underline{j}$$

$$\underline{R} = R \cos \phi \cos \theta \underline{i} + R \cos \phi \sin \theta \underline{j} + h \underline{k}$$

Vectors are perpendicular when

$$\underline{R} \cdot \underline{v}_{A/B} = 0 \text{ or } \underline{v}_{A/B} \cdot \underline{R} = 0, \text{ so}$$

$$(-1200)R \cos \phi \cos \theta + (-900)h \cos \phi \sin \theta + 0(h) = 0$$

$$\tan \theta = -\frac{1200}{900} = -1.333, \theta = 126.9^\circ$$

irrespective of h.

2/187 In positions A' & B', $\theta = 0, \phi = 0$

$\underline{a}_A = \underline{a}_{A/B}$ since $\underline{a}_B = 0$, where $\underline{a}_A = \underline{u}^2 / r$

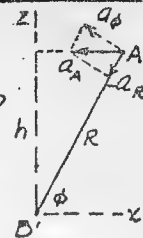
so $-\underline{a}_R = (\underline{u}^2 / r) \cos \phi$, $\underline{a}_\phi = (\underline{u}^2 / r) \sin \phi$, $\underline{a}_\theta = 0$

Thus from Eqs. 2B

$$\underline{a}_R = \ddot{R} - \dot{\theta}^2 R = -\frac{\underline{u}^2 \cos \phi}{r} = -\frac{\underline{u}^2}{\sqrt{h^2 + r^2}}$$

$$\underline{a}_\phi = R\ddot{\theta} \cos \phi + 0 - 0, \ddot{\theta} = 0$$

$$\underline{a}_\theta = R\dot{\phi} + 0 + 0, \dot{\phi} = \frac{\underline{u}^2 h}{rR^2} = \frac{h \underline{u}^2}{r(r^2 + h^2)}$$



2/188 \underline{v}_A = true velocity of airplane; \underline{v}_w = velocity of wind

$\underline{v}_{A/W}$ = velocity of airplane rel. to wind
x-y-z moves with wind

$$\underline{v}_{A/W} = 240 \text{ km/h}, \underline{v}_w = 50 \text{ km/h}$$

$$\underline{v}_{A/W} = \underline{v}_A - \underline{v}_w$$

$$= (0.9985 \underline{v}_A \underline{i} + 0.0555 \underline{v}_A \underline{j}) - 50(0.970)(\underline{i} - \underline{j})$$

$$240 = \sqrt{(0.9985 \underline{v}_A - 48.5)^2 + (35.4 + 0.0555 \underline{v}_A)^2}$$

$$\underline{v}_A^2 - 70.6 \underline{v}_A - 55\,100 = 0, \underline{v}_A = 273 \text{ km/h}$$

$$\text{so } \underline{v}_{A/W} = 237 \underline{i} + 35.4 \underline{j} + 15.13 \underline{k}$$

Transform to spherical coord. (Eq. 35)

$$\begin{Bmatrix} \underline{v}_{A/W} \\ \underline{v}_{A/W} \\ \underline{v}_{A/W} \end{Bmatrix} = [T_\phi][T_\theta] \begin{Bmatrix} \underline{v}_{A/W} \\ \underline{v}_{A/W} \\ \underline{v}_{A/W} \end{Bmatrix} = \begin{Bmatrix} \cos \phi \cos \theta & \cos \phi \sin \theta & \sin \phi \\ -\sin \theta & \cos \theta & 0 \\ -\sin \phi \cos \theta & -\sin \phi \sin \theta & \cos \phi \end{Bmatrix} \begin{Bmatrix} 237 \\ 35.4 \\ 15.13 \end{Bmatrix}$$

The 2nd Eq. is $0 = -237 \sin \theta + 35.4 \cos \theta$, $\theta = 8.49^\circ$

" 3rd " " $0 = -237(0.9890) \sin \phi - 35.4(0.1476) \sin \phi + 15.13 \cos \phi$

$$\tan \phi = \frac{15.13}{240}, \phi = 3.61^\circ$$

2/189 Attach x-y-z to missile whose coordinates are

$$X = 45 (\cos 12^\circ) (\cos 60^\circ) = 22.0 \text{ km}$$

$$Y = 45 (\cos 12^\circ) (\sin 60^\circ) = 38.1 \text{ km}$$

$$Z = 45 \sin 12^\circ = 9.36 \text{ km}$$

$$r = \underline{AB} = \sqrt{(30-22.0)^2 + (75-38.1)^2 + (15-9.36)^2} = 38.2 \text{ km}$$

Dir. cosines of \underline{AB} are

$$\ell = 7.99/38.2 = 0.2094, m = 36.9/38.2 = 0.9666, n = 5.64/38.2 = 0.1479$$

$$\text{Vel. ity of missile} = 3750(0.2094 \underline{i} + 0.9666 \underline{j} + 0.1479 \underline{k}) \text{ km/h}$$

$$= 785 \underline{i} + 3620 \underline{j} + 555 \underline{k} \text{ km/h}$$

\underline{v}_A = velocity of aircraft, $\underline{v}_{A/M}$ = vel. of aircraft rel. to missile

$$\underline{v}_{A/M} = \underline{v}_A - \underline{v}_M = 1200 \underline{j} - 785 \underline{i} - 3620 \underline{j} - 555 \underline{k}$$

$$= -785 \underline{i} - 2420 \underline{j} - 555 \underline{k} \text{ km/h}$$

$$\text{From Eq. 35, } \begin{Bmatrix} \underline{v}_{A/M} \\ \underline{v}_{A/M} \\ \underline{v}_{A/M} \end{Bmatrix} = \begin{Bmatrix} \cos \phi' \cos \theta' & \cos \phi' \sin \theta' & \sin \phi' \\ -\sin \theta' & \cos \theta' & 0 \\ -\sin \phi' \cos \theta' & -\sin \phi' \sin \theta' & \cos \phi' \end{Bmatrix} \begin{Bmatrix} 7.99 \\ 36.9 \\ 5.64 \end{Bmatrix}$$

$$\text{in } x, y, z \begin{Bmatrix} \underline{v}_{A/M} \\ \underline{v}_{A/M} \\ \underline{v}_{A/M} \end{Bmatrix} = \begin{Bmatrix} \cos \phi' \cos \theta' & \cos \phi' \sin \theta' & \sin \phi' \\ -\sin \theta' & \cos \theta' & 0 \\ -\sin \phi' \cos \theta' & -\sin \phi' \sin \theta' & \cos \phi' \end{Bmatrix} \begin{Bmatrix} 7.99 \\ 36.9 \\ 5.64 \end{Bmatrix}$$

This gives $\cos \theta' = 0.2118$, $\sin \theta' = 0.9773$, $\cos \phi' = 0.9890$, $\sin \phi' = 0.1479$

Then from Eq. 35 with $\underline{v}_{A/M}$

$$\begin{Bmatrix} \underline{v}_{A/M} \\ \underline{v}_{A/M} \\ \underline{v}_{A/M} \end{Bmatrix} = \begin{Bmatrix} 0.2094 & 0.9666 & 0.1479 \\ -0.9773 & 0.2118 & 0 \\ -0.0313 & -0.1446 & 0.9890 \end{Bmatrix} \begin{Bmatrix} -785 \\ -2420 \\ -555 \end{Bmatrix} \begin{Bmatrix} \underline{i} \\ \underline{j} \\ \underline{k} \end{Bmatrix} \text{ 1st Eq. gives } \underline{v}_{A/M} = -2590 \text{ km/h}$$

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$$\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$$

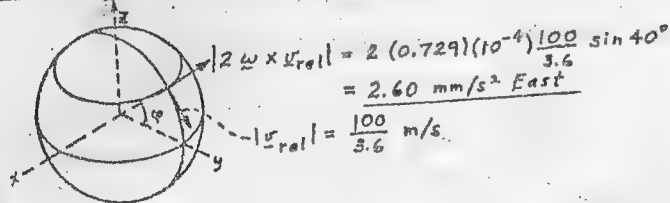
$$\underline{v}_A = -300 \underline{i} \text{ km/h}, \underline{v}_B = 210 \underline{j} \text{ km/h}$$

$$\underline{\omega} \times \underline{r} = \frac{1.0\pi}{180} (3600) \underline{i} \times 9 \underline{j} = 565 \underline{k} \text{ km/h}$$

$$\underline{v}_{rel} = -300 \underline{i} - 210 \underline{j} - 565 \underline{k} \text{ km/h}$$

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$$|\underline{\omega}| = 0.729 (10^{-4}) \text{ rad/s}$$



$$|\underline{\omega} \times \underline{v}_{rel}| = 2 (0.729) (10^{-4}) \frac{100}{3.6} \sin 40^\circ$$

$$= 2.60 \text{ mm/s}^2 \text{ East}$$

$$|\underline{v}_{rel}| = \frac{100}{3.6} \text{ m/s}$$

2/192

With \underline{P} invariant in $x-y-z$, Ex. 39 gives

$$\left(\frac{d\underline{P}}{dt}\right)_{xyz} = \underline{\omega} \times \underline{P} \quad \text{Thus} \quad \left(\frac{d^2\underline{P}}{dt^2}\right)_{xyz} = \dot{\underline{\omega}} \times \underline{P} + \underline{\omega} \times \dot{\underline{P}}$$

$$\text{so} \quad \left(\frac{d^2\underline{P}}{dt^2}\right)_{xyz} = \dot{\underline{\omega}} \times \underline{P} + \underline{\omega} \times (\underline{\omega} \times \underline{P})$$

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$$\left(\frac{d\underline{H}}{dt}\right)_{xyz} = \underline{\omega} \times \underline{H} + \left(\frac{d\underline{H}}{dt}\right)_{xyz}$$

$$\text{where } \underline{H} = 6 \underline{i} + 3 \underline{j} + 5 \underline{k} \text{ kg} \cdot \text{m}^2/\text{s}$$

$$\underline{\omega} = 2t \underline{i} + 3t^2 \underline{j} + 4t^3 \underline{k} \text{ rad/s}$$

$$\text{For } t = 2 \text{ s}, \underline{\omega} = 4 \underline{i} + 12 \underline{j} + 32 \underline{k} \text{ rad/s}$$

$$\left(\frac{d\underline{H}}{dt}\right)_{xyz} = 0 \text{ since } \underline{H} \text{ is fixed in } x-y-z$$

$$\text{so} \quad \left(\frac{d\underline{H}}{dt}\right)_{xyz} = (4 \underline{i} + 12 \underline{j} + 32 \underline{k}) \times (6 \underline{i} + 3 \underline{j} + 5 \underline{k})$$

$$= -36 \underline{i} + 172 \underline{j} - 60 \underline{k} \text{ kg} \cdot \text{m}^2/\text{s}^2$$

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$$\left(\frac{d}{dt}\left[\frac{d\underline{Y}}{dt}\right]_{xyz}\right)_{xyz} = \underline{\omega} \times \left[\underline{\omega} \times \underline{Y} + \left(\frac{d\underline{Y}}{dt}\right)_{xyz}\right] + \left(\frac{d}{dt}\left[\underline{\omega} \times \underline{Y} + \left(\frac{d\underline{Y}}{dt}\right)_{xyz}\right]\right)_{xyz}$$

$$\left(\frac{d^2\underline{Y}}{dt^2}\right)_{xyz} = \underline{\omega} \times (\underline{\omega} \times \underline{Y}) + \underline{\omega} \times \left(\frac{d\underline{Y}}{dt}\right)_{xyz} + \dot{\underline{\omega}} \times \underline{Y} + \underline{\omega} \times \left(\frac{d\underline{Y}}{dt}\right)_{xyz} + \left(\frac{d^2\underline{Y}}{dt^2}\right)_{xyz}$$

$$\text{Note that } \left(\frac{d\underline{\omega}}{dt}\right)_{xyz} = \left(\frac{d\underline{\omega}}{dt}\right)_{xyz} = \dot{\underline{\omega}} \text{ since } \underline{\omega} \times \underline{\omega} = 0$$

$\underline{\omega}$ being absolute & the same in both systems

$$\text{so} \quad \left(\frac{d^2\underline{Y}}{dt^2}\right)_{xyz} = \dot{\underline{\omega}} \times \underline{Y} + \underline{\omega} \times (\underline{\omega} \times \underline{Y}) + 2 \underline{\omega} \times \left(\frac{d\underline{Y}}{dt}\right)_{xyz} + \left(\frac{d^2\underline{Y}}{dt^2}\right)_{xyz}$$

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$$\left(\frac{d\underline{H}}{dt}\right)_{xyz} = \underline{\omega} \times \underline{H}, \quad \underline{H} \text{ invariant in } xyz$$

$$\left(\frac{d^2\underline{H}}{dt^2}\right)_{xyz} = \dot{\underline{\omega}} \times \underline{H} + \underline{\omega} \times \dot{\underline{H}} = \dot{\underline{\omega}} \times \underline{H} + \underline{\omega} \times (\underline{\omega} \times \underline{H})$$

$$\dot{\underline{\omega}} = \frac{d}{dt}(2t \underline{i} + 3t^2 \underline{j} + 4t^3 \underline{k}) = (2 \underline{i} + 6t \underline{j} + 12t^2 \underline{k}) + \underline{\omega} \times \underline{\omega}$$

$$\dot{\underline{\omega}}_{t=2} = 2 \underline{i} + 12 \underline{j} + 48 \underline{k} \text{ rad/s}^2$$

$$\dot{\underline{\omega}} \times \underline{H} = (2 \underline{i} + 12 \underline{j} + 48 \underline{k}) \times (6 \underline{i} + 3 \underline{j} + 5 \underline{k})$$

$$= -84 \underline{i} + 278 \underline{j} - 66 \underline{k} \text{ kg} \cdot \text{m}^2/\text{s}^2$$

$$\underline{\omega} \times \underline{H} = (4 \underline{i} + 12 \underline{j} + 32 \underline{k}) \times (6 \underline{i} + 3 \underline{j} + 5 \underline{k})$$

$$= -36 \underline{i} + 172 \underline{j} - 60 \underline{k} \text{ kg} \cdot \text{m}^2/\text{s}^2$$

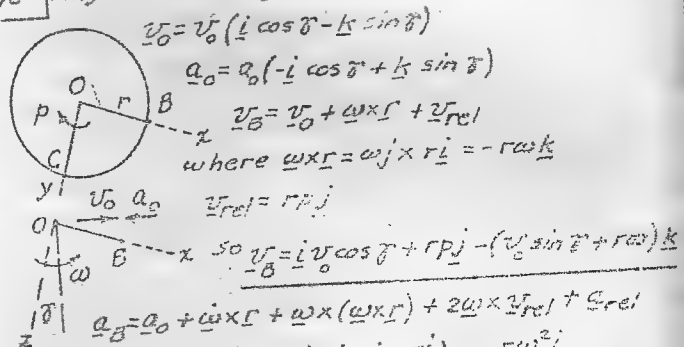
$$\underline{\omega} \times (\underline{\omega} \times \underline{H}) = (4 \underline{i} + 12 \underline{j} + 32 \underline{k}) \times (-36 \underline{i} + 172 \underline{j} - 60 \underline{k})$$

$$= -6224 \underline{i} - 912 \underline{j} + 1120 \underline{k} \text{ kg} \cdot \text{m}^2/\text{s}^2$$

$$\text{Thus} \quad \left(\frac{d^2\underline{H}}{dt^2}\right)_{xyz} = -6308 \underline{i} - 634 \underline{j} + 1034 \underline{k} \text{ kg} \cdot \text{m}^2/\text{s}^2$$

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Angular velocity of axes is $\underline{\omega} = -\dot{\theta} \underline{j} = \dot{\omega} \underline{j}$, $\dot{\omega} = \dot{\omega}$



$$\underline{v}_A = \underline{v}_O + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$$

$$\underline{a}_O = \underline{a}_O(-\underline{i} \cos \theta + \underline{k} \sin \theta)$$

$$\underline{v}_B = \underline{v}_O + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$$

$$\text{where } \underline{\omega} \times \underline{r} = \dot{\omega} \underline{j} \times r \underline{i} = -r \dot{\omega} \underline{k}$$

$$\underline{v}_{rel} = r \dot{\omega} \underline{j}$$

$$\text{so } \underline{v}_B = \underline{i} \dot{\omega} \cos \theta + r \dot{\omega} \underline{j} - (r \dot{\omega} \sin \theta + r \dot{\omega}) \underline{k}$$

$$\underline{a}_B = \underline{a}_O + \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2 \underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$$

$$\dot{\underline{\omega}} \times \underline{r} = 0; \underline{\omega} \times (\underline{\omega} \times \underline{r}) = \dot{\omega} \underline{j} \times (\dot{\omega} \underline{j} \times r \underline{i}) = -r \dot{\omega}^2 \underline{i}$$

$$2 \underline{\omega} \times \underline{v}_{rel} = 2 \dot{\omega} \underline{j} \times r \dot{\omega} \underline{j} = 0$$

$$\underline{a}_{rel} = -r \dot{\omega}^2 \underline{i}$$

$$\text{Thus } \underline{a}_B = -(a_O \cos \theta + r \dot{\omega}^2 + r \dot{\omega}^2) \underline{i} + \underline{k} a_O \sin \theta$$

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Refer to solution for Prob. 2/196 for $\underline{\omega}$, $\dot{\omega}$

For pt. C, $\underline{\omega} \times \underline{r} = \dot{\omega} \underline{j} \times r \underline{j} = 0$, $\underline{v}_{rel} = -r \dot{\omega} \underline{i}$

$$\underline{v}_C = \underline{v}_O + \underline{\omega} \times \underline{r} + \underline{v}_{rel} = (v_O \cos \theta - r \dot{\omega}) \underline{i} - \underline{k} v_O \sin \theta$$

$$\underline{a}_C = \underline{a}_O + \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2 \underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$$

$$\dot{\underline{\omega}} \times \underline{r} = 0, \underline{\omega} \times (\underline{\omega} \times \underline{r}) = 0$$

$$2 \underline{\omega} \times \underline{v}_{rel} = 2 \dot{\omega} \underline{j} \times (-r \dot{\omega}) \underline{i} = 2 r \dot{\omega} \underline{k}$$

$$\underline{a}_{rel} = -r \dot{\omega}^2 \underline{j}$$

$$\underline{a}_C = -\underline{i} a_O \cos \theta - r \dot{\omega}^2 \underline{j} + (a_O \sin \theta + 2 r \dot{\omega} \underline{k}) \underline{k}$$

$$\underline{a}_C = -\underline{i} a_O \cos \theta - r \dot{\omega}^2 \underline{j} + (a_O \sin \theta + 2 r \dot{\omega} \underline{k}) \underline{k}$$

$$\underline{a}_C = -\underline{i} a_O \cos \theta - r \dot{\omega}^2 \underline{j} + (a_O \sin \theta + 2 r \dot{\omega} \underline{k}) \underline{k}$$

$$\underline{a}_C = -\underline{i} a_O \cos \theta - r \dot{\omega}^2 \underline{j} + (a_O \sin \theta + 2 r \dot{\omega} \underline{k}) \underline{k}$$

$$\underline{a}_C = -\underline{i} a_O \cos \theta - r \dot{\omega}^2 \underline{j} + (a_O \sin \theta + 2 r \dot{\omega} \underline{k}) \underline{k}$$

$$\underline{a}_C = -\underline{i} a_O \cos \theta - r \dot{\omega}^2 \underline{j} + (a_O \sin \theta + 2 r \dot{\omega} \underline{k}) \underline{k}$$

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$$\underline{a}_C = -\underline{i} a_O \cos \theta - r \dot{\omega}^2 \underline{j} + (a_O \sin \theta + 2 r \dot{\omega} \underline{k}) \underline{k}$$

$$\underline{a}_C = -\underline{i} a_O \cos \theta - r \dot{\omega}^2 \underline{j} + (a_O \sin \theta + 2 r \dot{\omega} \underline{k}) \underline{k}$$

$$\underline{a}_C = -\underline{i} a_O \cos \theta - r \dot{\omega}^2 \underline{j} + (a_O \sin \theta + 2 r \dot{\omega} \underline{k}) \underline{k}$$

2/198 With origin fixed & $\dot{\omega} = 0$, acceleration of A is $\underline{a}_A = \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$$\underline{\omega} = k\omega; \underline{r} = s(j \cos \beta - k \sin \beta); \underline{\omega} \times \underline{r} = -(s\omega \cos \beta)\underline{i}$$

$$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -(s\omega^2 \cos \beta)\underline{j}; s = s_0 \sin 2\pi nt$$

$$\underline{v}_{rel} = \dot{s}(-k \sin \beta + j \cos \beta)$$

$$= 2\pi n s_0 \cos 2\pi nt (-k \sin \beta + j \cos \beta)$$

$$\underline{a}_{rel} = \ddot{s}(-k \sin \beta + j \cos \beta)$$

$$= -4\pi^2 n^2 s_0 \sin 2\pi nt (-k \sin \beta + j \cos \beta)$$

$$2\underline{\omega} \times \underline{v}_{rel} = -(4\pi n s_0 \omega \cos 2\pi nt \cos \beta)\underline{i}$$

For $s = s_0$ with \dot{s} negative, $\sin 2\pi nt = +1$, $\cos 2\pi nt = 0$

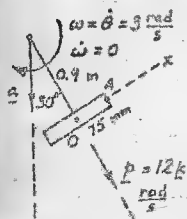
$$\underline{a}_A = -(s_0 \omega^2 \cos \beta)\underline{j} - 4\pi^2 n^2 s_0 (-k \sin \beta + j \cos \beta)$$

$$= s_0 [-j(\omega^2 + 4\pi^2 n^2) \cos \beta + k(4\pi^2 n^2 \sin \beta)]$$

For $s = 0$ & \dot{s} positive, $\sin 2\pi nt = 0$, $\cos 2\pi nt = +1$

$$\underline{a}_A = -(4\pi n \omega s_0 \cos \beta)\underline{i}$$

2/199 $\underline{a}_A = \underline{a}_0 + \underline{\omega} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$



$$\underline{a}_0 = \underline{a} + \underline{a}_0/g$$

$$= 9(-\underline{i} \sin 30^\circ + \underline{k} \cos 30^\circ) + 0.9(3^2)(-\underline{k})$$

$$= -4.5\underline{i} - 0.306 \underline{k} \text{ m/s}^2$$

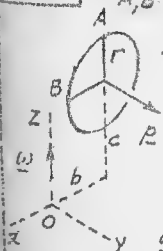
$$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -3\underline{j} \times (-3\underline{j} \times 0.075 \underline{i}) = -0.675 \underline{i} \text{ m/s}^2$$

$$2\underline{\omega} \times \underline{v}_{rel} = 2(-3\underline{j}) \times 0.075(12 \underline{j}) = 0$$

$$\underline{a}_{rel} = 0.075(12^2)(-\underline{i}) = -10.8 \underline{i} \text{ m/s}^2$$

Collect terms & get $\underline{a}_A = -(15.98 \underline{i} + 0.306 \underline{k}) \text{ m/s}^2$

2/200 $\underline{a}_{A,B} = \underline{a}_0 + \underline{\omega} \times \underline{r}_{A,B} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{A,B}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$



$$\underline{a}_0 = 0, \underline{\omega} = 0, \underline{v}_{rel,A} = r\omega \underline{i}, \underline{v}_{rel,B} = -r\omega \underline{k}$$

$$\underline{a}_{rel,A} = -r\omega^2 \underline{k}, \underline{a}_{rel,B} = -r\omega^2 \underline{i}$$

$$\underline{\omega} \times \underline{r}_A = \omega \underline{k} \times (-b \underline{i} + [c+r] \underline{k}) = -b\omega \underline{j}$$

$$\underline{\omega} \times \underline{r}_B = \omega \underline{k} \times (-[b-r] \underline{i} + c \underline{k}) = -(b-r)\omega \underline{j}$$

$$\underline{\omega} \times (\underline{\omega} \times \underline{r}_A) = \omega \underline{k} \times (-b\omega \underline{j}) = b\omega^2 \underline{i}$$

$$\underline{\omega} \times (\underline{\omega} \times \underline{r}_B) = \omega \underline{k} \times (-(b-r)\omega \underline{j}) = (b-r)\omega^2 \underline{i}$$

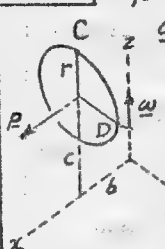
$$2\underline{\omega} \times \underline{v}_{rel,A} = 2\omega \underline{k} \times r\omega \underline{i} = 2rp\omega \underline{j}$$

$$2\underline{\omega} \times \underline{v}_{rel,B} = 2\omega \underline{k} \times (-rp\omega \underline{k}) = 0$$

Collect terms & get

$$\underline{a}_A = b\omega^2 \underline{i} + 2rp\omega \underline{j} - r\omega^2 \underline{k}, \underline{a}_B = [(b-r)\omega^2 - r\omega^2] \underline{i}$$

2/201 $\underline{a}_{C,D} = \underline{a}_0 + \underline{\omega} \times \underline{r}_{C,D} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{C,D}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$



$$\underline{a}_0 = 0, \underline{\omega} = 0, \underline{v}_{rel,C} = -rp \underline{j}, \underline{v}_{rel,D} = rp \underline{k}$$

$$\underline{a}_{rel,C} = -rp^2 \underline{k}, \underline{a}_{rel,D} = -rp^2 \underline{j}$$

$$\underline{\omega} \times \underline{r}_C = \omega \underline{k} \times [b \underline{i} + (c+r) \underline{k}] = b\omega \underline{j}$$

$$\underline{\omega} \times \underline{r}_D = \omega \underline{k} \times [b \underline{i} + r \underline{j} + c \underline{k}] = -r\omega \underline{i} + b\omega \underline{j}$$

$$\underline{\omega} \times (\underline{\omega} \times \underline{r}_C) = \omega \underline{k} \times b\omega \underline{j} = -b\omega^2 \underline{i}$$

$$\underline{\omega} \times (\underline{\omega} \times \underline{r}_D) = \omega \underline{k} \times [-r\omega \underline{i} + b\omega \underline{j}] = -b\omega^2 \underline{i} - r\omega^2 \underline{j}$$

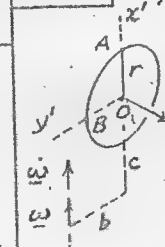
$$2\underline{\omega} \times \underline{v}_{rel,C} = 2\omega \underline{k} \times (-rp \underline{j}) = 2rp\omega \underline{i}$$

$$2\underline{\omega} \times \underline{v}_{rel,D} = 2\omega \underline{k} \times rp \underline{k} = 0$$

Collect terms & get

$$\underline{a}_C = -(b\omega^2 - 2rp\omega) \underline{i} - rp^2 \underline{k}, \underline{a}_D = -b\omega^2 \underline{i} - (r\omega^2 + rp^2) \underline{j}$$

2/202 $\underline{a}_{A,B} = \underline{a}_0 + \underline{\omega} \times \underline{r}_{A,B} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{A,B}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$



$$\underline{a}_0 = b\omega^2 \underline{j} - b\omega \underline{k}$$

$$\underline{\omega} \times \underline{r}_A = \omega \underline{i} \times r \underline{i} = 0, \underline{\omega} \times \underline{r}_B = \omega \underline{i} \times r \underline{j} = r\omega \underline{k}$$

$$\underline{\omega} \times \underline{r}_A = \omega \underline{i} \times r \underline{i} = 0, \underline{\omega} \times \underline{r}_B = \omega \underline{i} \times r \underline{j} = r\omega \underline{k}$$

$$\underline{\omega} \times (\underline{\omega} \times \underline{r}_A) = 0, \underline{\omega} \times (\underline{\omega} \times \underline{r}_B) = \omega \underline{i} \times r\omega \underline{k} = -r\omega^2 \underline{j}$$

$$2\underline{\omega} \times \underline{v}_{rel,A} = 2\omega \underline{i} \times rp \underline{j} = 2rp\omega \underline{k}$$

$$2\underline{\omega} \times \underline{v}_{rel,B} = 2\omega \underline{i} \times (-rp \underline{i}) = 0$$

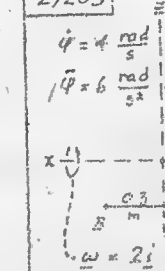
$$\underline{a}_{rel,A} = -rp^2 \underline{i}, \underline{a}_{rel,B} = -rp^2 \underline{j}$$

Collect terms & get

$$\underline{a}_A = -rp^2 \underline{i} + b\omega^2 \underline{j} + (2rp\omega - b\omega) \underline{k}$$

$$\underline{a}_B = (b\omega^2 - r\omega^2 - rp^2) \underline{j} + (r\omega - b\omega) \underline{k}$$

2/203 $\underline{a}_A = \underline{a} + \underline{\omega} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$



$$\underline{a} = -6 \underline{i} \text{ m/s}^2, \underline{\omega} = 0$$

$$\underline{\omega} \times \underline{r} = 2 \underline{i} \times (-1.5 \underline{i} + [2.4 \underline{j} - 0.2] \underline{k})$$

$$= -3.76 \underline{j} \text{ m/s}$$

$$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = 2 \underline{i} \times (-3.76 \underline{j}) = -7.52 \underline{k} \text{ m/s}^2$$

$$\underline{v}_{rel} = 2.4(4) \left(-\underline{i} \frac{\sqrt{3}}{2} - \underline{k} \frac{1}{2} \right) = -9.32 \underline{i} - 4.8 \underline{k} \text{ m/s}$$

$$2\underline{\omega} \times \underline{v}_{rel} = 2(2 \underline{i}) \times (-9.32 \underline{i} - 4.8 \underline{k}) = 19.2 \underline{j} \text{ m/s}^2$$

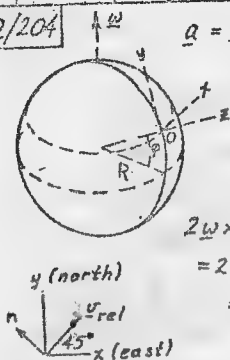
$$\underline{a}_{rel} = 2.4(4^2) \left(\underline{i} \frac{1}{2} - \underline{k} \frac{\sqrt{3}}{2} \right) + 2.4(6) \left(-\underline{i} \frac{\sqrt{3}}{2} - \underline{k} \frac{1}{2} \right)$$

$$= 6.73 \underline{i} - 40.5 \underline{k} \text{ m/s}^2$$

Collect terms & get

$$\underline{a}_A = 0.729 \underline{i} + 19.2 \underline{j} - 48.0 \underline{k} \text{ m/s}^2$$

2/204

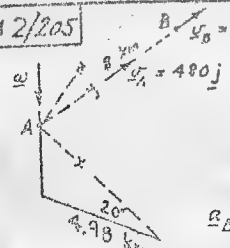


$$\begin{aligned} \underline{a} &= \underline{a}_0 + \underline{\omega} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel} \\ \underline{r} &= 0, \underline{a}_0 = R\omega^2 \cos \varphi (\underline{j} \sin \varphi - \underline{k} \cos \varphi) \\ &= 6370 (10^3) (0.729)^2 (10^{-4}) \cos 30^\circ \\ &\quad (\underline{j} \sin 30^\circ - \underline{k} \cos 30^\circ) \\ &= 0.0293 (0.5 \underline{j} - 0.866 \underline{k}) \text{ m/s}^2 \\ 2\underline{\omega} \times \underline{v}_{rel} &= 2\omega (\underline{j} \cos \varphi + \underline{k} \sin \varphi) \times \underline{v}_{rel} (\underline{i} \cos 45^\circ + \underline{j} \sin 45^\circ) \\ &= 2(0.729)(10^{-4}) (0.866 \underline{j} + 0.5 \underline{k}) \times \frac{600}{\sqrt{2}} (\underline{i} + \underline{j}) \\ &= 0.0617 (-0.5 \underline{i} + 0.5 \underline{j} - 0.866 \underline{k}) \text{ m/s}^2 \\ \text{Collecting } x-y \text{ terms gives} \\ \underline{a}_{xy} &= -0.0309 \underline{i} + 0.0456 \underline{j} \text{ m/s}^2 \end{aligned}$$

$$\underline{a}_n = \frac{0.0309}{\sqrt{2}} + \frac{0.0456}{\sqrt{2}} = 0.0541 \text{ m/s}^2 \text{ (north-west)}$$

Note: \underline{a}_{rel} would have no n-component if it existed due to vertical curvature of rails or due to accel. along rails.

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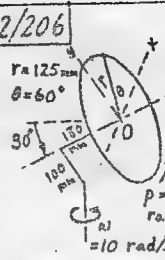
$$\begin{aligned} \underline{v}_B &= \underline{v}_A + \underline{\omega} \times \underline{r} + \underline{v}_{rel} \\ \underline{v}_B &= 640 \underline{j} \\ \underline{v}_A &= 480 \underline{j} \\ \omega &= \frac{480}{4.98} = 96.4 \text{ rad/h} \\ \underline{v}_{rel} &= (640 - 480) \underline{j} - 96.4 (\underline{i} \sin 20^\circ - \underline{k} \cos 20^\circ) \underline{j} \\ &= -725 \underline{i} + 160 \underline{j} - 264 \underline{k} \text{ km/h} \end{aligned}$$

$$\begin{aligned} \underline{a}_B &= \underline{a}_A + \underline{\omega} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel} \\ \underline{a}_B &= 0, \underline{a}_A = \frac{(480/3.6)^2}{4.98 (10^3)} (\underline{i} \cos 20^\circ + \underline{k} \sin 20^\circ) \\ &= 3.57 (0.940 \underline{i} + 0.342 \underline{k}) = 3.35 \underline{i} + 1.22 \underline{k} \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} \underline{\omega} &= 0, \underline{\omega} \times (\underline{\omega} \times \underline{r}) = -r\omega^2 \underline{j} = -8 (10^3) (96.4/3600)^2 \underline{j} = -5.73 \underline{j} \text{ m/s}^2 \\ 2\underline{\omega} \times \underline{v}_{rel} &= \frac{2 \cdot 96.4}{3600} (\underline{i} \sin 20^\circ - \underline{k} \cos 20^\circ) \times (-725 \underline{i} + 160 \underline{j} - 264 \underline{k}) \cdot 3.6 \\ &= 2.24 \underline{i} + 11.46 \underline{j} + 0.814 \underline{k} \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} \underline{a}_{rel} &= 0 - (3.35 \underline{i} + 1.22 \underline{k}) - 5.73 \underline{j} - (2.24 \underline{i} + 11.46 \underline{j} + 0.814 \underline{k}) \\ &= -5.59 \underline{i} - 5.73 \underline{j} - 2.03 \underline{k} \text{ m/s}^2 \end{aligned}$$

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$$\begin{aligned} \underline{v}_P &= \underline{v}_0 + \underline{\omega} \times \underline{r} + \underline{v}_{rel}; \underline{\omega} = 8.66 \underline{j} + 5.0 \underline{k} \text{ rad/s} \\ \underline{v}_0 &= (150 \cos 30^\circ - 100 \sin 30^\circ) 10 \underline{j} \\ &= 799 \underline{j} \text{ mm/s} \\ \underline{\omega} \times \underline{r} &= (8.66 \underline{j} + 5.0 \underline{k}) \times 125 (\underline{i} \cos 60^\circ + \underline{j} \sin 60^\circ) \\ &= -541 \underline{i} + 312 \underline{j} - 541 \underline{k} \text{ mm/s} \\ \underline{v}_{rel} &= 125 (20) (-\underline{i} \sin 60^\circ + \underline{j} \cos 60^\circ) \\ &= -2170 \underline{i} + 1250 \underline{j} \text{ mm/s} \end{aligned}$$

$$\text{So } \underline{v}_P = -1.907 \underline{i} + 1.562 \underline{j} - 0.541 \underline{k} \text{ m/s}$$

$$\begin{aligned} \underline{a}_P &= \underline{a}_0 + \underline{\omega} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}; \underline{\omega} = 0 \\ \underline{a}_0 &= (0.15 \cos 30^\circ - 0.1 \sin 30^\circ) 10^2 (\underline{j} \sin 30^\circ - \underline{k} \cos 30^\circ) = 4.77 \underline{j} - 6.92 \underline{k} \text{ m/s}^2 \\ \underline{\omega} \times (\underline{\omega} \times \underline{r}) &= (8.66 \underline{j} + 5.00 \underline{k}) \times (-541 \underline{i} + 312 \underline{j} - 541 \underline{k}) (10^{-3}) \\ &= -6.25 \underline{i} - 2.71 \underline{j} + 4.69 \underline{k} \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} 2\underline{\omega} \times \underline{v}_{rel} &= 2(8.66 \underline{j} + 5.0 \underline{k}) \times (-2.17 \underline{i} + 1.25 \underline{j}) = -12.5 \underline{i} - 2.17 \underline{j} + 37.5 \underline{k} \text{ m/s}^2 \\ \underline{a}_{rel} &= 0.125 (20)^2 (-\underline{i} \cos 60^\circ - \underline{j} \sin 60^\circ) = -25.0 \underline{i} - 43.3 \underline{j} \text{ m/s}^2 \\ \text{Collect terms \& get} \\ \underline{a}_P &= -43.75 \underline{i} - 63.7 \underline{j} + 35.3 \underline{k} \text{ m/s}^2 \end{aligned}$$

2/207



$$\begin{aligned} \text{Velocity of satellite } \underline{v} &= \underline{v}_A + \underline{\omega} \times \underline{r} + \underline{v}_{rel} \\ \text{Accel. " " } \underline{a} &= \underline{a}_A + \underline{\omega} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel} \\ \underline{\omega} \times \underline{r} &= \underline{\omega} \times h \underline{k} = h\omega \cos \varphi \underline{i}; \underline{v}_A = \underline{v} \cos \varphi \underline{i} \\ \underline{v} &= \underline{v} \underline{j} \text{ so } \underline{v}_{rel} = -(h+R)\omega \cos \varphi \underline{i} \\ h\omega \cos \varphi &= 640 (0.729) (10^{-4}) \frac{\sqrt{2}}{3600} = 145.5 \frac{\text{km}}{\text{h}} \\ R\omega \cos \varphi &= 6370 (0.729) (10^{-4}) \frac{\sqrt{2}}{3600} = 144.6 \frac{\text{km}}{\text{h}} \\ \underline{v}_{rel} &= -1593 \underline{i} + 27100 \underline{j} \text{ km/h} \\ \underline{v}_{rel} &= \sqrt{(1593)^2 + (27100)^2} = 27150 \frac{\text{km}}{\text{h}} \\ \theta &= 360^\circ - 3.36^\circ = 356.64^\circ \end{aligned}$$

$$\begin{aligned} \underline{a} &= (\underline{v}^2/[R+h]) (-\underline{k}); \underline{a}_A = R\omega^2 \cos \varphi (\underline{j} \sin \varphi - \underline{k} \cos \varphi) \\ \underline{\omega} \times (\underline{\omega} \times \underline{r}) &= \underline{\omega} (\underline{j} \cos \varphi + \underline{k} \sin \varphi) \times h\omega \cos \varphi \underline{i} = h\omega^2 \cos \varphi (\underline{j} \sin \varphi - \underline{k} \cos \varphi) \\ 2\underline{\omega} \times \underline{v}_{rel} &= 2\omega (\underline{j} \cos \varphi + \underline{k} \sin \varphi) \times (-(h+R)\omega \cos \varphi \underline{i}) \\ &= 2\omega^2 (-(h+R) \cos \varphi) (\underline{j} \sin \varphi - \underline{k} \cos \varphi) \\ \text{Thus } (\underline{a}_{rel})_{xy} &= \omega \sin \varphi [2\underline{v} \underline{i} + (h+R)\omega \cos \varphi \underline{j}] \\ &= 0.729 (10^{-4}) [2(27100) \underline{i} + (640+6370) (0.729) \underline{j}] = 2600 \underline{i} + 1000 \underline{j} \text{ m/s}^2 \\ &= 0.549 \underline{i} + 0.0161 \underline{j} \text{ m/s}^2 \\ (\underline{a}_{rel})_{xy} &= \sqrt{(0.549)^2 + (0.0161)^2} = 0.549 \text{ m/s}^2, \theta = \tan^{-1} \frac{0.0161}{0.549} = 1.68^\circ \end{aligned}$$

CHAPTER THREE

KINETICS OF PARTICLES

3/6



$$(a) \Sigma F = ma$$

$$150 \text{ kg} : T - 1472 = 150a$$

$$200 \text{ kg} : 1962 - T = 200a$$

$$\text{Solve \& get } a = 1.40 \text{ m/s}^2$$

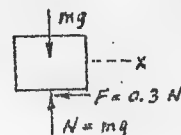
$$(b) \Sigma F = ma$$

$$1962 - 1472 = 150a$$

$$a = 3.27 \text{ m/s}^2$$

3/12

Let $m = \text{mass of crate}$



$$\Sigma F_x = ma_x; -0.3mg = ma_x$$

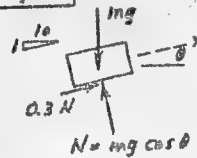
$$a_x = -0.3g = -2.94 \text{ m/s}^2$$

$$\int_0^v v dv = \int_0^s a_x dx, \frac{v^2}{2} = a_x s$$

$$s = \frac{(70/3.6)^2 / 2}{2.94} = 64.3 \text{ m}$$

3/13

$$\sin \theta = 0.0995, \cos \theta = 0.9995$$



$$\Sigma F_x = ma_x; 0.3(mg \cos \theta) - mg \sin \theta = ma_x$$

$$a_x = 9.81(0.2985 - 0.0995)$$

$$= 1.952 \text{ m/s}^2$$

$$\int_0^v v dv = \int_0^s a_x dx, v^2 = 2ax$$

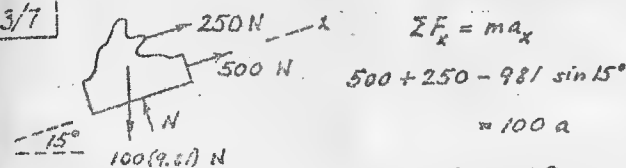
$$= 2(1.952)50$$

$$= 195.2 \text{ (m/s)}^2$$

$$v = 13.97 \text{ m/s}$$

$$= 50.3 \text{ km/h}$$

3/7



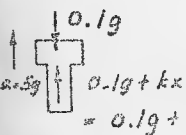
$$\Sigma F_x = ma_x$$

$$500 + 250 - 981 \sin 15^\circ$$

$$= 100 a$$

$$a = 4.96 \text{ m/s}^2$$

3/8



$$\Sigma F = ma; 0.1g + 0.005k - 0.1g$$

$$= 0.1(5g)$$

$$k = \frac{0.5}{0.005} g = 981 \text{ N/m}$$

3/14

$$\Sigma F_x = ma_x; -mg \sin 15^\circ - 0.3(mg \cos 15^\circ) = ma_x$$

$$a_x = -9.81(0.3[0.9659] + 0.2588)$$

$$= -5.38 \text{ m/s}^2$$

$$\int_0^v v dv = -5.38 \int_0^s dx$$

$$\frac{1}{2}(6^2 - 9^2) = -5.38(s - 0), s = 4.16 \text{ m}$$

3/9

$$F = ma; 2.5 = 70(10^3)a, a = 35.7 \mu\text{m/s}^2$$

$$\Delta v = \int a dt = at; t = \frac{(65 - 40)(10^6)}{35.7(10^{-6})(3600)(24)} = 2251 \text{ days}$$

$$= 6.16 \text{ years}$$

$$\Delta s = \int v dt = v_{av} t = \frac{65000 + 40000}{2}(2251)(24)$$

$$= 2.84(10^9) \text{ km}$$

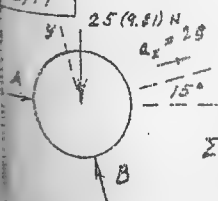
3/10

$$F = ma_y; 16\pi = 3.5 \ddot{y}$$

$$d(\dot{y}) = \frac{16}{3.5} dt, \int_0^{\dot{y}} d(\dot{y}) = \frac{16}{3.5} \int_0^t dt$$

$$\dot{y} + 3 = \frac{8}{3.5} t^2, \Delta y = \int_0^3 (\frac{8}{3.5} t^2 - 3) dt = 11.57 \text{ m}$$

3/11



$$\Sigma F_x = ma_x$$

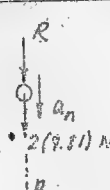
$$A \cos 15^\circ - 25(9.81) \sin 15^\circ = 25(2)(9.81)$$

$$A = \frac{25}{0.9659} (2 + 0.2588) 9.81 = 574 \text{ N}$$

$$\Sigma F_y = 0; B - 25(9.81) \cos 15^\circ - 574 \sin 15^\circ = 0$$

$$B = 385 \text{ N}$$

3/15

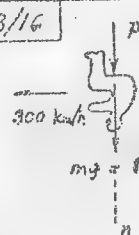


$$\Sigma F_n = ma_n; R + 2(9.81) = 2 \frac{(8.8)^2}{0.5}$$

$$R = 92.2 - 19.62 = 72.5 \text{ N}$$

Down

3/16



$$p = mg/4$$

$$\Sigma F_n = ma_n; 2n = v^2/p$$

$$mg + mg/4 = mv^2/p, p = \frac{4v^2}{5g}$$

$$\text{so } p = \frac{4(300/3.6)^2}{5(9.81)} = 566 \text{ N}$$

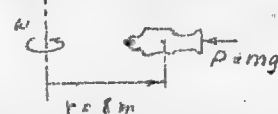
3/17

$g = \text{surface gravitational}$

$$\Sigma F_n = ma_n$$

acceleration on earth

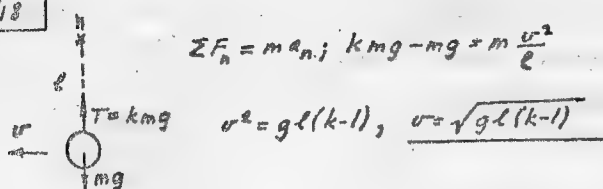
$$mg = mr\omega^2, \omega = \sqrt{\frac{g}{r}}$$



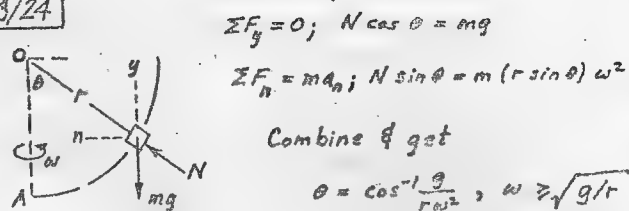
$$\omega = \sqrt{\frac{9.81}{8}} = 1.107 \text{ rad/s}$$

$$\text{or } N = 10.57 \text{ rev/min}$$

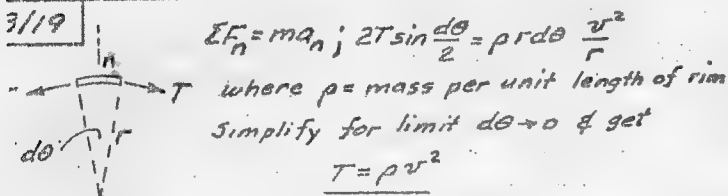
3/18



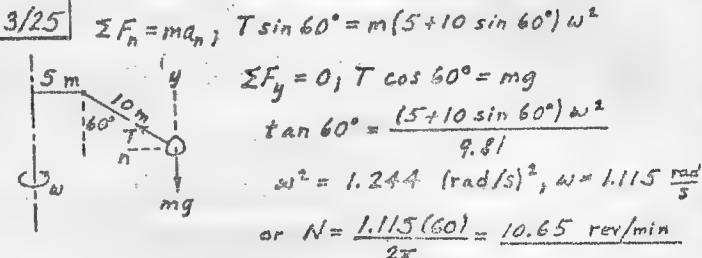
3/24



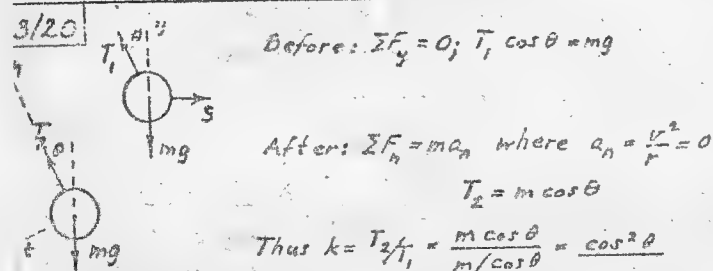
3/19



3/25



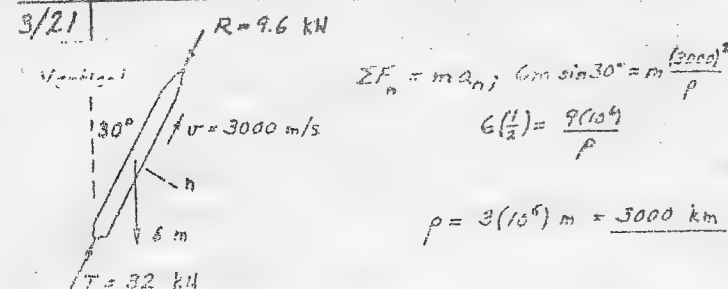
3/20



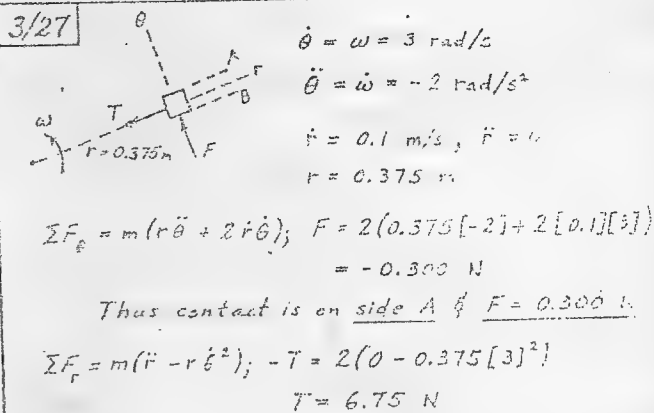
3/26

$r = 50t^3 - 75t + 100 \text{ mm}; \dot{r} = 150t^2 - 75 \text{ mm/s}; \ddot{r} = 300t \frac{\text{mm}}{\text{s}^2}$
 $\theta = 2 \sin \frac{\pi}{2} t; \dot{\theta} = \pi \cos \frac{\pi}{2} t; \ddot{\theta} = -\frac{\pi^2}{2} \sin \frac{\pi}{2} t$
 For $t = 2 \text{ s}$, $r = 350 \text{ mm}$, $\dot{r} = 525 \text{ mm/s}$, $\ddot{r} = 600 \text{ mm/s}^2$
 $\theta = 0, \dot{\theta} = -\pi \text{ rad/s}, \ddot{\theta} = 0$
 $F_r = m(\ddot{r} - r\dot{\theta}^2) = 2(0.6 - 0.35(-\pi)^2) = -5.71 \text{ N}$
 $F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = 2[0.35(0) + 2(0.525)(-\pi)] = -6.60 \text{ N}$
 $F = \sqrt{F_r^2 + F_\theta^2} = \sqrt{(-5.71)^2 + (-6.60)^2} = 8.72 \text{ N}$

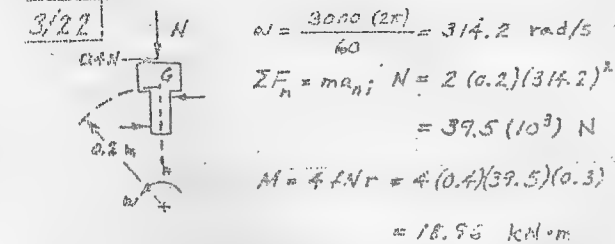
3/21



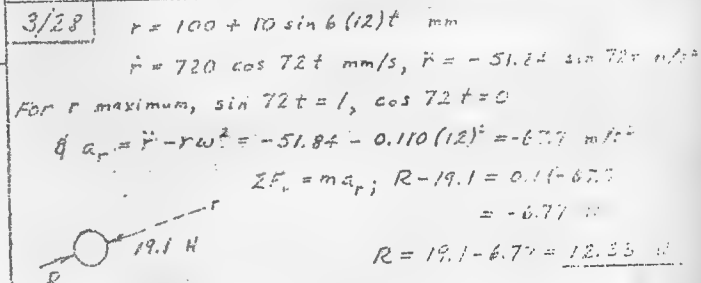
3/27



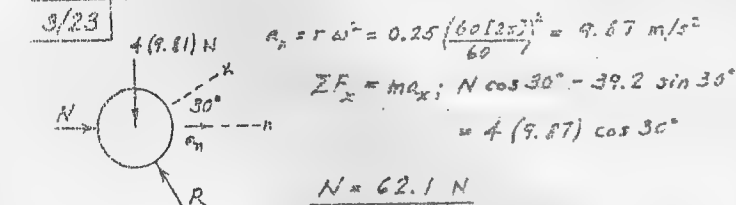
3/22



3/28



3/23



3/29 $v = 26730 \text{ km/h} = 7425 \text{ m/s}$, $F = 519 \text{ N}$

$v_r = \dot{r} = 7425 \sin 30^\circ = 3712 \text{ m/s}$
 $v_\theta = r\dot{\theta} = 7425 \cos 30^\circ = 6430 \text{ m/s}$
 $a_r = \ddot{r} - r\dot{\theta}^2 = \ddot{r} - \frac{(r\dot{\theta})^2}{r} = \ddot{r} - \frac{(6430)^2}{10.729(10^6)}$
 $= \ddot{r} - 3.85 \text{ m/s}^2$

$\Sigma F_r = ma_r$; $519 = 150(\ddot{r} - 3.85)$, $\ddot{r} = 0.39 \text{ m/s}^2$

3/30 From Prob. 2/115, $a_r = -\frac{5}{2}R\dot{\theta}^2 \sin \frac{\theta}{2}$

$\Sigma F_r = ma_r$, $T - mg \sin \theta = m(-\frac{5}{2}R\dot{\theta}^2 \sin \frac{\theta}{2})$
 $T = m(g \sin \theta - \frac{5}{2}R\dot{\theta}^2 \sin \frac{\theta}{2})$

or $T = m \sin \frac{\theta}{2} (2g \cos \frac{\theta}{2} - \frac{5}{2}R\dot{\theta}^2)$

(Note: $\dot{\theta}_{\max} = 2\sqrt{\frac{9 \cos \theta/2}{5R}}$ for T to be positive)

3/31 $\Sigma F_y = ma_y$; $0.3 = 0.2a_y$, $a_y = 1.5 \text{ m/s}^2$

$\Delta v = \int_0^2 a dt$; $v_y - 0 = 1.5(2) = 3.0 \text{ m/s}^2$

No change in v_x or v_z , so final velocity is

$\underline{v} = 1.0\mathbf{i} + 3.0\mathbf{j} + 4.5\mathbf{k}$, $v = \sqrt{1.0^2 + 3.0^2 + 4.5^2}$
 $= 5.5 \text{ m/s}$

3/32 a is constant for constant M , so

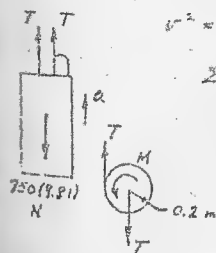
$v^2 = 2ay$; $a = \frac{3^2}{2(4)} = 1.125 \text{ m/s}^2$

$\Sigma F = ma$; $2T - 750(9.81) = 750(1.125)$

$T = 4100 \text{ N}$

$\Sigma M = 0$; $0.2(4100) - M = 0$

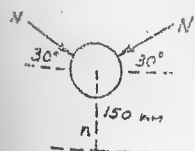
$M = 820 \text{ N}\cdot\text{m}$



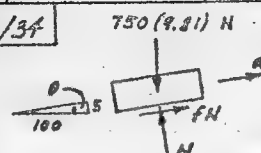
3/33 $\Sigma F_n = ma_n$; $2N \sin 30^\circ = 2.5(0.15)(\frac{600(2\pi)}{60})^2$

$N = 1480 \text{ N}$

$F = 4N \cos 30^\circ = 5130 \text{ N}$



3/34



$15 - 3 = 12 \text{ m}$ crate
 15 m truck

Since $s = \frac{1}{2}at^2$,

$a = \frac{12}{15}a_{\text{truck}}$
 $= 0.8 \frac{v^2}{2s} = 0.8 \frac{(40/3.6)^2}{2(15)} = 3.29 \frac{\text{m}}{\text{s}^2}$

$\cos \theta = 0.9988$

$\sin \theta = 0.0499$

$N = 750(9.81)0.9988$

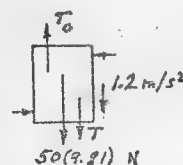
$= 7350 \text{ N}$

$\Sigma F = ma$; $f(7350) - 750(9.81)(0.0499) = 750(3.29)$

$f = 0.386$

3/35

$T_o = T e^{f\theta} = T e^{0.2(\pi + \pi)} = 3.51 T$

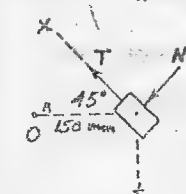


$\Sigma F = ma$; $490 + T - 3.51T = 50(1.2)$

$T = 171.3 \text{ N}$

3/36

$a_n = r\omega^2 = 0.15(300\frac{2\pi}{60})^2 = 148.0 \text{ m/s}^2$



$\Sigma F_x = ma_x$; $T = 3(148.0 \cos 45^\circ)$
 $= 314 \text{ N}$

Direction of rotation does not change accel., hence has no influence on T or N .

3/37

$a_t = r\alpha = 0.15(120) = 18 \text{ m/s}^2$

$\Sigma F_x = ma_x$; $T = 3(18 \cos 45^\circ) = 38.2 \text{ N}$



$\Sigma F_n = ma_n = 0$; $N \frac{1}{\sqrt{2}} = \frac{38.2}{\sqrt{2}} = 0$

$N = 38.2 \text{ N}$

3/38

$a_x = r\omega^2 \cos 45^\circ - r\alpha \sin 45^\circ$

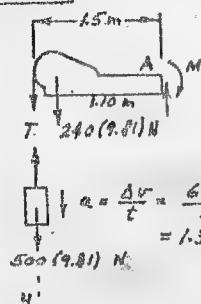
$= 0.1(0.7071)(9^2 - 24) = 4.03 \text{ m/s}^2$



$\Sigma F_x = ma_x$; $F = 10(4.03)$
 $= 40.3 \text{ N}$

contacts slot on right side

3/39



$$\Sigma F_y = ma_y; 4900 - T = 500(1.35)$$

$$T = 4900 - 675 = 4230 \text{ N}$$

$$\text{Beam: } \Sigma M_A = 0;$$

$$M = 240(9.81)(1.10) + 4230(1.5)$$

$$= 8930 \text{ N}\cdot\text{m}$$

$$a = \frac{\Delta v}{t} = \frac{6-0.6}{4}$$

$$= 1.35 \text{ m/s}^2$$

3/40

$$\Sigma F_x = ma_x; mg \sin 30^\circ - fmg \cos 30^\circ = ma_x$$

$$a_x = 9.81(0.5 - f\frac{\sqrt{3}}{2})$$

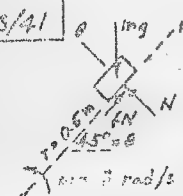


$$\int v dv = \int a_x dx; v_2^2 - v_1^2 = 2a_x(2)$$

$$1^2 - 0.7^2 = 2(9.81)(0.5 - f\frac{\sqrt{3}}{2})2 \text{ from which}$$

$$f = 0.553$$

3/41



$$\Sigma F_\theta = ma_\theta; N - mg \cos \theta = m(0)$$

$$N = mg \cos \theta$$

$$\Sigma F_r = ma_r; fN - mg \sin \theta = m(\theta - r\omega^2)$$

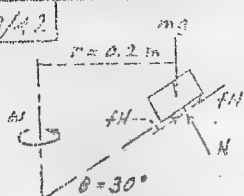
where $\ddot{r} = 0$ up to the point of slipping.

$$\text{Thus } fmg \cos \theta - mg \sin \theta = -mr\omega^2$$

$$f = \tan \theta - \frac{r\omega^2}{g \cos \theta} = 1 - \frac{0.5(3)^2}{9.81(1/\sqrt{2})}$$

$$= 1 - 0.649 = 0.351$$

3/42



fN is down for ω_{max} , up for ω_{min}

$$\Sigma F_y = 0; N \cos \theta \mp fN \sin \theta = mg$$

$$\Sigma F_n = m\omega^2 r; N \sin \theta \pm fN \cos \theta = mr\omega^2$$

upper sign for ω_{max}

lower " " ω_{min}

Combine & get

$$\frac{\sin \theta \pm f \cos \theta}{\cos \theta \mp f \sin \theta} = \frac{r\omega^2}{g}; \omega = \sqrt{\frac{g}{r}} \sqrt{\frac{\sin \theta \pm f \cos \theta}{\cos \theta \mp f \sin \theta}}$$

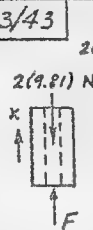
$$\text{So } \omega = \sqrt{\frac{9.81}{0.2}} \sqrt{\frac{0.5 \pm 0.3(0.866)}{0.866 \mp 0.3(0.5)}}$$

$$\text{upper sign } \omega_{max} = 7.21 \text{ rad/s}$$

$$\text{lower sign } \omega_{min} = 3.41 \text{ rad/s}$$

$$\text{or } 32.5 < N < 68.9 \text{ rev/min}$$

3/43



$$F = 262 - 1750x = 1750(0.15 - x), x \text{ in m}$$

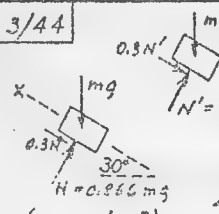
$$\Sigma F_x = ma_x; 1750(0.15 - x) - 19.62 = 2a$$

$$a = 121.4 - 875x \text{ m/s}^2$$

$$\int v dv = \int a dx; \int v dv = \int (121.4 - 875x) dx$$

$$\frac{v^2}{2} = 121.4x - 875\frac{x^2}{2} \Big|_0^{0.15} = 8.37, v = 4.09 \text{ m/s}$$

3/44



mg (just past B)

$$\Sigma F_n = ma_n; 0.866mg - 0.866mg/2$$

$$= m\frac{v_B^2}{2}$$

$$v_B^2 = 0.866g$$

$H = 0.866mg$

(prior to B)

$$\Sigma F_x = ma_x; -0.5mg - 0.3(0.866mg) = ma_x$$

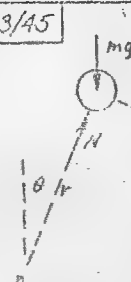
$$a_x = -0.760g$$

$$v_B^2 = u^2 + 2a_x x; u^2 = 0.866g - 2(-0.760g)2\sqrt{3}$$

$$= 6.13(9.81) = 60.1 \text{ (m/s)}^2$$

$$u = 7.75 \text{ m/s}$$

3/45



$$\Sigma F_t = ma_t; mg \sin \theta = m\alpha r; a_t = g \sin \theta$$

$$\omega d\omega = \alpha d\theta; \int \omega d\omega = \int \frac{g \sin \theta}{r} d\theta$$

$$\frac{\omega^2}{2} = \frac{g}{r} (-\cos \theta) \Big|_0^\theta; \omega^2 = \frac{2g}{r} (1 - \cos \theta)$$

$$\Sigma F_n = ma_n; mg \cos \theta - N = mr\omega^2$$

$$\text{so } N = mg \cos \theta - 2mg(1 - \cos \theta) = mg(3 \cos \theta - 2)$$

$$N = 0 \text{ when } 3 \cos \theta - 2 = 0, \theta = \cos^{-1} \frac{2}{3} = 48.19^\circ$$

3/46

$$\underline{r} = \underline{i} \sin 2\pi t - 16 \underline{j} (1 - e^{-t/4}) + \frac{1}{2} \underline{k} t^2$$

$$\underline{\dot{r}} = \underline{i} (2\pi \cos 2\pi t) - \underline{j} (4e^{-t/4}) + \underline{k} t$$

$$\underline{\ddot{r}} = -\underline{i} (4\pi^2 \sin 2\pi t) + \underline{j} e^{-t/4} + \underline{k}$$

$$\Sigma F_x = R_x = m(\ddot{r})_x; R_x = -2(4\pi^2 \sin 2\pi t)$$

$$R_x = -79.0 \sin 2\pi t \text{ N}$$

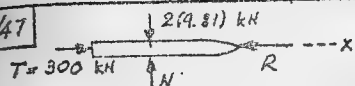
$$t = 4 \text{ s}, \underline{\ddot{r}} = -\underline{i} (0) + \underline{j} e^{-1/4} + \underline{k}$$

$$= 0.368 \underline{j} + 0.5 \underline{k} \text{ m/s}^2$$

$$|\underline{\ddot{r}}| = \sqrt{0.368^2 + 0.5^2} = 0.621 \text{ m/s}^2$$

$$\underline{F} = m \underline{\ddot{r}}; |\underline{F}| = 2(0.621) = 1.242 \text{ N}$$

3/47



$$R = 0.5v \text{ kN}$$

$$\Sigma F_x = ma_x; 300 - 0.5v = 2a$$

$$\text{or } (300 - 0.5v) dx = 2v dv$$

$$\int_0^x dx = 2 \int_0^v \frac{v dv}{300 - 0.5v}$$

$$x = 2 \left(\frac{1}{0.5} \right) \left[300 - 0.5v - 300 \ln(300 - 0.5v) \right]_{0.400}$$

$$= 8.00 \left[-200 - 300 \ln \frac{100}{300} \right]$$

$$= 8.00 [200 + 300 (1.099)]$$

$$= 1037 \text{ m}$$

$$3/48 \quad \Sigma F = ma; 2F \cos \theta = m(-\ddot{x}), \quad \frac{2K}{m} \frac{l-b}{b} \frac{x}{l} = -\ddot{x}$$

$$\text{but } \dot{x} dx = \ddot{x} dt, \text{ so } \frac{1}{2} \int \dot{x}^2 dx = -\frac{2K}{m} \int \left(\frac{l-b}{b} - \frac{1}{2} \right) x dx$$

$$\text{if } l = \sqrt{x^2 + b^2}, \text{ hence}$$

$$\frac{v^2}{2} = \frac{2K}{m} \int_0^x \left(\frac{x}{b} - \frac{x}{\sqrt{x^2 + b^2}} \right) dx = \frac{2K}{m} \left[\frac{x^2}{2b} - \sqrt{x^2 + b^2} + b \right]$$

$$v^2 = \frac{2K}{mb} (\sqrt{b^2 + x^2} - b)^2; \quad v = (\sqrt{b^2 + x^2} - b) \sqrt{\frac{2K}{mb}}$$

3/49

$$\Sigma F_y = ma_y; T - mg = m(-\ddot{y})$$

$$y^2 + b^2 = s^2 \text{ where } s = \overline{CB}$$

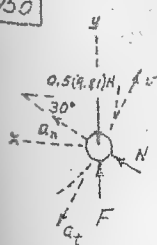
$$y \dot{y} + 0 = s \dot{s}; \text{ but } 2s = \frac{d}{2} \omega$$

$$\text{so } y \dot{y} = s \frac{\omega d}{4}; y \dot{y} + \dot{y}^2 = \left(\frac{\omega d}{4} \right)^2 + 0$$

$$\dot{y} = \frac{1}{y} \left[\left(\frac{\omega d}{4} \right)^2 - \left(\frac{s}{y} \frac{\omega d}{4} \right)^2 \right] = \frac{1}{y} \frac{\omega^2 d^2}{16} \left(1 - \frac{s^2}{y^2} \right)$$

$$\text{Thus } T = mg(1 - \dot{y}^2/g) = mg \left[1 + \frac{\omega^2 b^2 d^2}{16g y^3} \right]$$

3/50



$$v = v_y / \cos 30^\circ = 2/0.866 = 2.31 \text{ m/s}$$

$$a_n = v^2/r = 2.31^2/0.25 = 21.3 \text{ m/s}^2$$

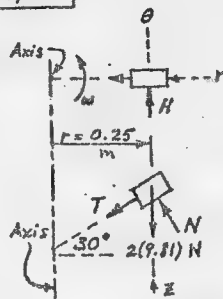
$$a_y = 0 \text{ so } a_t(0.866) = 21.3(0.5)$$

$$a_t = 12.52 \text{ m/s}^2$$

$$\Sigma F_x = ma_x; 0.866N = 0.5(21.3(0.866) + 12.52(0.5))$$

$$N = 14.22 \text{ N}$$

3/51



$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$= 0 - 0.25(3\pi)^2 = -22.2 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$= 0 + 2(0.1)\frac{\sqrt{3}}{2}(3\pi) = 0.3\sqrt{3}\pi \text{ m/s}^2$$

$$a_E = 0; a_N = -a_r \sin 30^\circ$$

$$= 22.2(0.5) = 11.1 \text{ m/s}^2$$

$$\Sigma F_N = ma_N; N - 19.62 \cos 30^\circ = 2(11.1)$$

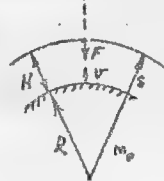
$$N = 39.2 \text{ N}$$

$$\omega = \frac{90(2\pi)}{60} = 3\pi \text{ rad/s}$$

$$\Sigma F_\theta = ma_\theta; H = 2(0.3)\sqrt{3}\pi = 3.26 \text{ N}$$

3/52

$$F = \gamma \frac{mm_0}{s^2} \text{ where } m_0 = \text{mass of earth}$$



$$\text{When } s = R, F = mg, \text{ so } \gamma m_0 = gR^2$$

$$\text{Thus } F = g m \frac{R^2}{s^2}$$

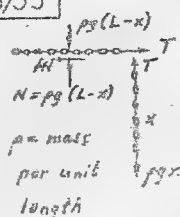
$$\Sigma F_s = ma_s; -g m \frac{R^2}{s^2} = m a_s; a_s = -g R^2/s^2$$

$$v dv = a ds; \int_0^v v dv = -g R^2 \int_{R^2}^{\infty} \frac{ds}{s^2}$$

$$-\frac{v^2}{2} = -g R^2 \left(0 + \frac{1}{R} \right); v = \sqrt{2gR}$$

$$v = \sqrt{2(9.82)(10^{-3})(6370)} = 11.19 \text{ km/s}$$

3/53



Initial value of x for impending slipping given by equil.

$$fpg(L-b) = pgb, b = \frac{f}{1+f} L$$

$$\Sigma F = ma; T - fpg(L-x) = p(L-x)a$$

$$pgx - T = pxa$$

$$\text{Eliminate } T \text{ \& get } a = \frac{g}{L} [x(1+f) - fL]$$

$$v dv = a dx; \int_0^v v dv = \int_{\frac{fL}{1+f}}^L \frac{g}{L} [x(1+f) - fL] dx$$

$$\text{Integrate \& get } v = \sqrt{\frac{gL}{1+f}}$$

3/54

$$\Sigma F_r = ma_r; 0 = m(\ddot{r} - r\dot{\theta}^2)$$

$$\text{sol. is } r = r_0 \cosh \omega t; \dot{r} = r_0 \omega \sinh \omega t$$

$$\Sigma F_\theta = m a_\theta; N = m(\dot{\theta} + 2\dot{r}\dot{\theta}) \sin \omega t$$

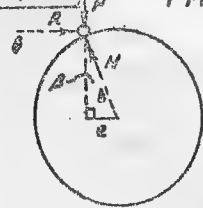
$$= 2m r_0 \omega^2 \sinh \omega t$$

$$\text{But } \cosh^2 \omega t - \sinh^2 \omega t = 1, \sinh \omega t = \sqrt{\cosh^2 \omega t - 1}$$

$$\text{So } N = 2m r_0 \omega^2 \sqrt{r^2 - r_0^2}$$

3/55

From sol. to Prob. 2/121



$$a_r = \frac{2e^2 - b^2}{\sqrt{b^2 - e^2}} \omega^2, a_\theta = -2e\omega^2$$

$$\Sigma F_r = ma_r; N \cos \beta - P = m a_r$$

$$\Sigma F_\theta = ma_\theta; N \sin \beta - R = m a_\theta$$

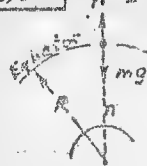
Eliminate N & get

$$-R = m(a_\theta - a_r \tan \beta) - P \tan \beta$$

$$\text{So } R = m \left(+2e + \frac{2e^2 - b^2}{\sqrt{b^2 - e^2}} \frac{e}{\sqrt{b^2 - e^2}} \right) \omega^2 + \frac{Pe}{\sqrt{b^2 - e^2}}$$

$$R = m e \omega^2 \frac{b^2}{b^2 - e^2} + \frac{Pe}{\sqrt{b^2 - e^2}}$$

3/56

 W' = apparent gravity force (force to support) mg = true gravitational attractionwhere g = absolute accel. of gravity

$$\Sigma F_n = ma_n; mg - W' = m R \omega^2, W' = m(g - R\omega^2)$$

$$k = \frac{W'}{mg} = 1 - \frac{R\omega^2}{g}$$

For values given

$$W' = 100 \left(1 - \frac{6378 (10^3) (0.729)^2 (10^{-9})}{9.815} \right)$$

$$= 100 (1 - 0.00345) = 99.655 \text{ N}$$

3/57

$$F = K \frac{m_1 m_2}{L^2} \text{ But } K m_e = g R^2 \text{ so } F = \frac{g R^2 m_1}{L^2}$$

where g = gravitational accel.

at earth's surface

$$\text{For moon, } \Sigma F_n = ma_n;$$

$$\frac{g R^2 m_1}{L^2} = m_1 \frac{v^2}{L}, v^2 = \frac{g R^2}{L}$$

$$\text{Period } T = \frac{2\pi L}{v} = \frac{2\pi L}{\sqrt{\frac{g R^2}{L}}} = \frac{2\pi \sqrt{L^3}}{R \sqrt{g}}$$

$$L = 384,398 (10^3) \text{ m}$$

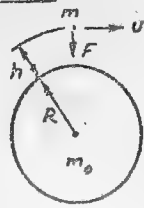
$$R = 6370 (10^3) \text{ m}$$

$$T = \frac{2\pi (384,398)}{6370} \sqrt{\frac{384,398 (10^3)}{9.824}} = 23.72 (10^5) \text{ s}$$

$$\text{or } T = \frac{23.72 (10^5)}{(3600)(23.93)} = 27.5 \text{ days}$$

3/58

$$F = K \frac{m m_0}{(R+h)^2}, \text{ but } K m_0 = g R^2, \text{ so } F = mg \frac{R^2}{(R+h)^2}$$



$$\Sigma F = ma_n; mg \frac{R^2}{(R+h)^2} = m \frac{v^2}{R+h}$$

$$\text{so } v = R \sqrt{\frac{g}{R+h}}$$

$$T = \frac{2\pi (R+h)}{v} = \frac{2\pi (R+h)}{R \sqrt{g/(R+h)}} = 2\pi \frac{(R+h)^{3/2}}{R \sqrt{g}}$$

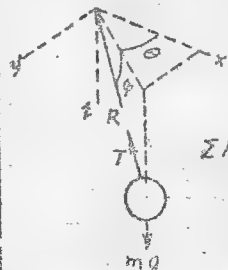
$$\text{For } h = 640 \text{ km, } v = 6370 \sqrt{\frac{9.824 (10^{-3})}{(6370+640)}} (3600)$$

$$= 27,147 \text{ km/h or } v = 7541 \text{ m/s}$$

$$\text{and } T = 2\pi \frac{(6370+640)^{3/2}}{6370 \sqrt{9.824 (10^{-3})}} = 5842 \text{ s or } T = 1 \text{ h } 37 \text{ min } 22 \text{ s}$$

3/59

$$\Sigma F_r = ma_r; mg \sin \varphi - T = -m R (\ddot{\theta}^2 + \dot{\theta}^2 \cos^2 \varphi)$$



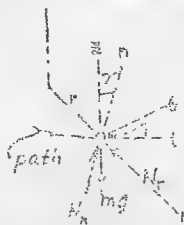
$$\Sigma F_\theta = ma_\theta; 0 = \ddot{\theta} \cos \varphi - 2\dot{\theta} \dot{\varphi} \sin \varphi$$

$$\Sigma F_\varphi = ma_\varphi; mg \cos \varphi = m R (\ddot{\varphi} + \dot{\theta}^2 \sin \varphi \cos \varphi)$$

where a_r, a_θ, a_φ come fromEqs. 2.8 with R constant

3/60

$$\Sigma F_t = ma_t; mg \sin \gamma = ma_t, a_t = g \sin \gamma$$



$$\Sigma F_n = ma_n; N - mg \cos \gamma = ma_n = 0$$

since $a_n = \ddot{r} \cos \gamma + r \ddot{\theta} \sin \gamma$ where

$$r = \frac{1}{2\pi} \frac{2\pi R}{2\pi} \dot{\theta} = R \dot{\theta} \sin \gamma, \ddot{r} = -R \dot{\theta} \dot{\gamma} \sin \gamma$$

$$\Sigma F_n = ma_n; N = m(2\pi \dot{\theta} \sin \gamma)$$

where a_t is obtained as in Prob. 2/16.

$$\text{so } N = mg \sqrt{\cos^2 \gamma + 4\pi^2 \sin^2 \gamma}$$

3/61

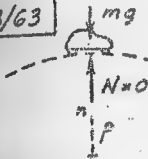
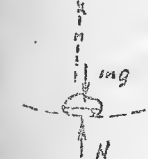
Force on ion is eEx which is always in $x-y$ plane and normal to v .

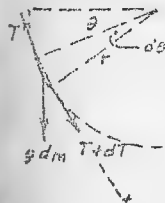
Hence curvilinear motion in $x-y$ plane with tangential acceleration equal to zero. Thus magnitude of v remains constant and is therefore v_0 . Motion must be circular in $x-y$. At A , $\Sigma F = ma_c$, $eEx = m \frac{v_0^2}{r}$ where r is the radius. so $r = \frac{m v_0^2}{eEx}$

Center of circle lies at

$$\left(\frac{m v_0^2}{eEx}, 0, 0 \right)$$

3/62 From $\Sigma F_r = ma_r$, $\ddot{r} - r\omega^2 = g \sin \omega t$, $\theta = \omega t$
 Complementary solution is $r = A \cosh \omega t + B \sinh \omega t$
 For particular solution try $r = K \sin \omega t$ which gives
 $K = -\frac{g}{2\omega^2}$; thus $r = A \cosh \omega t + B \sinh \omega t - \frac{g}{2\omega^2} \sin \omega t$
 When $t=0$, $r=0$ so $0 = A + 0 - 0$, $A = 0$
 " " , $\dot{r}=0$ so $0 = B\omega - \frac{g}{2\omega}$, $B = \frac{g}{2\omega^2}$
 Thus $r = \frac{g}{2\omega^2} (\sinh \theta - \sin \theta)$
 $\Sigma F_\theta = ma_\theta$ gives $N = mg (\cos \theta - \frac{2\omega^2}{g} \dot{r})$
 $N = mg (2 \cos \theta - \cosh \theta)$

3/63  $\Sigma F_n = ma_n$; $mg = m \frac{v^2}{r}$, $v^2 = gr$, $v = \sqrt{gr}$
 At top of hump $dy/dx = 0$
 & radius of curvature is
 $\rho = (d^2y/dx^2)^{-1}$ where $y = b \sin \frac{2\pi x}{L}$
 $\frac{dy}{dx} = \frac{2\pi b}{L} \cos \frac{2\pi x}{L}$, $\frac{d^2y}{dx^2} = -\frac{4\pi^2 b}{L^2} \sin \frac{2\pi x}{L}$
 At top of hump $x = \frac{L}{4}$, $\sin \frac{2\pi x}{L} = 1$
 & $|\frac{d^2y}{dx^2}| = \frac{1}{\rho} = \frac{4\pi^2 b}{L^2}$; thus $v = \frac{L}{2\pi} \sqrt{\frac{g}{b}}$

 At bottom of dip $\Sigma F_n = ma_n$
 $N - mg = m \frac{v^2}{\rho}$, $N = mg(1+1) = 2mg$

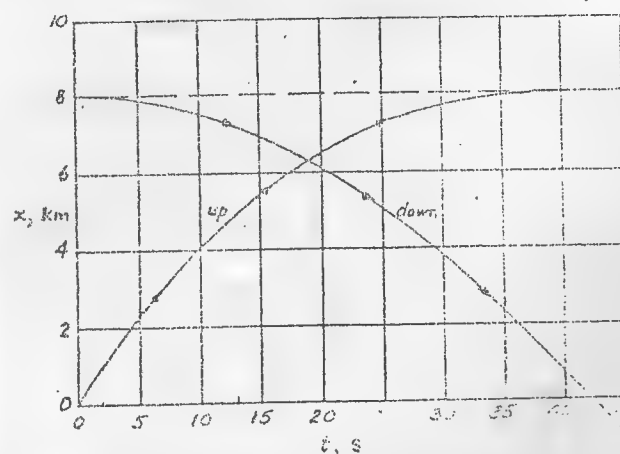
3/64 $g dm = \rho g r d\theta$; $\Sigma F_t = ma_t$

 $(T + dT) \cos \frac{d\theta}{2} - T \cos \frac{d\theta}{2} + \rho g r d\theta (\cos \theta) = \rho r d\theta (a_t)$
 $dT + \rho g r \cos \theta d\theta = \rho r a_t d\theta$
 $dT = \rho r (a_t - g \cos \theta) d\theta$
 $\int dT = \rho r \int_0^\theta (a_t - g \cos \theta) d\theta$
 $T = \rho r (a_t \theta - g \sin \theta)$
 When $\theta = \frac{\pi}{2}$, $T = 0$, so $0 = \rho r (a_t \frac{\pi}{2} - g)$, $a_t = \frac{2g}{\pi}$
 & $T = \rho g r [\frac{2\theta}{\pi} - \sin \theta]$

3/65 $\Sigma F_n = ma_n$; $N - mg \sin \theta = m \frac{v^2}{r}$
 $\Sigma F_t = ma_t$; $mg \cos \theta - fN = m \dot{v}$
 Eliminate N , substitute $v dv = \dot{r} d\theta$ & get
 $\frac{1}{2} \frac{d(v^2)}{d\theta} + f v^2 = gr (\cos \theta - f \sin \theta)$ or let $u = v^2$ so that
 $\frac{du}{d\theta} + 2fu = 2gr (\cos \theta - f \sin \theta)$ which is a linear
 non homogeneous diff. eq. with known solution of
 $u = C e^{-2f\theta} + e^{-2f\theta} \int 2gr (\cos \theta - f \sin \theta) e^{2f\theta} d\theta$
 $= C e^{-2f\theta} + e^{-2f\theta} \frac{2gr e^{2f\theta}}{1+4f^2} (2f \cos \theta + \sin \theta - 2f^2 \sin \theta + f \cos \theta)$
 $u = v^2 = C e^{-2f\theta} + \frac{2gr}{1+4f^2} (3f \cos \theta + [1-2f^2] \sin \theta)$
 When $\theta = 0$, $v = 0$ & $C = -\frac{2gr}{1+4f^2} 3f$
 $v^2 = \frac{2gr}{1+4f^2} (1 - 2f^2 - 3f e^{-f\theta})$; for $f = \frac{1}{5}$, $r = 3m$,
 $v^2 = 30.4 \text{ (m/s)}^2$, $v = 5.52 \text{ m/s}$

3/66 $mg = 4(9.81)$ $\Sigma F_z = ma_z$
 $= 39.2 \text{ N}$ (up) $-(R + 39.2) = 4 \frac{\Delta \dot{x}}{\Delta t}$, $\Delta \dot{x} \approx -\frac{1}{4}(R + 39.2) \Delta t$
 (down) $R - 39.2 = 4 \frac{\Delta \dot{x}}{\Delta t}$, $\Delta \dot{x} \approx \frac{1}{4}(R - 39.2) \Delta t$
 & $\Delta x = \dot{x}_{av} \Delta t$

UP					DOWN				
t	Δt	R+39.2	$\Delta \dot{x}$	\dot{x}	t	Δt	R-39.2	$\Delta \dot{x}$	\dot{x}
0	2	95.4	-4.7	500	0	5	-39.2	-4.9	0
2	2	80.8	-4.0	452	5	5	-38.3	-4.8	-4.9
4	3	71.7	-3.4	412	10	5	-37.3	-4.7	-9.7
7	3	61.5	-2.6	358	15	5	-35.6	-4.5	-14.4
10	3	55.3	-2.2	312	20	5	-33.4	-4.2	-18.6
13	3	51.1	-1.8	270	25	5	-30.8	-3.9	-23.0
16	5	47.9	-1.6	232	30	5	-27.3	-3.4	-26.9
21	5	44.2	-1.5	172	35	5	-24.2	-3.0	-30.8
26	5	41.9	-1.2	117	40	5	-20.3	-2.5	-33.3
31	5	40.4	-1.1	64	45				-35.8
36	1.4	39.2	-1.4	10					
37.4				0					

$x = \Sigma \Delta x = 7990 \text{ m}$



$t = \text{total time } 37.4 + 42.2 \approx 80.5$
 $x = \text{Max. altitude} \approx 8.00 \text{ km}$

13/67

From Prob. 2/86,

$$a_r = (2c \cos \theta - b) \omega^2 = (0.15[0.5] - 0.1)(20)^2 = -10.00 \text{ m/s}^2$$

$$a_\theta = 2c \omega^2 \sin \theta = 0.15(20)^2 \sqrt{3}/2 = 52.0 \text{ m/s}^2$$

$b = 100 \text{ mm}$ $C = k \Delta r = 5.4([100 - 75(\frac{1}{2})] - [100 - 75(1)]) = 202 \text{ N}$
 $c = 75 \text{ mm}$ $\Sigma F_r = m a_r; F_r - 202 = 0.5(-10)$
 $\omega = 20 \text{ rad/s}$ $F_r = -5 + 202 = 197 \text{ N}$
 $\theta = 60^\circ$

Since F is normal to smooth cam surface,

$$F_\theta / F_r = dr / r d\theta; F_\theta = \frac{dr}{r d\theta} F_r$$

Thus $F_\theta = \frac{r}{F_r} \sin \theta (F_r) = \frac{75}{100 - 75(1/2)} \frac{\sqrt{3}}{2} (197) = 205 \text{ N}$

$$\Sigma F_\theta = m a_\theta; P - 205 = 0.5(52.0)$$

$$P = 205 + 26.0 = 231 \text{ N}$$

13/68

$$\Sigma F_x = m \ddot{x}; T \sin \theta = m \ddot{x}, T \sin kt = m \ddot{x}$$

$$\ddot{x} = -\frac{T}{km} \cos kt + C_1$$

$t = 0, \ddot{x} = 0, C_1 = \frac{T}{km}, \ddot{x} = \frac{T}{mk} (1 - \cos kt)$

$$\dot{x} = \frac{T}{mk} (t - \frac{1}{k} \sin kt)$$

$$\Sigma F_y = m \ddot{y}; T \cos \theta - mg = m \ddot{y}$$

$$\ddot{y} = \frac{T}{m} \cos kt - g; \ddot{y} = \frac{T}{mk} \sin kt - g + C_2$$

$u = C_2$ so $\ddot{y} = \frac{T}{mk} \sin kt - g + u$

$$y = -\frac{T}{mk^2} \cos kt - \frac{1}{2} g t^2 + u t + C_3, C_3 = \frac{T}{mk^2}$$

$$y = \frac{T}{mk^2} (1 - \cos kt) - \frac{1}{2} g t^2 + u t$$

13/69

$R = kv$

$$\Sigma F_x = m \ddot{x}; -kv \cos \theta = m \ddot{x}$$

or $\ddot{x} + \frac{k}{m} \dot{x} = 0$

$$\Sigma F_y = m \ddot{y}; -mg - kv \sin \theta = m \ddot{y}$$

or $\ddot{y} + \frac{k}{m} \dot{y} = -g$

Solution of these diff. eqs. gives

$$x = A + B e^{-kt/m}$$

$$y = C + D e^{-kt/m} - \frac{mg}{k} t$$

Boundary $\dot{x} = u \cos \alpha$ & $\dot{y} = u \sin \alpha$ for $t = 0$

Conditions $x = y = 0$ for $t = 0$

give $A = -D = \frac{um}{k} \cos \alpha$

$$C = -D = \frac{um}{k} \sin \alpha + g \frac{m}{k^2}$$

so $x = \frac{mu}{k} \cos \alpha (1 - e^{-kt/m}), x_{\max} = \frac{mu}{k} \cos \alpha$

$$y = \frac{m}{k} \left(u \sin \alpha + \frac{gm}{k} \right) (1 - e^{-kt/m}) - \frac{gm}{k} t$$

13/70

For $R = \text{const.}, \psi = 60^\circ, \dot{\psi} = 0.3 \frac{\text{rad}}{\text{s}}, \dot{\theta} = \omega = 0.2 \frac{\text{rad}}{\text{s}}$

$$a_r = -R(\dot{\psi}^2 + \dot{\theta}^2 \cos^2 \psi)$$

$$= -15(0.3^2 + 0.2^2 [\frac{1}{2}]^2) = -1.5 \text{ m/s}^2$$

$$a_\theta = R \cos \psi \ddot{\theta} - 2R \dot{\psi} \dot{\theta} \sin \psi$$

$$= 0 - 2(15)(0.3)(0.2) \frac{\sqrt{3}}{2} = -0.9\sqrt{3} \text{ m/s}^2$$

$$a_\psi = R \ddot{\psi} + R \dot{\theta}^2 \sin \psi \cos \psi$$

$$= 0 + 15(0.2)^2 \frac{\sqrt{3}}{2} \frac{1}{2} = 0.15\sqrt{3} \text{ m/s}^2$$

$$\Sigma F_r = m a_r; F_r - 294 \frac{\sqrt{3}}{2} = 30(-1.5), F_r = 209.9 \text{ N}$$

$$\Sigma F_\theta = m a_\theta; F_\theta = 30(-0.9\sqrt{3}) = -46.8 \text{ N}$$

$$\Sigma F_\psi = m a_\psi; F_\psi - 294 \frac{1}{2} = 30(0.15\sqrt{3}), F_\psi = 154.8 \text{ N}$$

$$F = \sqrt{209.9^2 + 46.8^2 + 154.8^2} = \sqrt{70211} = 265 \text{ N}$$

13/71

Use spherical coordinates with

$$\omega = \dot{\theta}, \rho = 90^\circ - \psi, \beta = 60^\circ, R = L = 0.9 \text{ m}$$

From Eq. 26

$$a_\psi = 2R \dot{\psi} \dot{\theta} \sin \psi \cos \psi$$

$$= -2L \dot{\beta} + L \omega^2 \sin \beta \cos \beta$$

$$= -2(0.6)(45 \frac{\pi}{180}) + 0.9(60 \frac{2\pi}{60})^2 \frac{1}{2} \frac{1}{2}$$

$$= 14.44 \text{ m/s}^2$$

$$a_\rho = 2R \dot{\theta} \cos \psi - 2R \dot{\psi} \sin \psi = 2u(L \sin \psi + L \dot{\beta} \cos \psi)$$

$$= 2(2\pi)(0.6 \frac{\pi}{180}) + 0.9 \frac{45\pi}{180} \frac{1}{2} = 10.97 \text{ m/s}^2$$

$$\Sigma F_\psi = m a_\psi; -39.2 \frac{\sqrt{3}}{2} + Q_\psi = 4(14.44), Q_\psi = 96.8 \text{ N}$$

$$\Sigma F_\rho = m a_\rho; Q_\rho = 4(10.97) = 43.9 \text{ N}$$

Total shear $Q = \sqrt{96.8^2 + 43.9^2} = 107 \text{ N}$

In space, omit gravity force, so

$$Q = \sqrt{57.8^2 + 43.9^2} = 72.5 \text{ N}$$

13/72

$x = \text{Spring deformation, m}$

$$g' = 1.62 \text{ m/s}^2 \text{ (moon gravity)}$$

$$\Sigma F_x = m \ddot{x}; mg' - 3kx = m \ddot{x}, \ddot{x} + 5.5 \frac{x}{\text{m}} = g'$$

Sol. is $x = A \sin pt + B \cos pt + \frac{g'}{5.5}$, $p = \frac{3\pi}{\text{m}}$

$$\ddot{x} = A p^2 \cos pt - B p^2 \sin pt$$

When $x = 0$, let $t = 0; 0 = 0 + B - \frac{g'}{5.5}, B = \frac{g'}{5.5} = -\frac{1}{3}$

$x = 0, \dot{x} = v_0 = 1.5 \text{ m/s}; v_0 = A p, A = v_0 / p$

"Jerk" $J = \ddot{\ddot{x}} = -\frac{3\pi}{\text{m}} \ddot{x} = -p^2 (v_0 \cos pt + \frac{g'}{5.5} \sin pt)$

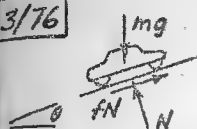
For max, $\frac{dJ}{dt} = -p^2 (-v_0 p \sin pt + g' \cos pt) = 0$

so $\tan pt = \frac{v_0 p}{g'}$, $\sin pt = \frac{v_0 p}{\sqrt{v_0^2 p^2 + g'^2}} = \frac{1.5 \pi}{\sqrt{1.5^2 \pi^2 + 1.62^2}}$

$$J = -\frac{p}{\sqrt{v_0^2 p^2 + g'^2}} (v_0 p^2 + g') = -p \sqrt{v_0^2 p^2 + g'^2}$$

so $|J| = \frac{3(1.5\pi)}{\sqrt{1.5^2 \pi^2 + 1.62^2}} (1.5 \pi + 1.62) = 17.6 \text{ m/s}^3$

3/76



$$\tan \theta = \frac{1}{10}, \cos \theta = 0.995, \sin \theta = 0.0995$$

$$U = \Delta T; (mg \sin \theta - f) s = -\frac{1}{2} m v^2$$

$$(0.976 - 9.76 f) 15 = -\frac{1}{2} (50/3.6)^2$$

$$f = 0.759$$

3/77

$$\Delta T + \Delta V_g = 0; -\frac{1}{2} m (1)^2 + m (9.81) (0.5) (1 - \cos \theta) = 0$$

$$\cos \theta = 0.8981, \theta = 26.1^\circ$$

3/78

$$U = \Delta T; U = \int (\underline{F} + m\mathbf{g}) \cdot d\mathbf{r}$$

$$= \int (F_x dx + F_y dy + [F_z - mg] dz)$$

$$= \int (-15 dx + 10 dy + [15 - 2(9.81)] dz) = -15(-0.4) + 10(0.8) - 4.62(-0.5) = 19.31 \text{ J}$$

$$\Delta T = \frac{1}{2} 2 (v_B^2 - 0) = v_B^2$$

$$19.31 = v_B^2, v_B = 4.39 \text{ m/s}$$

3/79

$$\Delta T + \Delta V_g = 0; \frac{1}{2} m (v_C^2 - 0) - mg (0.6 - 0.9/2) = 0$$

$$v_C^2 = 2.94 \text{ (m/s)}^2, v_C = 1.716 \text{ m/s}$$

3/80

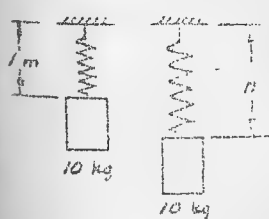
$$U = \Delta T + \Delta V_g; 250(\sqrt{2.7^2 + 1.8^2} - 0.6) = \frac{1}{2} 10 v^2$$

$$+ 10(9.81) 2.4$$

$$250(3.0 - 0.6) = 5v^2 + 255, v^2 = 72.9 \text{ (m/s)}^2, v = 8.54 \text{ m/s}$$

3/81

$$\Delta T = 0 \text{ so } \Delta V_g + \Delta V_s = 0$$



$$-10(9.81)h - 10(9.81)(-1) + \frac{1}{2} 450(h-1)^2 = 0$$

$$22.5h^2 - 54.8h + 32.3 = 0$$

$$h = \frac{54.8 \pm \sqrt{(54.8)^2 - 4(22.5)(32.3)}}{2(22.5)}$$

$$= 1.436 \text{ m or } 1 \text{ m}$$

3/82

$$\Delta T + \Delta V_g = 0; \text{ When } y = 0, z = 0$$

$$\frac{1}{2} m v_A^2 + mg (0 - \frac{0.8}{\sqrt{2}}) = 0, v_A^2 = 2(9.81) \frac{0.8}{\sqrt{2}}$$

$$v_A = 3.53 \text{ m/s}$$

3/83

$$\text{Power } P = \dot{V}_g; 0.70 P = 70(9.81)(6)(100)/60$$

$$= 6867 \text{ W}$$

$$P = 9.81 \text{ kW}$$

3/84

$$P = \underline{F} \cdot \underline{\dot{r}} = (60\mathbf{i} - 25\mathbf{j} - 40\mathbf{k}) \cdot (1.2\mathbf{i} + 1.8\mathbf{j} - 1.8\mathbf{k})$$

which for $t = 4 \text{ s}$ becomes

$$P = (60\mathbf{i} - 25\mathbf{j} - 40\mathbf{k}) \cdot (1.2\mathbf{i} + 7.2\mathbf{j} - 28.8\mathbf{k})$$

$$= 72 - 180 + 1152 = 1044 \text{ W}$$

$$\text{or } P = 1.044 \text{ kW}$$

3/85



$$\sum F_y = 0; N - mg \cos \theta = 0$$

$$U = \Delta T; (mg \sin \theta - 0.3 mg \cos \theta) \frac{1.8}{\sin \theta} = \frac{1}{2} m (0.15^2 - 0.45^2)$$

$$v_1 = 0.45 \text{ m/s}$$

$$v_2 = 0.15 \text{ m/s}$$

$$1.8(9.81)(1 - \frac{0.3}{\tan \theta}) = \frac{-0.180}{2}$$

$$\tan \theta = 0.2985, \theta = 16.62^\circ$$

3/86

$$\text{For } x = 75 \text{ mm, } U = \Delta T$$

$$\frac{1}{2} 0.075 R_m = \frac{1}{2} 0.25 (600)^2, R_m = 1.2 \text{ MN}$$

$$\text{For } x = 25 \text{ mm, } R = \frac{1}{3} 1.2 (10^6) = 0.4 (10^6) \text{ N}$$

$$U = \Delta T; \frac{1}{2} (0.025) 0.4 (10^6) = \frac{1}{2} 0.25 (600^2 - v^2)$$

$$v^2 = 320 000 \text{ (m/s)}^2, v = 566 \text{ m/s}$$

3/87

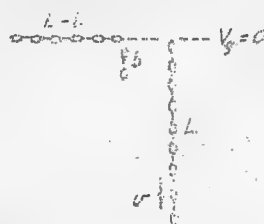
$$\text{With no friction, } \Delta V_g + \Delta T = 0,$$

$$-mgh + \frac{1}{2} m v_B^2 = 0, v_B = v_B' = \sqrt{2gh}$$

With some friction, the friction force will be greater in case (a) than in case (b), because the normal reaction between bead and wire is greater in (a) than in (b). Thus more negative work will be done & kinetic energy will be less at B than at B' so $v_B < v_{B'}$.

3/88

$$\Delta T + \Delta V_g = 0; (\frac{1}{2} \rho L v^2 - 0) + (-\rho g L - (-\rho g \frac{L}{2})) = 0$$

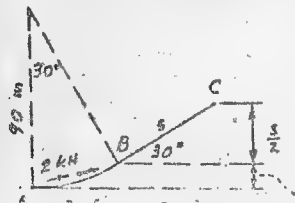


where $\rho = \text{mass per unit length}$

$$\frac{1}{2} v^2 = g \frac{L - \frac{L}{2}}{1}, v = \sqrt{gL}$$

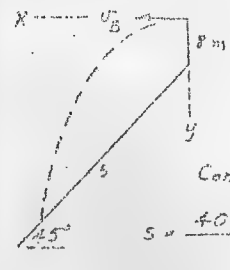
3/89 $U = \Delta T + \Delta V_g + \Delta V_e, \Delta T = 0$
 $U = 200(\sqrt{0.45^2 + 0.225^2} - 0.225) = 200(0.278) = 55.6 \text{ J}$
 $\Delta V_g = 7(9.81)(0.45) = 30.9 \text{ J}$
 $\Delta V_e = \frac{1}{2}k(0.075)^2 = 0.00281k \text{ J}$
 so $55.6 = 30.9 + 0.00281k, k = 8.79 \text{ kN/m}$

3/90 $\overline{AB} = r\theta = 90 \frac{\pi}{6} = 47.1 \text{ m}$
 $U = \Delta T + \Delta V_g$
 But $\Delta T = 0$ from A to B
 $U = 2000(47.1) = 94.2(10^3) \text{ J}$
 $\Delta V_g = 100(9.81)(\frac{5}{2} + 12.06)$
 Thus,
 $94.2(10^3) = 981(\frac{5}{2} + 12.06)$
 $S = 168.0 \text{ m}$



3/91 Neglect ΔV_g ; $\Delta V_e = 0 - 2(\frac{1}{2})k(\ell - b)^2$
 $= -\frac{2}{2}k(\sqrt{b^2 + x^2} - b)^2$
 $\Delta T = \frac{1}{2}mv^2 - 0$; Also, stiffness of entire spring
 AB is $k = K/b$
 Substitute in $\Delta V_e + \Delta T = 0$ & get
 $mv^2 = \frac{2K}{b}(\sqrt{b^2 + x^2} - b)^2, v = \sqrt{\frac{2K}{mb}(\sqrt{b^2 + x^2} - b)}$

3/92 From A to B, $\Delta V_g + \Delta T = 0$
 $-mg(\frac{32}{\sqrt{2}} - 2) + \frac{1}{2}mv_B^2 = 0, v_B = 16.94 \text{ m/s}$
 x -dir: $a_x = 0, x = \frac{1}{2}at^2$, so
 $\frac{32}{\sqrt{2}} = \frac{1}{2}(16.94)t$
 y -dir: $a_y = g, y = \frac{1}{2}gt^2$, so
 $\frac{32}{\sqrt{2}} + 8 = \frac{9.81}{2}t^2$
 Combine & get $4.905t^2 - 406.5 - 4590 = 0$
 $S = \frac{406.5 \pm \sqrt{164700 + 90100}}{9.81} = \frac{406.5 \pm 505}{9.81}$
 $= 92.8 \text{ m or } -10.08 \text{ m} \quad S = 92.8 \text{ m}$



3/93 $\Delta T + \Delta V_g + \Delta V_e = 0$
 $\frac{1}{2}10v^2 - 10(9.81)(h-1) + \frac{1}{2}450(h-1)^2 = 0$
 $v^2 = 9.81(2)(h-1) - 45(h-1)^2$
 $\frac{d(v^2)}{dh} = 9.81(2) - 90(h-1) = 0$ for max. $h = 1.218 \text{ m}$
 $v_{max}^2 = 9.81(2)(1.218-1) - 45(1.218-1)^2 = 2.14 \text{ (m/s)}^2$
 $v_{max} = \sqrt{2.14} = 1.46 \text{ m/s}$

3/94 (a) $\Delta T = 0; \Delta V_g + \Delta V_e = 0; 2(9.81)h - \frac{1}{2}120(0.5)^2 = 0$
 $h = 0.744 \text{ m}$



(b) $\Delta T + \Delta V_g + \Delta V_e = 0$;
 $\frac{1}{2}2v^2 + 2(9.81)x - \frac{1}{2}120[(0.5)^2 - (0.5-x)^2] = 0$
 $v^2 = 40.4x - 60x^2$ assuming $x \leq 0.5 \text{ m}$

$d(v^2)/dx = 40.4 - 120x = 0$ for max. v ; $x = 0.336 \text{ m}$

$v_{max} = \sqrt{(40.4 - 60(0.336))0.336} = 2.61 \text{ m/s}$

3/95 Let $V_g = 0$ in initial position ($x = 0$)
 $x = \text{deformation of spring}$

$\Delta V_g + \Delta V_e + \Delta T = 0; \Delta V_g = -mgx, \Delta V_e = \frac{1}{2}kx^2, \Delta T = \frac{1}{2}mv^2$
 $-mgx + \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = 0$

For max. $v, \frac{1}{2}m \frac{d(v^2)}{dx} = mg - kx = 0, x = \frac{mg}{k}$

so $v^2 = \frac{2}{m} \left(\frac{(mg)^2}{k} - \frac{1}{2}k \left[\frac{(mg)^2}{k^2} \right] \right) = \frac{mg^2}{k}, v = \sqrt{\frac{mg}{k}}$

Max δ & R occur for $\Delta T = 0$

$-mgx + \frac{1}{2}kx^2 = 0, x = \delta = \frac{2mg}{k}; R = k\delta = 2mg$

3/96 $x^2 + y^2 = 0.9^2, x\dot{x} + y\dot{y} = 0$

$\Delta T + \Delta V_g = 0; \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + mg(y - \frac{0.9}{\sqrt{2}}) = 0$

$\dot{x}^2(1 + \frac{y^2}{x^2}) = 2(9.81)(\frac{0.9}{\sqrt{2}} - y); 2\dot{x}^2 = 19.62(\frac{0.9}{\sqrt{2}} - y^2)$

For max. $\dot{x}, \frac{d(\dot{x}^2)}{dy} = \frac{19.62}{0.81}(\frac{1.8}{\sqrt{2}} - 2y) = 0$

so $y = \frac{0.9}{\sqrt{2}} \text{ m} \quad \dot{x}^2 = \frac{19.62}{0.81}(\frac{0.9}{\sqrt{2}} - \frac{0.9}{\sqrt{2}}) = \frac{19.62}{0.81}$

$v_{x,max} = \dot{x} = \sqrt{\frac{19.62 \times 2}{0.81}} = 0.962 \text{ m/s}$

3/97 Driving force $F = m \frac{dv}{dt}$

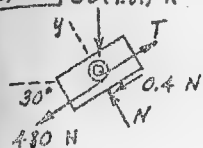
Power $P = Fv$ constant

Thus $\frac{F}{v} = m \frac{dv}{dt}$ or $Pds = m v \frac{dv}{ds} ds = m v^2 ds$

$\int_0^s Pds = \int_0^s m v^2 ds; Ps = \frac{m}{2}(v^2 - v_0^2)$

$s = \frac{m}{2P}(v^2 - v_0^2)$

3/98 50(9.81) N For 1.2-m displ. of block, P moves 1.2 m



down incline. T does no work.

$$\Sigma F_y = 0; N = 50(9.81) \frac{\sqrt{3}}{2} = 425 \text{ N}$$

$$0.4 N = 169.9 \text{ N}$$

$$U = \Delta T;$$

$$480(1.2) - 169.9(1.2) - 490(\frac{1}{2})(1.2) = \frac{1}{2} 50 v^2$$

$$v^2 = \frac{77.8}{25} = 3.11 \text{ (m/s)}^2$$

$$v = 1.764 \text{ m/s}$$

3/99



$$\Delta T + \Delta V_g = 0; \frac{1}{2} m v^2 - m g (h - \frac{r}{\sqrt{2}}) = 0$$

$$v^2 = 2g(h - \frac{r}{\sqrt{2}})$$

$$\Sigma F_n = m a_n; N + \frac{m g}{\sqrt{2}} = m a_n$$

$$\text{where } a_n = \frac{v^2}{r} = 2g(\frac{h}{r} - \frac{1}{\sqrt{2}})$$

$$\text{so } N = m g \left[2(\frac{h}{r} - \frac{1}{\sqrt{2}}) - \frac{1}{\sqrt{2}} \right] = m g \left[2\frac{h}{r} - \frac{3}{\sqrt{2}} \right]$$

$$\text{For } m = 0.25 \text{ kg, } r = 0.15 \text{ m, } h = 0.6 \text{ m,}$$

$$N = 0.25(9.81) \left[2 \frac{0.60}{0.15} - \frac{3}{\sqrt{2}} \right] = 14.42 \text{ N}$$

3/100

$$\Delta T + \Delta V_g = 0, \quad V_g = -\frac{m g R^2}{r}$$

$$\frac{1}{2} m (v_B^2 - [7100]^2) = 9.81(10^3)(3600)m(6370) \left(\frac{1}{1200} - \frac{1}{16000} \right)$$

$$\frac{1}{2} v_B^2 - 25.2(10^6) = 394(10^6) \text{ (km/h)}^2$$

$$v_B = 29000 \text{ km/h}$$

3/101

$$\Delta T = \frac{1}{2} m (v_B^2 - v_A^2) = \frac{1}{2} 22(13400^2 - 13000^2)(26)^{-2}$$

$$= -12.94(10^6) \text{ J}$$

$$\Delta V_g = -m g R^2 \left(\frac{1}{r} - \frac{1}{R} \right) = -22(9.81)(6370) \left(\frac{1}{6450} - \frac{1}{6410} \right)(10^3)$$

$$= 2.47(10^6) \text{ J}$$

$$U = -P(400)(10^3) \text{ J}$$

$$U = \Delta T + \Delta V_g; -400(10^3)P = (-12.94 + 2.47)(10^6), P = 11.16 \text{ N}$$

3/102

$$\Delta T + \Delta V_g = 0, \Delta T = \frac{1}{2} m (v^2 - [3000/3.6]^2)$$

$$\Delta V_g = -m g R^2 \left(\frac{1}{R} - \frac{1}{2R} \right) = -m g R/2$$

$$= -m \frac{1.62}{2} (1738)(10^3)$$

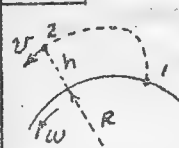
$$\text{Thus } v^2 - \left(\frac{3000}{3.6} \right)^2 - 1.62(1738)(10^3) = 0$$

$$v^2 = 3.51(10^6) \text{ (m/s)}^2$$

$$v = 1873 \text{ m/s, or } v = 1873(3.6) = 6740 \text{ km/h}$$

3/103

From Prob. 3/58 $v = R \sqrt{\frac{g}{R+h}}$



$$E_2 = T_2 + V_2 = \frac{1}{2} m R^2 \frac{g}{R+h} - \frac{g R^2 m}{R+h}$$

$$E_1 = T_1 + V_1 = \frac{1}{2} m R^2 \omega^2 - \frac{g R^2 m}{R}$$

$$E = E_2 - E_1 = \frac{1}{2} m R \left[g \frac{R+2h}{R+h} - R \omega^2 \right]$$

3/104

$$\Delta T = \frac{1}{2} (2.5)(2.4^2 - 1.8^2) = 3.15 \text{ J}$$

$$\Delta V_g = \frac{1}{2} 30 \left[(\sqrt{1.5^2 + 1.2^2} - 0.9)^2 - (\sqrt{0.9^2 + 1.2^2} - 0.9)^2 \right]$$

$$= 10.23 \text{ J}$$

$$\Delta V_g = 2.5(9.81)(-0.9) = -22.1 \text{ J}$$

$$U = U_f = \Delta T + \Delta V_g + \Delta V_g; U_f = 3.15 + 10.23 - 22.1$$

$$U_f = -8.69 \text{ J}$$

$$|U_f| = F_{av} \Delta s; 8.69 = F_{av} \sqrt{0.9^2 + 1.5^2}, F_{av} = 4.97 \text{ N}$$

3/105

$$\Delta T + \Delta V_g = 0; \Delta V_g = m g (-0.45 + h) + m g (-0.2 + 0.2 \frac{1}{2})$$

$$\text{where } h = 0.1 + \sqrt{0.25^2 - (0.2 \sin 60^\circ)^2}$$

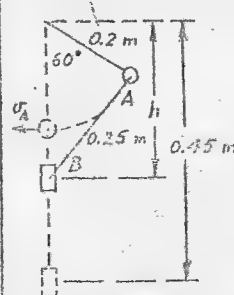
$$= 0.280 \text{ m}$$

$$\Delta V_g = -m g (0.45 - 0.280) - m g (0.2 - 0.1)$$

$$= -0.1697 m g + 0.1 m g = -0.270 m g$$

$$\Delta T = \frac{1}{2} m (v_A^2 - 0)$$

$$\text{Thus } \frac{1}{2} v_A^2 - 0.270(9.81) = 0$$



$$v_A^2 = 5.29 \text{ (m/s)}^2, v_A = 2.30 \text{ m/s}$$

3/106

F

Let x = stretch of spring from position of zero force

At equil. position $\Sigma F = 0; 2.5(9.81) + k x_0 = 0$

$$x_0 = -\frac{2.5(9.81)}{1.6} = -15.62 \text{ mm}$$

$$\Delta V_g + \Delta V_e + \Delta T = 0$$

$$\Delta V_g = -m g h = -2.5(9.81)(0.042) = -1.030 \text{ J}$$

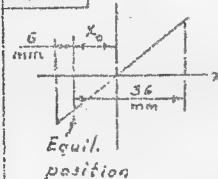
$$\Delta V_e = \frac{1}{2} k (x_2^2 - x_1^2) = \frac{1.6}{2} [(0.04)^2 - (-0.04 + 0.042)^2]$$

$$= 0.9[-84 x_0 - 1260] \times 1020 = 11340 - 1040 \text{ mJ}$$

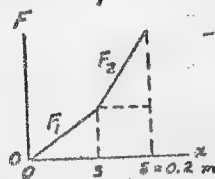
$$\Delta T = \frac{1}{2} m v^2 = \frac{1}{2} 2.5 v^2 = 1.25 v^2$$

$$\text{Thus } 1.25 v^2 - 1.030 - 0.104 = 0, v^2 = 0.967 \text{ (m/s)}^2$$

$$v = 0.982 \text{ m/s}$$



3/107



$$F_1 = 300 \text{ N}, F_2 = 300 \text{ N} + 150(x-s) = 450x - 150 \text{ s}$$

$$-U = \frac{1}{2} 300 s^2 + \int_s^{0.2 \text{ m}} (450x - 150s) dx$$

$$= 150 s^2 + 225 x^2 \Big|_s^{0.2} - 150 s x \Big|_s^{0.2}$$

$$= 75 s^2 - 30 s + 9 \text{ J}$$

$$U = \Delta T;$$

$$75 s^2 - 30 s + 9 = \frac{1}{2} 0.5 (5)^2$$

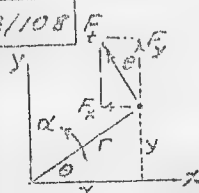
$$\text{or } 75 s^2 - 30 s + 2.75 = 0$$

$$s = \frac{30 \pm \sqrt{50^2 - 4(75)(2.75)}}{2(75)} = \frac{30 \pm 8.66}{150}$$

$$= 0.1423 \text{ m or } 0.257 \text{ m}$$

0.257 m > 200 mm so impossible; Thus $s = 142.3 \text{ mm}$

3/108



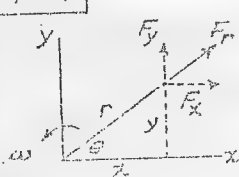
$$F_x = -F \sin \theta = -r \alpha \frac{y}{r} = -\alpha y$$

$$F_y = F \cos \theta = r \alpha \frac{x}{r} = \alpha x$$

$$\left(\frac{\partial F_x}{\partial y} = -\alpha \right) \neq \left(\frac{\partial F_y}{\partial x} = +\alpha \right)$$

so field is nonconservative & no V exists

3/109



$$F_x = F \cos \theta = -r \omega^2 \frac{x}{r} = -x \omega^2$$

$$F_y = F \sin \theta = -r \omega^2 \frac{y}{r} = -y \omega^2$$

$$\frac{\partial F_x}{\partial y} = 0 = \frac{\partial F_y}{\partial x} \text{ so field is conservative}$$

$$dV = -\mathbf{F} \cdot d\mathbf{r} = -(-r \omega^2) dr, V = \int_0^r r \omega^2 dr = \frac{1}{2} r^2 \omega^2$$

$$V = \frac{1}{2} (x^2 + y^2) \omega^2, F_x = -\frac{\partial V}{\partial x} = -x \omega^2, F_y = -\frac{\partial V}{\partial y} = -y \omega^2$$

3/110

Power developed by spring is $P = Fv = 120(0.5-x) \dot{x}$
 P_{max} occurs where P^2 is max. & from Prob. 3/94

$$P^2 = (120)^2 (0.5-x)^2 60 (0.673x - x^2)$$

$$= 0.864 (10^6) (0.1682x - 0.923x^2 + 1.673x^3 - x^4)$$

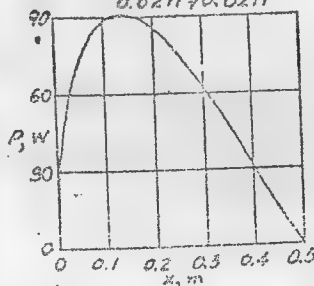
$$\frac{d(P^2)}{dx} = 0.864 (10^6) (0.1682 - 1.846x + 5.02x^2 - 4x^3) = 0 \text{ for max. } P^2$$

$$\text{To solve cubic } x^3 - \frac{5.02}{4} x^2 + \frac{1.846}{4} x - \frac{0.1682}{4} = 0, \text{ let } x = y + \frac{5.02}{12}$$

$$\text{which gives } y^3 = 0.0633y - 0.00463$$

$$P^2 = 0.0211, P = -0.00231; P^2 - P^3 \text{ is neg. so}$$

$$\cos u = \frac{-0.00231}{0.0211/0.0211} = -0.755, u = 139.0^\circ$$



$$y_1 = 2\sqrt{0.0211} \cos \frac{u}{3} = 0.201$$

$$y_2 = 2\sqrt{0.0211} \cos \left(\frac{u}{3} + 120^\circ \right) = -0.292$$

$$y_3 = 2\sqrt{0.0211} \cos \left(\frac{u}{3} + 240^\circ \right) = 0.0817$$

$$\text{so } x_1 = 0.619 \text{ m}$$

$$x_2 = 0.1359 \text{ m}$$

$$x_3 = 0.500 \text{ m}$$

$$P_{\text{max}} = 71.4 \text{ W}$$

43/111

ρ = mass density; ϕ = angle of rotation around OA

Due to an element of ring of mass dm , $\frac{dF}{r^2}$

$$dF = \gamma \frac{dm}{s^2} = \gamma \frac{\rho r d\phi (r \sin \theta) d\phi}{R^2 + r^2 - 2Rr \cos \theta}$$

Integrate over ϕ , 0 to 2π

$$dF = dF' \cos \theta = 2\pi \gamma \rho r^2 \frac{\sin \theta d\theta}{R^2 + r^2 - 2Rr \cos \theta}$$

$$= 2\pi \gamma \rho r^2 \int_0^\pi \frac{(2 - r \cos \theta) \sin \theta d\theta}{(R^2 + r^2 - 2Rr \cos \theta)^{3/2}}$$

$$\text{Let } u = 2 - r \cos \theta, du = r \sin \theta d\theta$$

$$F = 2\pi \gamma \rho r^2 \int_{u=2}^{u=2+r} \frac{u du}{(u^2 - R^2 + 2Rr)^{3/2}} = \pi \gamma \rho r^2 \left[\frac{2r(1 - R \cos \theta)}{(R^2 + r^2 - 2Rr \cos \theta)^{1/2}} \right]_{\theta=0}^{\theta=\pi}$$

$$= \pi \gamma \rho r^2 \left[\frac{2r}{R+r} - \frac{2r}{R-r} \right] = \frac{2\pi \gamma \rho r^2}{R^2 - r^2} \left[\frac{R-r}{R+r} - \frac{R+r}{R-r} \right]$$

$$= \frac{2\pi \gamma \rho r^2}{R^2 - r^2} \left[\frac{R^2 - R^2 - r^2 + r^2}{(R+r)(R-r)} \right] = \frac{2\pi \gamma \rho r^2}{R^2 - r^2} \left[\frac{-2r^2}{(R+r)(R-r)} \right]$$

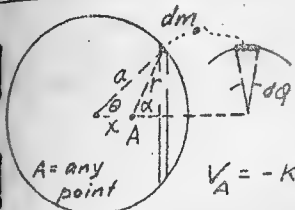
$$= \frac{2\pi \gamma \rho r^2}{R^2 - r^2} \left[\frac{-2r^2}{R^2 - r^2} \right] = \frac{4\pi \gamma \rho r^4}{(R^2 - r^2)^2}$$

Replace m by dm mass of differential shell of solid sphere & F by dF & write $d\phi = 2\pi r \sin \theta d\theta$

which yields $F = \gamma m / R^2$ same as if m were concentrated at a point, distant R from A .

3/112 $dm_0 = (r \sin \theta) d\varphi d\theta \rho$, ρ = mass per unit area of shell.

$dV_A = -K \frac{dm_0}{r}$ where $K dm_0$ is equivalent to gR^2 of Eq. 55




$V_A = -K \rho a \int_0^{2\pi} \int_0^\pi \sin \theta d\theta d\varphi = -2\pi K \rho a \int_0^\pi \sin \theta d\theta$

$= -2\pi K \rho a \int_0^\pi \frac{a \sin \theta d\theta}{\sqrt{a^2 + x^2 - 2ax \cos \theta}} = 2\pi K \rho a^2 \int_{-1}^1 \frac{du}{\sqrt{C + Du}}$ where $\begin{cases} u = \cos \theta \\ C = a^2 + x^2 \\ D = -2ax \end{cases}$

$= 2\pi K \rho a^2 \left[\frac{2\sqrt{a^2 + x^2 - 2ax \cos \theta}}{-2ax} \right]_0^\pi = 2\pi K \rho a^2 \frac{(a+x) - (a-x)}{-2ax}$

$= -K \frac{4\pi \rho a^2}{a} = -K \frac{m_0}{a}$ independent of x , so constant throughout

3/113 Portion II of earth consists of concentric shells within which potential, by Prob 3/112, is constant - hence no gradient & no force. Force is, then, $F = K \frac{m_1 m}{r^2}$



where $m_1 = \int_0^r 4\pi x^2 \rho dx$ & m = particle mass

If $\rho = f(x)$ were known, then $U = \Delta T$ would give

$F(-dr) = d(\frac{1}{2} m v^2)$

so $-\int_0^R \left[\frac{K m}{r^2} \int_0^r 4\pi x^2 \rho dx \right] dr = \int_0^v d(\frac{1}{2} m v^2)$

or $4\pi K m \int_0^R \int_0^r x^2 dx dr = \frac{1}{2} m v^2$

where ρ = density of earth at radius x (concentric symmetry assumed)
 v = velocity reached by m at center of earth

3/118 $\int \Sigma F dt = m \Delta v$



$[2(8000) - \Delta R] 9 = 10000 (1050 - 1000) / 3.6$

$16000 - \Delta R = 15432, \Delta R = 568 \text{ N}$

3/119 Impulse $I = m \Delta v = m v_{\max}$

v_{\max} occurs where $\dot{v} = 0$ or $P = R = kv^2$
 $v_{\max} = \sqrt{P/k}$ so $I = m \sqrt{P/k}$

3/120 For system of disk & projectile $\Delta G = 0$

so $180(3000) + 0 = 180(1500) + 960 v', v' = 291 \text{ m/s}$

3/121 $\underline{F} = \underline{\dot{G}} = \frac{d}{dt} (6t^3 \underline{i} + 15t^2 \underline{j})$
 $= 18t^2 \underline{i} + 30t \underline{j}$

For $t = 3 \text{ s}$, $\underline{F} = 18(3^2) \underline{i} + 30(3) \underline{j} = 162 \underline{i} + 90 \underline{j} \text{ N}$

& $F = \sqrt{162^2 + 90^2} = 185.3 \text{ N}$

3/122 6th rocket burns out after $2\frac{3}{4} \text{ s}$,

so total impulse on sled during 3 s is

$6(100)(10^3) - 3R \text{ N}\cdot\text{s}$

$\int \Sigma F dt = m \Delta v; 600(10^3) - 3R = 3(10^3)(150)$

$R = 50 \text{ kN}$

3/123



$\int \Sigma F dt = m \Delta v$
 $-\frac{0.09 - 0.05}{2} 6000 = 5(v - 20), v = -4.00 \text{ m/s}$

or $v = 4.00 \text{ m/s}$ to the left.

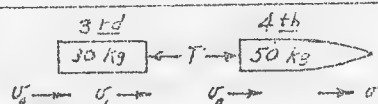
3/124

Impact velocity $v_0 = \sqrt{2gh} = \sqrt{2(9.81)(1.2)} = 4.85 \text{ m/s}$

$\Delta G = 0; 400(4.85) + 0 = (400 + 250)v$

$v = 2.99 \text{ m/s}$

3/125



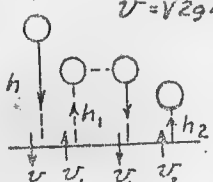
$\int \Sigma F dt = \Delta G; -T(0.5) = 30(v_1 - v_0)$


$T(0.5) = 50(v - v_0)$

$0.5T(\frac{1}{30} + \frac{1}{50}) = (v - v_0) - (v_1 - v_0), v - v_1 = 0.0267 T$

But $v - v_1 = 10 \text{ m/s}$, so $T = \frac{10}{0.0267} = 375 \text{ N}$

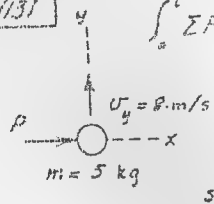

3/126 $v = \sqrt{2gh}$, $v' = \sqrt{2gh'}$, $e = \frac{v'}{v} = \sqrt{\frac{h'}{h}}$
 $n = \frac{mgh - mgh'}{mgh} = 1 - \frac{h'}{h}$, $n = 1 - e^2$

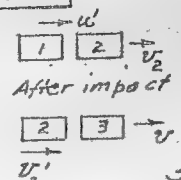
3/127 $v = \sqrt{2gh}$, $v_1 = \sqrt{2gh_1}$, $v_2 = \sqrt{2gh_2}$

 $e = \frac{v_1}{v}$ & $e = \frac{v_2}{v_1}$
 so $v_1^2 = v v_2$
 $e = \frac{\sqrt{v v_2}}{v} = \sqrt{\frac{v_2}{v}} = \left(\frac{h_2}{h}\right)^{1/4}$

3/128 
 $mg = 200(1.62) = 324 \text{ N}$
 $T = 18000/10 = 1800 \text{ N}$
 $\int \Sigma F_y dt = m \Delta v_y$
 $(324 - 1800)t = 200(1.5 - 45)$
 $t = 5.89 \text{ s}$


3/129 $\int \Sigma F_y dt = m \Delta v_y$; $0.4t = 0 - \left(-\frac{10}{2}\right)$
 or $v_y = \frac{dy}{dt} = 0.8t - 5$, $y = \int_0^t (0.8t - 5) dt$
 $= 0.4t^2 - 5t$
 For $y = 0$, $(0.4t - 5)t = 0$, $t = 12.5 \text{ s}$

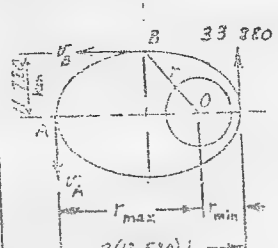
3/130 $v_y = 600 \cos \theta$; $\dot{\theta} = \pi/10 \text{ rad/s}$, so $dt = \frac{10}{\pi} d\theta$
 $\int F_y dt = m \Delta v_y$; $\int_0^{\pi/2} 600 \cos \theta \left(\frac{10}{\pi} d\theta\right) = 260(v_y - 0)$
 $\frac{6000}{\pi} \sin \theta \Big|_0^{\pi/2} = 260 v_y$, $v_y = \frac{6000}{260\pi} = 7.35 \text{ m/s}$

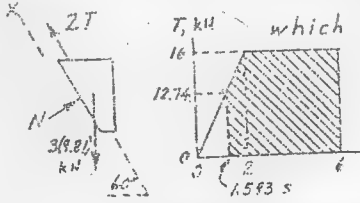
3/131 
 $\int_0^t \Sigma F_x dt = m \Delta v_x = m(v_{2x} - v_{1x})$
 From graph $\int_0^2 \Sigma F_x dt = \frac{40}{2} + \frac{40}{2} + \frac{20}{2} = 50 \text{ N}\cdot\text{s}$
 so $50 = 5(v_{2x} - 0)$, $v_{2x} = 10 \text{ m/s}$
 $v = \sqrt{10^2 + 0^2} = 10 \text{ m/s}$
 $\theta = \tan^{-1} \frac{t}{10} = 35.7^\circ$


3/132 $\Delta G = 0$; $mu = m(u' + v'_2)$; $e = \frac{v'_2 - u'}{u}$

 solve for $v'_2 = \frac{u}{2}(1+e)$
 After impact $\Delta G = 0$; $m \frac{u}{2}(1+e) = m(v'_2 + v'_1)$; $v'_1 = \frac{v - v'_2}{2}$
 solve for $v = \frac{u}{4}(1+e)^2$

3/133 Let v_s & v_b stand for rebound velocities from steel & brass plates, respectively.
 Impact velocity $= \sqrt{2gh} = \sqrt{19.62(0.15)} = 1.716 \text{ m/s}$
 $0.6 = \frac{v_s}{1.716}$, $v_s = 1.029 \text{ m/s}$ } $\omega = \frac{1.029 - 0.686}{0.60}$
 $0.4 = \frac{v_b}{1.716}$, $v_b = 0.686 \text{ m/s}$ } $= 0.572 \text{ rad/s CCW}$

3/134 Final velocity of m is to be zero
 so for $\Delta G = 0$, $mv_1 + 0 = 300v + 0$
 $e = \frac{v - 0}{v_1 - 0}$, so $0.3 = \frac{m}{300}$, $m = 90 \text{ kg}$
 Thus for $v_1 = \sqrt{19.62(4)} = 8.86 \text{ m/s}$
 $v = ev_1 = 0.3(8.86) = 2.66 \text{ m/s}$

 $\Delta E = mgh - \frac{1}{2}m'v^2 = 90(9.81)(4) - \frac{1}{2}300(2.66)^2$
 $\Delta E = 3530 - 1059 = 2470 \text{ J}$

3/135 $\Sigma M_O = \dot{H}_O = 0$ so $H_O = \text{constant}$

 $r_{\min} = 4370 + 390 = 4760 \text{ km}$
 $r_{\max} = 2(13520) - 4760 = 20280 \text{ km}$
 For H_O constant,
 $4760(33880) = 11720 v_1 = 20280 v_2$
 $v_1 = 11290 \text{ km/h}$
 $v_2 = 15540 \text{ km/h}$

3/136 $\int \Sigma F_x dt = m \Delta v_x$; Skip begins to move when
 $2T = 3(9.81)\frac{\sqrt{2}}{2}$, $T = 12.74 \text{ kN}$
 which occurs at $t = \frac{12.74}{1.593} = 7.99 \text{ s}$

 $\int \Sigma F_x dt = 2 \left[\frac{16 + 12.74}{2} (2 - 1.593) + 16(6 - 2) \right] - 3(9.81)\frac{\sqrt{2}}{2} (6 - 1.593)$
 $= 27.4 \text{ kN}\cdot\text{s}$
 Thus $27.4 = 3(v - 0)$, $v = 9.13 \text{ m/s}$

3/137

During impact $\Sigma F_x = 0$ so no change in x-component of velocity

$$v \cos \theta = 24(0.5) = 12 \text{ m/s}$$

In y-dir, $e = \frac{v \sin \theta}{24 \cos 30^\circ} = 0.8$

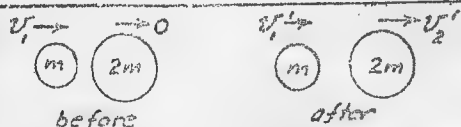
Combine & get

$$\tan \theta = \frac{16.63}{12} = 1.3856$$

$$\theta = 54.2^\circ$$

$$v = \frac{12}{\cos 54.2^\circ} = \frac{12}{0.5852} = 20.5 \text{ m/s}$$

3/138



before

after

Let e = coefficient of restitution

$$\Delta G = 0; m v_1' + 2m v_2' = m v_1$$

$$\& v_2' - v_1' = e(v_1)$$

Combine & get $v_1' = \frac{1-2e}{3} v_1, v_2' = \frac{1+e}{3} v_1$

Kinetic energy loss is

$$\Delta T = \frac{1}{2} m v_1'^2 - \left[\frac{1}{2} m \left(\frac{1-2e}{3} v_1 \right)^2 + \frac{1}{2} 2m \left(\frac{1+e}{3} v_1 \right)^2 \right]$$

$$= \frac{1}{3} m v_1^2 (1-e^2), \quad \frac{\Delta T}{de} = \frac{1}{3} m v_1^2 (-2e) = 0$$

$$\text{so } e = 0$$

$$\& v_1' = \frac{v_1}{3}, \quad v_2' = \frac{v_1}{3}$$

3/139

Let v_r = ram velocity after impact

$$\Delta G = 0; 400 \sqrt{19.62(1.2)} + 0 = 400 v_r + 250 v$$

$$\& e = \frac{v - v_r}{\sqrt{19.62(1.2)}}$$

Combine & get $v_r = \frac{(400 - 250e) \sqrt{19.62(1.2)}}{400 + 250} = 0.373(8-5e)$

Max. transfer of energy occurs for

v_r a minimum which occurs for $e = e_{\max} = 1$

$$\text{Thus } v_{r_{\min}} = 0.373(8-5) = 1.120 \text{ m/s}$$

$$\text{so } v = (8 \sqrt{19.62(1.2)} - 8(1.120))/5 = 5.47 \text{ m/s}$$

$e=1$ is larger than could be realized with steel on wood.

3/140

$H_{\text{vertical}} = \text{constant}; m r_1^2 \omega_1 = m r_2^2 \omega_2$

$$\text{so } r_1^2 N_1 = r_2^2 N_2, \left(300 \frac{\sqrt{3}}{2} \right)^2 (90) = \left(300 \frac{1}{2} \right)^2 N, N = 270 \text{ rev/min}$$

$$U = \Delta T + \Delta V_g; U = 2 \frac{1}{2} 4 \left[\left(0.3 \frac{1}{2} 270 \frac{2\pi}{60} \right)^2 - \left(0.3 \frac{\sqrt{3}}{2} 90 \frac{2\pi}{60} \right)^2 \right] + 2(4)(9.81) \left[0.3 \frac{\sqrt{3}}{2} - 0.3 \frac{1}{2} \right] = 48.0 + 8.62 = 56.6 \text{ J}$$

3/141

Angular momentum about central axis is conserved so.

$$H = \frac{d}{dt} (m r^2 \omega) = 0$$

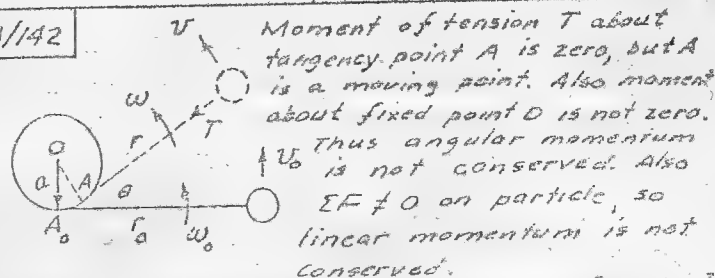
$$m(2r\omega dr + r^2 d\omega) = 0, \quad \frac{d\omega}{dr} = \frac{-2\omega}{r}$$

$$\Sigma F_n = m a_n; F = m r \omega^2$$

$$dU = dT; -F dr = d\left(\frac{1}{2} m r^2 \omega^2\right)$$

$$-m r \omega^2 dr = m(r \omega^2 dr + r^2 \omega d\omega) = m(r \omega^2 dr + r^2 \omega \left[\frac{-2\omega}{r} dr \right]) = -r \omega^2 dr \quad (\text{check})$$

3/142



Moment of tension T about tangency point A is zero, but A is a moving point. Also moment about fixed point O is not zero. Thus angular momentum is not conserved. Also $\Sigma F \neq 0$ on particle, so linear momentum is not conserved.

But energy is conserved so $\frac{1}{2} m_0 (r_0 \omega_0)^2 = \frac{1}{2} m (r \omega)^2$

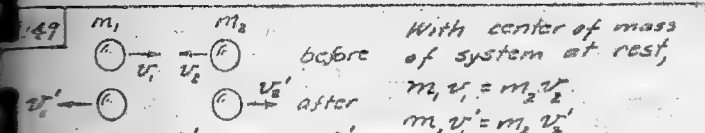
$$\& \omega = \frac{r_0 \omega_0}{r} = \frac{r_0}{r_0 - a\theta} \omega_0; \quad \omega = \frac{\omega_0}{1 - \frac{a}{r_0} \theta}$$

$$\Sigma M_O = H_O; -T a = \frac{d}{dt} (m r v) = m v \dot{r} \quad \text{since } v = \text{const}$$

$$\text{But } \dot{r} = \frac{d}{dt} (r_0 - a\theta) = -a\dot{\theta} = -a\omega$$

$$\text{So } -T a = m v (-a\omega), \quad T = m v \omega \quad \text{or } T = m r_0 \omega_0 \omega$$

$$\text{since } v = v_0 = r_0 \omega_0$$



us $v_1' = v_1 \frac{v_2'}{v_2}$, $v_2' = v_2 \frac{v_1'}{v_1}$

so $e = \frac{v_1' + v_2'}{v_1 + v_2}$ or $v_1' = (1 + \frac{v_2}{v_1}) = e(v_1 + v_2)$ which gives $\begin{cases} v_1' = ev_1 \\ v_2' = ev_2 \end{cases}$

by $\Delta E = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 - \frac{1}{2} m_1 v_1^2 - \frac{1}{2} m_2 v_2^2$

$= \frac{1}{2} \{ m_1 v_1^2 (1 - e^2) + m_2 v_2^2 (1 - e^2) \} = \frac{1 - e^2}{2} (m_1 v_1^2 + m_2 v_2^2)$

let $m_1 v_1 + [m_2 v_2] = m_2 v_2' + [m_1 v_1']$, $v_1' = \frac{m_2}{m_1 + m_2} (v_1 + v_2)$

$m_2 v_2 + [m_1 v_1] = m_1 v_1' + [m_2 v_2']$, $v_2' = \frac{m_1}{m_1 + m_2} (v_1 + v_2)$

$\Delta E = \frac{1 - e^2}{2} \left\{ \frac{m_1 m_2^2}{(m_1 + m_2)^2} + \frac{m_2 m_1^2}{(m_1 + m_2)^2} \right\} (v_1 + v_2)^2$

$\Delta E = \frac{1 - e^2}{2} \frac{m_1 m_2}{m_1 + m_2} (v_1 + v_2)^2$

3/150 Maximum advantage of the earth's rotation is achieved for launch at the equator with trajectory headed east, in direction of velocity of earth's surface.

3/151 For circular orbit $a = R + H$, so from

Eq. 82 $v^2 = 2gR^2 \left(\frac{1}{R+H} - \frac{1}{2(R+H)} \right)$, $v = R \sqrt{\frac{g}{R+H}}$

3/152 Angular momentum is conserved, so

$v_a (R + H_a) = v_p (R + H_p)$ where $a = \text{apogee}$
 $p = \text{perigee}$

$H_a = \frac{v_p}{v_a} (R + H_p) - R = 1.5 (6370 + 389) - 6370$
 $= 3768 \text{ km}$

3/153 From Eq. 82 with $r = a = R + H$,

$v^2 = 2gR^2 \left(\frac{1}{a} - \frac{1}{2a} \right) = \frac{gR^2}{R+H}$

$= \frac{1.62 (3600)^2 (1738)^2}{1738 + 80} = 34.88 (10^6)$

$v = 5910 \text{ km/h}$

3/154 From Eq. 79 with $r = a$, $T = 2\pi \frac{r^{3/2}}{R\sqrt{g}}$

For the moon (Appen. C), $R = 1738 \text{ km}$

$g = 1.62 \text{ m/s}^2$
 $r = 1738 + 80 \text{ km}$

so $T = 2\pi \frac{(1818)^{3/2}}{1738 \sqrt{1.62 (10^{-3})}} = 6962 \text{ s}$
or $T = 1 \text{ h } 56 \text{ min } 2 \text{ s}$

3/155 From Eq. 82 with $a = \infty$, $r = R + H$, escape velocity is $v^2 = 2gR^2 \left(\frac{1}{R+H} - \frac{1}{\infty} \right)$

$v = R \sqrt{\frac{2g}{R+H}}$

3/156 From Eq. 78, $a = \frac{r_{\min}}{1 - e} = \frac{6370 + 145}{1 - 0.2} = 8144 \text{ km}$

From Eq. 82, $v^2 = 2(9.824 (10^{-3}) (3600)^2 (6370)^2 \left(\frac{1}{6370} - \frac{1}{16288} \right))$
where $r = 6370 + 320 = 6690 \text{ km}$

Solve & get $v^2 = 910 (10^6)$, $v = 30170 \text{ km/h}$

3/157 Min. velocity occurs at Apogee A at which $r = 4R$. Also $2a = 6R$ so from

Eq. 82, $v^2 = 2gR^2 \left(\frac{1}{4R} - \frac{1}{6R} \right) = \frac{gR}{6}$

$v^2 = \frac{9.824 (3600)^2 6370}{10^3 \cdot 6} = 135.17 (10^6)$

$v = 11630 \text{ km/h}$

3/158 From Eq. 78 $a = \frac{r_{\min}}{1 - e} = \frac{640 + 620}{1 - 0.25} = 9120 \text{ km}$

& Appendix C

From Eq. 83 $v_{\max} = v_p = 6200 \sqrt{\frac{2.44 (10^{-3}) (11 - 0.25)}{9120 (1 - 0.25)}}$

$= 6200 (0.962) (10^{-3}) (1.291)$

$= 7.70 \text{ km/s}$

3/159 From the equation for the elliptical orbit, Eq. 78, when $\theta = \pi/2$, $r = a(1 - e^2)$

But $r_{\max} = a(1 + e)$ & $r_{\min} = a(1 - e)$, &

$r = a(1 - e)(1 + e) = \frac{r_{\max} r_{\min}}{a}$

3/160 From Eq. 82 at injection point
 $\frac{1}{2a} = \frac{1}{R+H} - \frac{v_0^2}{2gR^2}$ so Eq. 82 gives for

any position $v^2 = 2gR^2 \left(\frac{1}{r} - \frac{1}{R+H} + \frac{v_0^2}{2gR^2} \right)$

$$\text{or } v^2 = v_0^2 + 2gR^2 \left(\frac{1}{r} - \frac{1}{R+H} \right)$$

3/161 For given orbit injection conditions

Eq. 82 gives $a = \frac{(R+H)gR^2}{2gR^2 - v_0^2(R+H)}$

Substitute in Eq. 79 & get $T = \frac{2\pi gR^2}{\sqrt{\left(\frac{2gR^2}{R+H} - v_0^2 \right)^3}}$

3/162 $r_{\min} = 6370 + 240 = 6610 \text{ km}$
 $r_{\max} = 6370 + 400 = 6770 \text{ km}$

From Eq. 78, $\frac{r_{\min}}{r_{\max}} = \frac{1-e}{1+e}$

so $(1+e)(6610) = (1-e)(6770)$, $e = 0.01196$

Eq. 79 with $a = \frac{1}{2}(r_{\max} + r_{\min}) = 6690 \text{ km}$

$T = 2\pi \frac{(6690)^{3/2}}{6370 \sqrt{9.821(10^{-8})}} = 5146.6 \text{ s or}$

$T = 1 \text{ h } 30 \text{ min } 46 \text{ s}$

3/163



$F = k \frac{m m_0}{r^2} = \frac{g R^2 m}{r^2}$

$2F_n = m a_n$; $\frac{g R^2 m}{r^2} \sin \alpha = m \frac{v^2}{p}$

so $p = \frac{r^2 v^2}{g R^2 \sin \alpha}$

3/164

Eq. 79, with $R = 6370 \text{ km}$, $g = 9.821 \text{ m/s}^2$,
 $T = 90(60) \text{ s}$,

$90(60) = 2\pi \frac{a^{3/2}}{6370 \sqrt{9.821(10^{-8})}}$

$a^3 = 294(10^9) \text{ km}^3$, $a = 6652 \text{ km}$

Eq. 82, $v^2 = 2(9.821)(10^{-8})(3600)^2(6370)^2 \left(\frac{1}{6370+150} - \frac{1}{2(6652)} \right)$

$= 808(10^6) \text{ (km/h)}^2$

$v = 28420 \text{ km/h}$

3/165

$r_{\min} = a(1-e) = R+H = 6370+500 = 6870 \text{ km}$
 $e = 0.7$

From Eq. 78, for $\theta = \pi/2$, $\frac{1}{r} = \frac{1}{a(1-e^2)} = \frac{1}{(R+H)(1+e)}$

so from Eq. 82, $v^2 = 2gR^2 \left(\frac{1}{(R+H)(1+e)} - \frac{1-e}{2(R+H)} \right)$

or $v^2 = \frac{gR^2}{R+H} \frac{1+e^2}{1+e} = \frac{9.821(10^{-8})(3600)^2(6370)^2}{6370+500} \frac{1+0.7^2}{1+0.7}$

$= 659(10^6) \text{ (km/h)}^2$

$v = 25670 \text{ km/h}$

3/166

From Eq. 78, $a = \frac{6370+300}{1-0.2} = 8358 \text{ km}$

From Eq. 79 the period is

$T = 2\pi \frac{(8358)^{3/2}}{(6370) \sqrt{9.821(10^{-8})}} = 7577 \text{ s}$

Longitude change $\Delta\beta = \omega_{\text{earth}} T = (0.7292)(10^{-4})(7577) = 0.5525 \text{ rad}$

or $\Delta\beta = 0.5525 \frac{180}{\pi} = 31.7^\circ \text{ west}$

3/167

$\Delta E = E_{\text{orbit}} - E_{\text{launch}}$

From Eq. 81 $E_{\text{orbit}} = -\frac{gR^2 m}{2(R+H)}$

$E_{\text{launch}} = T + V = \frac{1}{2} m(R\omega)^2 - \frac{m g R^2}{R}$

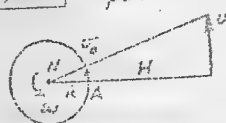
so $\Delta E = m g R \left(1 - \frac{R}{2(R+H)} - \frac{2\omega^2}{2g} \right)$

$= 450(9.821)(6370)(10^3) \left(1 - \frac{6370}{2(6370+250)} - \frac{6370(0.7292)^2}{2(9.821)(10^{-8})} \right)$

$= 28.2(10^9)(0.5172) = 14.56 \text{ GJ}$

3/168

Spot A must be on the equator (latitude zero) and v must be in the easterly direction.



$\frac{v}{R+H} = \omega = 0.729(10^{-4}) \text{ rad/s}$

But $v = R \sqrt{\frac{g}{R+H}}$ for circular orbit. Combine & get

$\frac{gR^2}{R+H} = (R+H)\omega^2$, $R+H = \sqrt{\frac{gR^2}{\omega^2}} = \sqrt{\frac{9.821(10^{-8})(6370)^2}{(0.729)^2(10^{-8})}}$

$= \sqrt{75.0(10^{12})} = 42170 \text{ km}$

$H = 42170 - 6370 = 35800 \text{ km}$, $\text{or } 11.07 \text{ Earth radii}$

3/169 $a = \frac{1}{2}(2[6370] + 150 + 1500) = 7195 \text{ km}$

From Eq. 82, at perigee P,

$$v^2 = \frac{2(9.824)(3600)^2(6370)^2}{10^3} \left(\frac{1}{6370+150} - \frac{1}{2(7195)} \right)$$

$$v = 29\,440 \text{ km/h} = 8.18 \text{ km/s}$$

$R\omega = 6370(0.729)(10^{-4}) = 0.464 \text{ km/s}$

Absolute angular velocity of dish $P_a = \frac{v - R\omega}{H}$

Relative " " " " $p = P_a - \omega$

$$p = \frac{v - R\omega}{H} - \omega = \frac{8.18 - 0.464}{150} - 0.729(10^{-4}) = 0.0513 \text{ rad/s}$$

3/170 From Eq. 82 for $a = \infty$, $r = R$, $v = \sqrt{2gR}$

$$v = \sqrt{2(9.824)(6370)(10^3)} = 11\,190 \text{ m/s}$$

or $v = 11.19 \text{ km/s}$

Relative projectile velocity u is a minimum when fired horizontally at equator in E-direction.

Absolute velocity v is independent of its direction or point of origin.

3/171 $\tan \beta = \frac{r}{R^2}$; From Eq. 78, $r = \frac{a(1-e^2)}{1+e \cos \theta}$

so $r = \frac{a(1-e^2) \cos \theta}{(1+e \cos \theta)^2}$

Thus $\tan \beta = \frac{a(1-e^2) \cos \theta}{(1+e \cos \theta)^2} \cdot \frac{1+e \cos \theta}{a(1-e^2)} = \frac{e \sin \theta}{1+e \cos \theta}$

3/172 Absolute orbital velocity $v = R \sqrt{\frac{g}{R+H}}$

$$= 6370 \sqrt{\frac{(9.824)(10^3)}{6370+800}} = 7460 \text{ m/s}$$

or $v = 26.800 \text{ km/h}$

Tangential velocity of tower is

$$v_o = (R+H)\omega = (6370+800)(0.729)(10^{-4})$$

$$= 0.5227 \text{ km/s or } 1882 \text{ km/h}$$

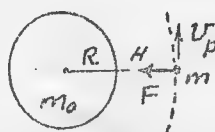
(a) $u = \sqrt{v^2 - v_o^2} = \sqrt{(26\,800)^2 - (1882)^2}$

$$= 26\,770 \text{ km/h}$$

(b) $u = v_o + v = 1882 + 26\,800 = 28\,680 \text{ km/h}$

3/173 $\Sigma F_n = m a_n$; $F = m \frac{v^2}{\rho}$ where $F = K \frac{mm_o}{(R+H)^2}$

$$= 9 \frac{R^2}{(R+H)^2} m$$



Thus $\frac{9R^2}{(R+H)^2} = \frac{v^2}{\rho}$

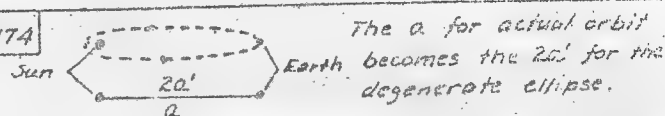
But at perigee $v^2 = \frac{9R^2}{a} \frac{1+e}{1-e}$ from Eq. 83

Also $R+H = r_{\min} = a(1-e)$, so $v^2 = \frac{9R^2}{R+H} (1+e)$

Combine & get $\frac{9R^2}{(R+H)^2} = \frac{9R^2}{R+H} \frac{1+e}{\rho}$

Hence $\rho = (R+H)(1+e)$

3/174



The a for actual orbit becomes the $2a'$ for the degenerate ellipse.

Thus time $t = \frac{1}{2} T'$, T' = period for degenerate ellipse

$T = \text{" " actual earth orbit}$

From Eq. 79, $T'/T = (a'/a)^{3/2} = (1/2)^{3/2}$

So $t = \frac{1}{2} T' = \frac{T}{4\sqrt{2}} = \frac{365}{4\sqrt{2}} = 64.6 \text{ days}$

3/175 From Eq. 82, $a = \frac{gR^2(R+H)}{2gR^2 - v_p^2(R+H)}$

& from Eq. 78, $r_{\min} = a(1-e)$ or $a = \frac{R+H}{1-e}$

Eliminate a & get $1-e = 2 - v_p^2(R+H)/gR^2$

So $1+e = v_p^2(R+H)/gR^2$

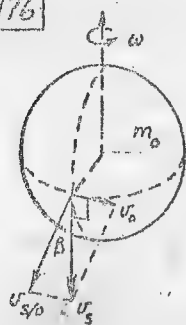
Substitute in Eq. 78 $r_{\max} = a(1+e) = \frac{v_p^2(R+H)^2}{2gR^2 - v_p^2(R+H)}$

Substitute in Eq. 83

& simplify to get

$$v_a = \frac{2gR^2}{v_p(R+H)} - v_p$$

3/176

 v_s = velocity of satellite v_o = velocity of point on equator $v_{s/o}$ = apparent velocity of satellite

$$v_o = R\omega$$

For satellite, $\Sigma F_n = ma_n$

$$K \frac{mm_0}{(R+H)^2} = m \frac{v_s^2}{R+H}, \text{ so } v_s^2 = \frac{gR^2}{R+H}$$

$$\text{where } Km_0 = gR^2; \therefore v_s = R \sqrt{\frac{g}{R+H}}$$

$$\text{Hence } \beta = \tan^{-1} \frac{R\omega}{R \sqrt{\frac{g}{R+H}}} = \tan^{-1} \omega \sqrt{\frac{R+H}{g}}$$

$$\beta = \tan^{-1} \left(0.729(10^{-4}) \sqrt{\frac{6370+300}{9.824(10^{-3})}} \right) = \tan^{-1} 0.0601 = 3.44^\circ$$

3/177

$$R = 6370 \text{ km, so } r_{\min} = 6370 + 320 = 6690 \text{ km}$$

For circular orbit $e = 0$

$$\& \text{Eq. 83 gives } v_{\text{cir}} = R \sqrt{g/a} = 6370 \sqrt{\frac{9.824(10^{-3})}{6690}} = 7.72 \text{ km/s}$$

$$\text{For elliptical orbit, } v_p = 7.72 + 0.200 = 8.02 \text{ km/s}$$

From Eq. 82,

$$8.02^2 = 2(9.824)(10^{-3})(6370)^2 \left(\frac{1}{6690} - \frac{1}{2a} \right)$$

$$\text{Solve & get } 2a = r_{\min} + r_{\max} = 14530 \text{ km}$$

$$\text{so } r_{\max} = R + H = 2a - r_{\min}$$

$$\text{Hence } H = 14530 - 6690 - 6370 = 1470 \text{ km}$$

3/178

From Eq. 83 the apogee velocity at A is

$$v_a = R \sqrt{\frac{g}{a}} \sqrt{\frac{r_{\min}}{r_{\max}}} \quad \text{where } R \& g \text{ are given for Mars in Appendix C}$$

$$v_a = 5388 \sqrt{\frac{3.73(10^{-3})}{13000/2}} \sqrt{\frac{5000}{8000}} = 2.03 \text{ km/s relative to planet}$$

$$\Delta v_p = 5 - 2.03 = 2.97 \text{ km/s}$$

$$\int_0^t F_p dt = 300(2.97)(10^3) \text{ where } \int F_p dt = 500t$$

$$\text{so } t = 1783 \text{ s or } t = 29 \text{ min } 43 \text{ s}$$

3/179

$$\text{For 1, } v_1 = R \sqrt{\frac{g}{r_1}}; \text{ for 2, } v_2 = R \sqrt{\frac{g}{r_2}}$$

$$\text{For transfer ellipse at A, } v_1' = R \sqrt{\frac{g}{a}} \sqrt{\frac{r_2}{r_1}} \quad \text{where } a = \frac{r_1 + r_2}{2} \quad (\text{see Eq. 83})$$

$$\text{So at A, } \Delta v_A = v_1' - v_1 = R \sqrt{\frac{g}{a}} \sqrt{\frac{r_2}{r_1}} - R \sqrt{\frac{g}{r_1}} = R \sqrt{\frac{g}{r_1}} \left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1 \right)$$

$$\& \text{ at B, } \Delta v_B = v_2 - v_2' = R \sqrt{\frac{g}{r_2}} - R \sqrt{\frac{g}{a}} \sqrt{\frac{r_1}{r_2}} = R \sqrt{\frac{g}{r_2}} \left(1 - \sqrt{\frac{2r_1}{r_1 + r_2}} \right)$$

$$\Delta v_A = 6370 \sqrt{\frac{9.824(10^{-3})}{6870}} \left(\sqrt{\frac{2(7370)}{6870+7370}} - 1 \right) = 132.6 \text{ m/s}$$

$$\Delta v_B = 6370 \sqrt{\frac{9.824(10^{-3})}{7370}} \left(1 - \sqrt{\frac{2(6870)}{6870+7370}} \right) = 130.2 \text{ m/s}$$

3/180

For P, time $t = A$, from Eq. 78, is

$$T/2 = \pi \frac{a^{3/2}}{R \sqrt{g}} = \pi \frac{[(7195)(10^3)]^{3/2}}{6370(10^3) \sqrt{9.824}} = 3037 \text{ s}$$

$$\text{where } a = \frac{1}{2}(2[6570] + 1200 + 450) = 7195 \text{ km}$$

$$\text{Period for S would be } (3037)(\frac{7195}{7195})^{3/2} = 6556 \text{ s}$$

$$\text{Thus } \frac{180^\circ - \theta}{360^\circ} = \frac{3037}{6556}, \theta = 13.5^\circ$$

$$v_s = \frac{2\pi(7370)(10^3)}{6556} = 7255 \text{ m/s}$$

$$(v_p)_A = v_{A, \text{planet}} = R \sqrt{\frac{g}{a}} \sqrt{\frac{r_{\min}}{r_{\max}}} = 6370 \sqrt{\frac{9.824}{7195(10^3) + 7370}} \sqrt{\frac{6690}{14530}} = 7064 \text{ m/s}$$

$$\int F dt = m \Delta v; 900t = 800(7255 - 7064) \\ t = 170 \text{ s}$$

3/181

From Eq. 77 with $C = 1$ & $a = \frac{sa}{1-e^2}$, $Km_0 = gR^2$

$$\text{there results } e = \frac{n^2}{gR^2} \frac{c}{a(1-e^2)} \text{ or } e^2 = 1 - \frac{h^2}{gR^2 a}$$

$$\text{From Eq. 82 for starting conditions } \frac{1}{a} = \frac{2}{R+H} - \frac{v_o^2}{gR^2}$$

$$\text{Also } h = r^2 \dot{\theta} = r(r\dot{\theta}) = (R+H)v_o \cos \beta = \text{constant}$$

Thus

$$e = 1 - \sqrt{1 - \frac{(R+H)v_o^2 \cos^2 \beta}{gR^2} \left(2 - \frac{(R+H)v_o^2}{gR^2} \right)}$$

Prob. 3/151 or 3/58, $v_c = R\sqrt{\frac{g}{R+H}}$

Derivation of Prob. 3/153 & substitute a from Eq. 82

$r_{\max} = a(1 \pm e)$ to get

$$r = \frac{R+H}{2 - \frac{(R+H)v_0^2}{gR^2}} \left\{ 1 \pm \sqrt{1 - \frac{(R+H)v_0^2 \cos^2 \beta}{gR^2}} \left(2 - \frac{(R+H)v_0^2}{gR^2} \right) \right\}$$

Substitute $\eta = v_0/v_c$ & get

For $\beta = 0$, $r = \frac{R+H}{2 - \eta^2} \left\{ 1 \pm \sqrt{1 - \eta^2(2 - \eta^2)} \right\}$

$$= \frac{R+H}{2 - \eta^2} (1 \pm [1 - \eta^2])$$

$r_{\max} = R+H$ if $\eta < 1$ $r_{\min} = R+H$ if $\eta > 1$
 $r = (R+H) \frac{\eta^2}{2 - \eta^2}$ if $\eta > 1$ $r_{\min} = (R+H) \frac{\eta^2}{2 - \eta^2}$ if $\eta < 1$

For $\eta = 1$, $r_{\max} = (R+H)(1 \pm \sin \beta)$
 r_{\min}

3/153 Let T_F = period for fixed sun

T_M = " " moving "

From Eq. 84

$$\frac{T_F}{T_M} = \frac{\sqrt{m_s + m}}{\sqrt{m_s}} = \sqrt{1 + \frac{m}{m_s}} = \sqrt{1 + \frac{1}{333000}}$$

$$= (1 + 3.003003(10^{-6}))^{1/2}$$

$$= 1 + \frac{1}{2}(3.003003)(10^{-6}) - \frac{1}{8}(9.218)(10^{-12}) + \dots$$

$$= 1.000001502 \dots$$

Error is $1.502(10^{-6})(365.26)(23.9344)(60)^2$

$$= 47.3 \text{ s/y}$$

Thus uncorrected period is 47 s longer
 than corrected period

24 Let T_F = period for fixed earth

T_M = " " moving "

From Eq. 84

$$\frac{T_F}{T_M} = \frac{\sqrt{m_0 + m}}{\sqrt{m_0}} = \sqrt{1 + \frac{m}{m_0}} = \sqrt{1 + 0.0123}$$

$$= (1 + 0.0123)^{1/2} = 1 + \frac{0.0123}{2} - \frac{0.0123^2}{8} + \dots$$

$$= 1 + 0.00615 - 0.000189 + \dots$$

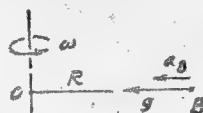
$$= 1.00613$$

Error is $0.00613(27.32)(23.9344) = 4.0090 \text{ h}$
 or 4 h 0 min 33 s too great

3/186

O = center of earth, B = point on equator

From Eq. 89, $g' = g - a_B$



$$a_B = R\omega^2$$

$$= 6378(10^3)(0.7292)(10^{-4})^2$$

$$= 0.0339 \text{ m/s}^2$$

Equatorial radius of earth (Appen. C) is $R = 12755/2 = 6378 \text{ km}$

$$\omega = 0.7292(10^{-4}) \text{ rad/s}$$

$$g' = 9.815 - 0.0339 = 9.781 \text{ m/s}^2$$

3/187

Relative to carrier, $U_r = AT_r$

$$(22 + P)75 = \frac{1}{2}3([240/3.6]^2 - 0)$$

$$22 + P = 88.9 \text{ kN}, \quad P = 66.9 \text{ kN}$$

3/188

Relative to the constant-velocity reference of B, the motion of A is

governed by $F_t = m\Delta v$

$$F = 5 \frac{1}{3}(16/3.6) = 4.44 \text{ kN}$$

$$G_r = m\dot{v}_r = 5(16/3.6) = 22.2 \text{ kN/s or } 22.2(10^3) \text{ kgm/s}$$

3/189

Let B = pt. on plate coincident with slider. $a_B = 8 \text{ m/s}^2$

From Eq. 95, $\Sigma F = m(a_B + a_{\text{rel}})$

In n-dir.

$$2(9.81) \cos 30^\circ - N_c = 2(8 \sin 30^\circ)$$

$$N_c = 8.99 \text{ N}$$

In t-dir, $2(9.81) \sin 30^\circ = 2(a_{\text{rel}} - 8 \cos 30^\circ)$

$$a_{\text{rel}} = 11.85 \text{ m/s}^2$$

3/190

x-y attached to the disk. $\omega = 20 \text{ k}$

$$\dot{\omega} = 0, \quad \dot{r} = 0, \quad a_B = a_O = 0, \quad v_{\text{rel}} = 0.9 \text{ j m/s}$$

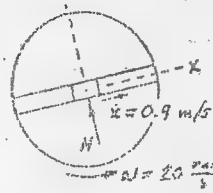
$$a_{\text{rel}} = \ddot{x} \text{ j}, \quad \text{so Eq. 86 becomes}$$

$$\Sigma F = m(2\omega \times v_{\text{rel}} + a_{\text{rel}})$$

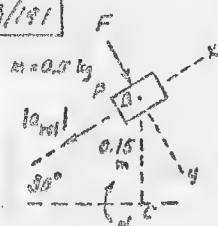
$$N_j = 0.5(2(20)(0.9)) + \ddot{x} \text{ j}$$

so $\ddot{x} = 0$ if spring force = kx , &

$$N = 0.5(36) = 18 \text{ N}$$



3/191

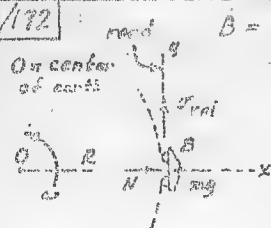
With $\omega = 0$, $\dot{\omega} = 0$, $\Gamma = 0$, $\mathbf{v}_{rel} = 0$ Eq. 86 gives $\Sigma \mathbf{F} = m(\mathbf{a}_B + \mathbf{a}_{rel})$ where $\mathbf{a}_B = 0.15(40)(i \cos 30^\circ + j \sin 30^\circ)$
 $= 3.0(i\sqrt{3} + j) \text{ m/s}^2$ $\mathbf{a}_{rel} = i\ddot{x}$, $\Sigma \mathbf{F} = jF$

Thus

$$jF = 0.5(3.0[i\sqrt{3} + j] + i\ddot{x})$$

from which $F = 0.5(3) = 1.5 \text{ N}$
 $\ddot{x} = \ddot{x} = -3.0/\sqrt{3} = -5.20 \text{ m/s}^2$; $|\mathbf{a}_{rel}| = 5.20 \text{ m/s}^2$,
 direction shown

3/192

B = origin of coordinates
attached to earthEq. 86 with $\Gamma = 0$, $\dot{\omega} = 0$, $\mathbf{a}_{rel} = 0$ is $\Sigma \mathbf{F} = m(\mathbf{a}_B + 2\boldsymbol{\omega} \times \mathbf{v}_{rel})$ $\mathbf{a}_B = R\omega^2$

$$= \frac{12.756}{R} (0.7292)^2 (10^{-4})^2$$

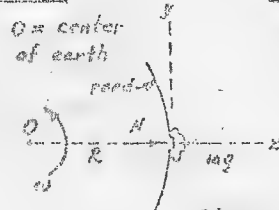
$$= 33.9 (10^{-6}) \text{ km/s}^2 = 0.0339 \text{ m/s}^2$$

$$2\boldsymbol{\omega} \times \mathbf{v}_{rel} = 2(0.7292)(10^{-4})(150/3.6)(-j) = -0.00608 j \text{ m/s}^2$$

$$\text{So in x-dir, } N - 1500(9.815) = 1500(-0.0339 - 0.00608)$$

$$N = 14,560 \text{ N}$$

3/193

For $\Gamma = 0$, $\dot{\omega} = 0$, Eq. 86 becomes

$$\Sigma \mathbf{F} = m(\mathbf{a}_B + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel})$$

where $\mathbf{a}_{rel} = \mathbf{v}_{rel}^2 / R$

$$= (150/3.6)^2 / (6378 \times 10^3)$$

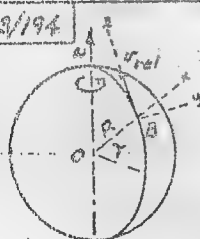
$$= 0.000272 \text{ m/s}^2 \text{ (-x-dir)}$$

Change from straight road (Prob. 3/192)

to curved road produces increment

$$\Delta N = 1500(-0.000272) = -0.408 \text{ N}$$

3/194

Attach axes to earth with z north
and y east. Terms in Eq. 86in y-dir. (normal to rails)
are

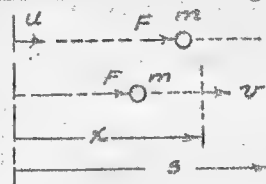
$$2\boldsymbol{\omega} \times \mathbf{v}_{rel} = 2\omega v_{rel} \sin \gamma (-j)$$

 $(\Sigma \mathbf{F} = m\mathbf{a})_y$ gives

$$T = 2m\omega v_{rel} \sin \gamma \text{ (west dir.)}$$

$$\text{so } T = 2(1500)(0.7292)(10^{-4})(2100/3.6) \sin 30^\circ = 72.9 \text{ N}$$

3/195

During launch plane goes distance
s for moving carrier and
distance x for
stationary carrierFor moving carrier,
relative to land $dU = d$

$$Fds = m\dot{s}ds = m\dot{s}d$$

$$\int_0^s Fds = \int_0^x Fds + \int_x^s Fds = \int_0^x m\dot{s}d + \int_x^s m\dot{s}d$$

$$Fx + F(s-x) = \frac{1}{2}mv^2 + muv$$

For stationary carrier $Fx = \frac{1}{2}mv^2$ Subtract & get $F(s-x) = muv$, sodifference between two cases is due to
initial kinetic energy of plane $\frac{1}{2}mv^2$ as
work done muu during added displacement
for moving carrier.

3/196

$$a_B = R\omega^2 \cos \gamma$$

$$I + m\ddot{y} = m\ddot{y}_B$$

From geometry of figure
 $\frac{R}{\sin \gamma} = \frac{m\ddot{y}_B}{\sin \theta}$, $\sin \theta = \frac{mR\omega^2 \cos \gamma}{2T}$

$$T \cos \theta + mR\omega^2 \cos^2 \gamma = mg$$

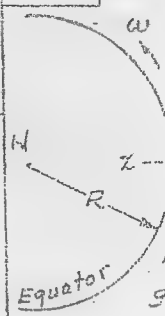
Combine & get

$$\tan \theta = \frac{R\omega^2 \sin \gamma}{2g - \frac{R\omega^2 \cos^2 \gamma}{\sin \theta}}$$

$$R\omega^2 = 0.0339 \text{ m/s}^2$$

$$\text{so } \tan \theta = \frac{0.0339}{2(9.82)} \frac{0.866}{1 - \frac{0.0339}{9.821} \frac{3}{4}} = 0.001499, \theta = 5^\circ$$

3/197

Attach x-y to tube at position of ball
at instant considered

$$\mathbf{r} = i\mathbf{x} = 0, \dot{\omega} = 0, \boldsymbol{\omega} \times \mathbf{r} = 0, \mathbf{a}_B = iR\omega^2$$

$$\mathbf{v}_{rel} = i\dot{x}, \mathbf{a}_{rel} = i\ddot{x}, \Sigma \mathbf{F} = i\mathbf{mg} - N\mathbf{j}$$

$$\boldsymbol{\omega} = -\omega \mathbf{k}, \text{ so Eq. 86 becomes}$$

$$i\mathbf{mg} - N\mathbf{j} = m(iR\omega^2 - 2\omega \dot{x} i + i\ddot{x})$$

$$= m(iR\omega^2 - 2\omega \dot{x} i + i\ddot{x})$$

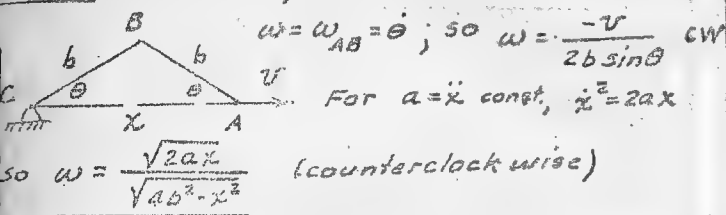
Equate i & j components & get

$$g = R\omega^2 + \ddot{x}, \ddot{x} = g - R\omega^2 \text{ so } \ddot{x} = (g - R\omega^2)$$

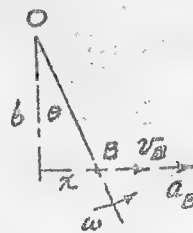
$$N = m(2\omega \dot{x}) = 2m\omega (g - R\omega^2)$$

 $g \gg R\omega^2$ so contact is on east side of tube
as shown

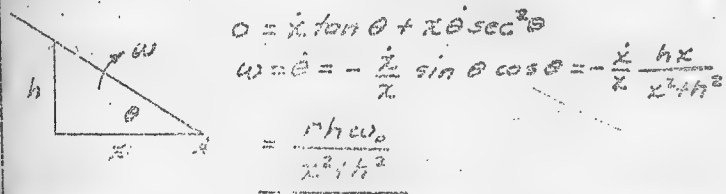
5/8 $x = 2b \cos \theta$, $\dot{x} = -2b\dot{\theta} \sin \theta$, $v = \dot{x}$



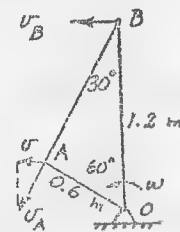
5/13 $x = b \tan \theta$; $v_B = \dot{x} = b\dot{\theta} \sec^2 \theta$



5/9 $\dot{x} = -v_A = -r\omega$, $h = x \tan \theta$



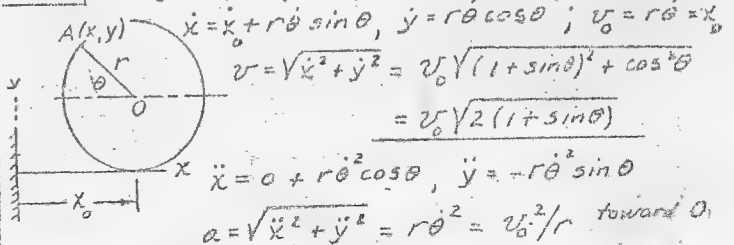
5/14 Given, $v = 0.15$ m/s,



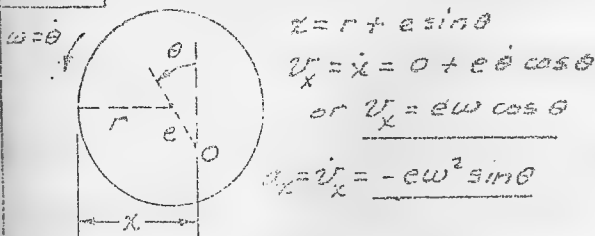
5/10 $v = r\omega$, $\dot{v} = 0 = r\dot{\omega} + \dot{r}\omega$



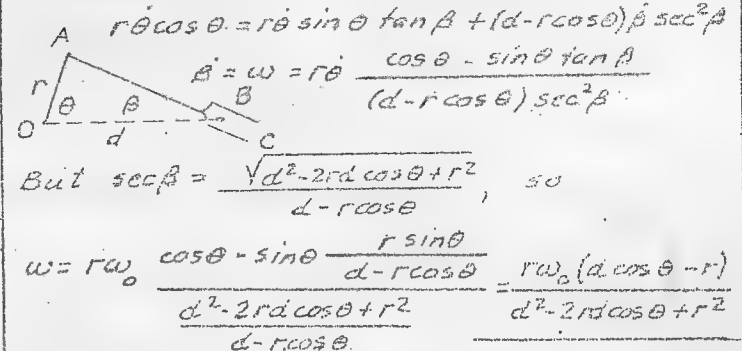
5/15 $x = x_0 - r \cos \theta$, $y = r(1 + \sin \theta)$



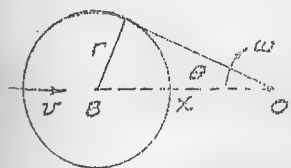
5/11 Displacement of fork of rod is



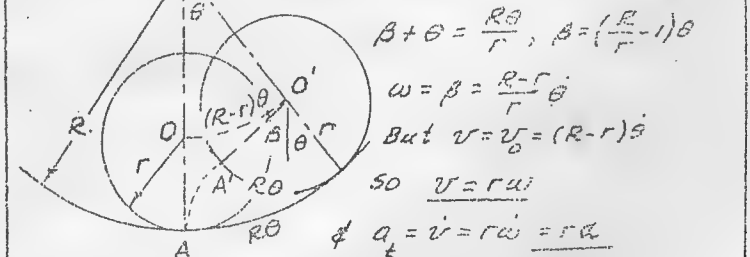
5/16 $\dot{\theta} = \omega_0$ constant; $r \sin \theta = (d - r \cos \theta) \tan \beta$



5/12 $v_B = v = -\dot{x}$



5/17 β = absolute angle through which wheel turns



5/18

$x = 200 \tan \theta$, $v_A = \dot{x} = 200 \dot{\theta} \sec^2 \theta$
 $a_A = \ddot{x} = 200 \ddot{\theta} \sec^2 \theta + 400 \dot{\theta}^2 \sec^2 \theta \tan \theta$
 $= 200 \ddot{\theta} \sec^2 \theta + 400 \frac{v_A^2}{200^2 \sec^4 \theta} \tan \theta$
 $= 200 \ddot{\theta} \sec^2 \theta + \frac{v_A^2}{100} \sin \theta \cos \theta$
 But for $\ddot{x} = \text{const } a_A$, $v_A^2 = 2a_A x$. Thus
 $a_A = 200 \ddot{\theta} \sec^2 \theta + \frac{a_A x}{50} \sin \theta \cos \theta$
 Substitute $a_A = 100 \text{ mm/s}^2$, $\sec \theta = \frac{5}{4}$, $\cos \theta = \frac{4}{5}$,
 $\sin \theta = \frac{3}{5}$ for $x = 150 \text{ mm}$
 $100 = 200 \ddot{\theta} \left(\frac{5}{4}\right)^2 + \frac{(100)(150)}{50} \frac{3}{5} \frac{4}{5}$, $\alpha = \ddot{\theta} = -0.1408 \text{ rad/s}^2$
 or $\alpha = 0.1408 \text{ rad/s}^2 \text{ CCW}$

5/19

$l \sin \beta = r \sin \theta$, $l \dot{\beta} \cos \beta = r \dot{\theta} \cos \theta$
 so $\omega = \dot{\beta} = \frac{r \dot{\theta} \cos \theta}{l \cos \beta} = \frac{r \omega_{\theta} \cos \theta}{l \sqrt{1 - \left(\frac{r}{l} \sin \theta\right)^2}}$
 $l \ddot{\beta} \cos \beta - l \dot{\beta}^2 \sin \beta = -r \dot{\theta}^2 \sin \theta$, $\ddot{\theta} = \dot{\omega}_{\theta} = 0$
 $\alpha = \ddot{\beta} = \frac{l \dot{\beta}^2 \sin \beta - r \dot{\theta}^2 \sin \theta}{l \cos \beta} = \frac{r \omega_{\theta}^2 \sin \theta}{l \left(1 - \frac{r^2}{l^2} \sin^2 \theta\right)^{3/2}}$

5/23

Length of AB is invariant, so A must have component of velocity along bar of $10 \cos 45^\circ$. Thus
 $(v_A)_{\min} = 10 \cos 45^\circ = 7.07 \text{ m/s}$
 $\omega_{AB} = v_{B/A} / AB = \frac{10/\sqrt{2}}{1.2} = 5.89 \text{ rad/s CCW}$

5/24

$v_A = 2 \text{ m/s}$
 $\frac{2}{0.1 - b} = \frac{4}{0.1 + b}$, $b = \frac{0.1}{3} \text{ m}$
 $v_D = \frac{CD}{CA} v_A = \frac{\sqrt{(0.1/3)^2 + 0.1^2}}{0.1 - (0.1/3)} \cdot 2$
 $= \sqrt{10} = 3.16 \text{ m/s}$

3/25

$OE = \frac{1}{3} \sqrt{3} \text{ m}$
 $OC = \sqrt{3^2 + (\sqrt{3}/3)^2} = 2\sqrt{7/3}$
 $= 3.06 \text{ m}$
 $v_O = OC \omega = 3.06(2)$
 $= 6.11 \text{ m/s}$

5/26

$\omega = v_{A/B} / AB = \frac{0.08/\sqrt{2}}{0.15} = 0.377 \text{ rad/s}$
 $v_A = 0.08 \text{ m/s}$
 $a_B = a_A + (a_{B/A})_n + (a_{B/A})_t$
 $a_{B/A} j = 0 + AB \omega^2 j + AB \alpha i$
 $a_B = 0.15 (0.377)^2 = 0.0213 \text{ m/s}^2$
 $= 21.3 \text{ mm/s}^2$

5/27

$(a_{C/B})_t = CB \alpha_{CB}$
 $a_{C/B} = 16 \text{ m/s}^2$
 $\alpha_{CB} = \frac{16(\sqrt{5}/2)}{2} = 6.93 \text{ rad/s}^2 \text{ CW}$
 $(a_{C/B})_n = CB \omega_{CB}^2$
 $\omega_{CB} = \omega_{CA} = \omega_{CB} = \frac{\sqrt{16(0.5)^2}}{2} = 2 \text{ rad/s}$
 CW or CCW

5/28

$C = \text{instantaneous velocity of ABD}$
 $v_F = 2 \text{ m/s}$, $v_D = \frac{2}{\sqrt{5}/2} \text{ m/s}$
 $\omega_{AD} = \omega_{CD} = \frac{v_D}{CD} = \frac{2}{\sqrt{5}/2 \cdot 2(0.1\sqrt{3}/2)} = 11.55 \text{ rad/s}$
 $v_A = AC \omega_{AC} = AC \omega_{CD}$
 $= 0.2 \frac{\sqrt{5}}{2} 11.55 = 2.31 \text{ m/s}$

5/29

$C_1 = \text{instantaneous center for CB}$
 $C_2 = \text{instantaneous center for AB}$
 $v_D = v_C$, $C_2 B = 0.1732 \text{ m}$
 $\omega_{AB} = v_D / C_2 B = \frac{0.2}{0.1732}$
 $\omega_{AB} = 1.155 \text{ rad/s CCW}$

5/30

$a_O = v_O^2 / (r+R)$; $a_{C/O} = r \omega^2 = r \left(\frac{v_O}{r}\right)^2$
 $a_C = a_O + a_{C/O}$
 $a_C = \frac{v_O^2}{r} - \frac{v_O^2}{r+R} = \frac{R v_O^2}{r(r+R)}$

5/18

$$x = 200 \tan \theta, v_A = \dot{x} = 200 \dot{\theta} \sec^2 \theta$$

$$a_A = \ddot{x} = 200 \ddot{\theta} \sec^2 \theta + 400 \dot{\theta}^2 \sec^2 \theta \tan \theta$$

$$= 200 \ddot{\theta} \sec^2 \theta + 400 \frac{v_A^2}{200^2 \sec^4 \theta} \tan \theta$$

$$= 200 \ddot{\theta} \sec^2 \theta + \frac{v_A^2}{100} \sin \theta \cos \theta$$

But for $\ddot{x} = \text{const}$, $v_A^2 = 2a_A x$. Thus.

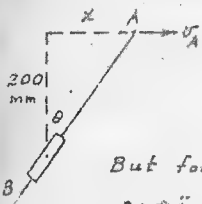
$$a_A = 200 \ddot{\theta} \sec^2 \theta + \frac{a_A x}{50} \sin \theta \cos \theta$$

$$\text{Substitute } a_A = 100 \text{ mm/s}^2, \sec \theta = \frac{5}{4}, \cos \theta = \frac{4}{5},$$

$$\sin \theta = \frac{3}{5} \text{ for } x = 150 \text{ mm}$$

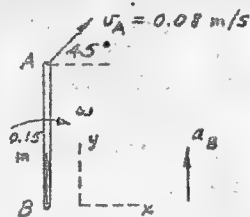
$$100 = 200 \ddot{\theta} \left(\frac{5}{4}\right)^2 + \frac{(100)(150)}{50} \frac{3}{5} \frac{4}{5}, \alpha = \ddot{\theta} = -0.1408 \text{ rad/s}^2$$

$$\text{or } \alpha = 0.1408 \text{ rad/s}^2 \text{ CCW}$$



5/26

$$\omega = v_{A/B} / AB = \frac{0.08 / \sqrt{2}}{0.15} = 0.377 \text{ rad/s}$$



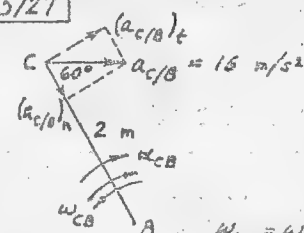
$$a_B = a_A + (a_{B/A})_n + (a_{B/A})_t$$

$$a_B j = 0 + AB \omega^2 j + AB \alpha i$$

$$a_B = 0.15 (0.377)^2 = 0.0218 \frac{\text{m}}{\text{s}^2}$$

$$= 21.3 \text{ mm/s}^2$$

5/27



$$(a_{C/B})_t = CB \alpha_{CB}$$

$$\alpha_{CA} = \alpha_{CB} = \frac{16 (\sqrt{3}/2)}{2}$$

$$= 6.93 \text{ rad/s}^2 \text{ CW}$$

$$(a_{C/B})_n = CB \omega_{CB}^2$$

$$\omega_{CA} = \omega_{CB} = \sqrt{16(0.5)/2} = 2 \text{ rad/s}$$

$$\text{CW or CCW}$$

5/19

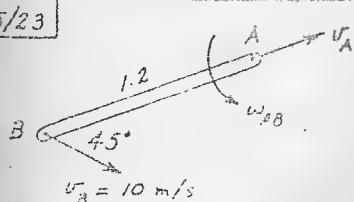
$$l \sin \beta = r \sin \theta, l \dot{\beta} \cos \beta = r \dot{\theta} \cos \theta$$

$$\text{So } \omega = \dot{\beta} = \frac{r \dot{\theta} \cos \theta}{l \cos \beta} = \frac{r \omega_{\theta} \cos \theta}{l \sqrt{1 - (r/l \sin \theta)^2}}$$

$$l \dot{\beta} \cos \beta - l \dot{\beta}^2 \sin \beta = -r \dot{\theta}^2 \sin \theta, \ddot{\theta} = \omega_{\theta} = 0$$

$$\alpha = \dot{\beta} = \frac{l \dot{\beta}^2 \sin \beta - r \dot{\theta}^2 \sin \theta}{l \cos \beta} = \frac{r \omega_{\theta}^2 \sin \theta}{l (1 - r^2 \sin^2 \theta / l^2)^{3/2}}$$

5/23

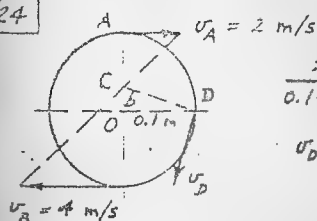


Length of AB is invariant, so A must have component of velocity along bar of $10 \cos 45^\circ$. Thus

$$(v_A)_{\min} = 10 \cos 45^\circ = 7.07 \text{ m/s}$$

$$\omega_{AB} = v_{B/A} / AB = \frac{10/\sqrt{2}}{1.2} = 5.89 \text{ rad/s CCW}$$

5/24

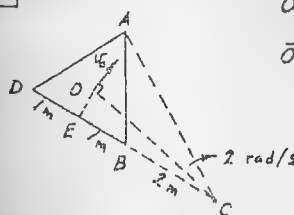


$$\frac{2}{0.1 - b} = \frac{4}{0.1 + b}, b = \frac{0.1}{3} \text{ m}$$

$$v_D = CD \frac{v_A}{CA} = \frac{\sqrt{(0.1/3)^2 + (0.1)^2}}{0.1 - (0.1/3)} 2$$

$$= \sqrt{10} = 3.16 \text{ m/s}$$

5/25



$$OE = \frac{1}{3} \sqrt{3} \text{ m}$$

$$OC = \sqrt{3^2 + (\sqrt{3}/3)^2} = 2\sqrt{7/3}$$

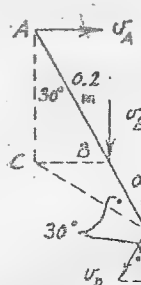
$$= 3.06 \text{ m}$$

$$v_O = OC \omega = 3.06 (2)$$

$$= 6.11 \text{ m/s}$$

5/28

C = instantaneous velocity of ABD



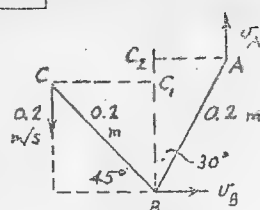
$$v_F = 2 \text{ m/s}, v_D = \frac{2}{\sqrt{3}/2} \text{ m/s}$$

$$\omega_{AD} = \omega_{CD} = \frac{v_D}{CD} = \frac{2}{\sqrt{3}/2} \frac{1}{2(0.1 + \sqrt{3}/2)} = 13.33 \frac{\text{rad/s}}{\text{CW}}$$

$$v_A = AC \omega_{AC} = AC \omega_{CD}$$

$$= 0.2 \frac{\sqrt{3}}{2} 13.33 = 2.31 \text{ m/s}$$

5/29



C₁ = instantaneous center for CB

C₂ = instantaneous center for AB

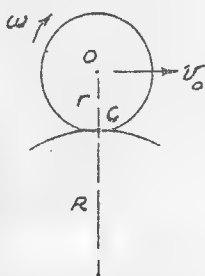
$$v_B = v_C, C_2 B = 0.1732 \text{ m}$$

$$\omega_{AB} = v_B / C_2 B = \frac{0.2}{0.1732}$$

$$\omega_{AB} = 1.155 \text{ rad/s CCW}$$

5/30

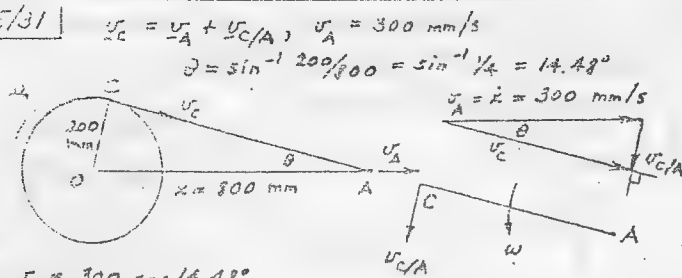
$$a_O = v_O^2 / (r + R) \downarrow; a_{C/O} = r \omega^2 = r \left(\frac{v_O}{r}\right)^2 = \frac{v_O^2}{r} \uparrow$$



$$a_C = a_O + a_{C/O}$$

$$a_C = \frac{v_O^2}{r} - \frac{v_O^2}{r + R} = \frac{R v_O^2}{r(r + R)} \text{ up}$$

5/31



$$v_C = 300 \cos 14.48^\circ$$

$$= 300(0.9682) = 290 \text{ mm/s}$$

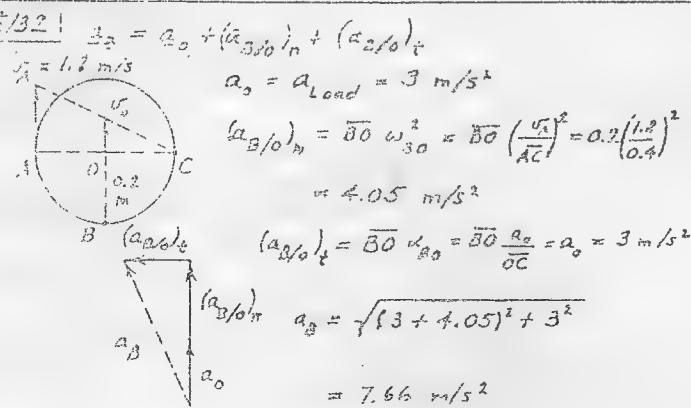
$$v_{CA} = 300 \sin 14.49^\circ = 300(1/4) = 75 \text{ mm/s}$$

$$\overline{CA} = 800 \cos 14.49^\circ = 775 \text{ mm}$$

$$\omega = v_{CA}/\overline{CA} = 75/775 = 0.0968 \text{ rad/s CCW}$$

$$\omega_3 = v_C/\overline{CO} = 290/200 = 1.452 \text{ rad/s CW}$$

5/32



$$a_3 = a_{\text{load}} = 3 \text{ m/s}^2$$

$$(a_{B/O})_n = \overline{BO} \omega_3^2 = \overline{BO} \left(\frac{v_A}{\overline{AC}} \right)^2 = 0.2 \left(\frac{1.2}{0.4} \right)^2$$

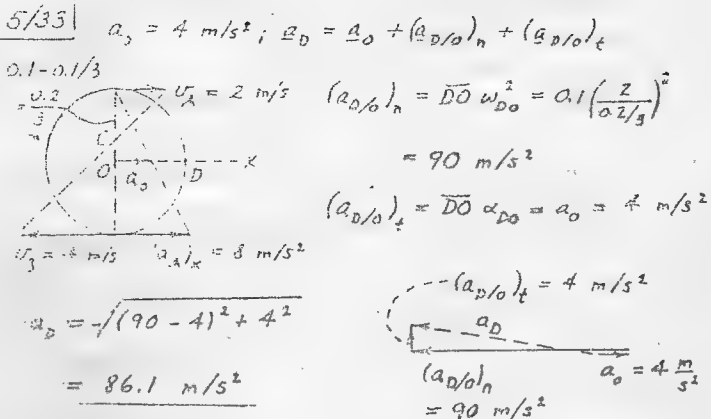
$$= 4.05 \text{ m/s}^2$$

$$(a_{B/O})_t = \overline{BO} \alpha_3 = \overline{BO} \frac{a_3}{\overline{OC}} = a_3 = 3 \text{ m/s}^2$$

$$a_B = \sqrt{(3 + 4.05)^2 + 3^2}$$

$$= 7.66 \text{ m/s}^2$$

5/33



$$a_3 = 4 \text{ m/s}^2; a_D = a_0 + (a_{D/O})_n + (a_{D/O})_t$$

0.1-0.1/3

$$(a_{D/O})_n = \overline{DO} \omega_{D/O}^2 = 0.1 \left(\frac{2}{0.2/3} \right)^2$$

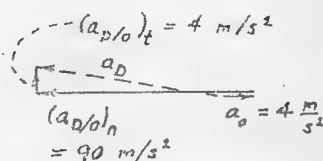
$$= 90 \text{ m/s}^2$$

$$(a_{D/O})_t = \overline{DO} \alpha_{D/O} = a_0 = 4 \text{ m/s}^2$$

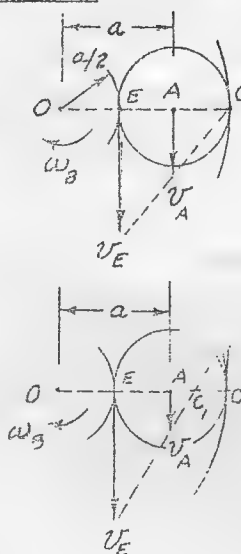
$$v_3 = 1 \text{ m/s} \quad (a_A)_n = 8 \text{ m/s}^2$$

$$a_D = \sqrt{(90 + 4)^2 + 4^2}$$

$$= 86.1 \text{ m/s}^2$$



5/34



$$(a) \quad v_A = \omega_0 a$$

$$v_E = 2v_A = 2a\omega_0$$

$$\omega_B = \frac{v_E}{a/2} = \frac{2a\omega_0}{a/2} = 4\omega_0 = 4(90) = 360 \text{ rad/s}$$

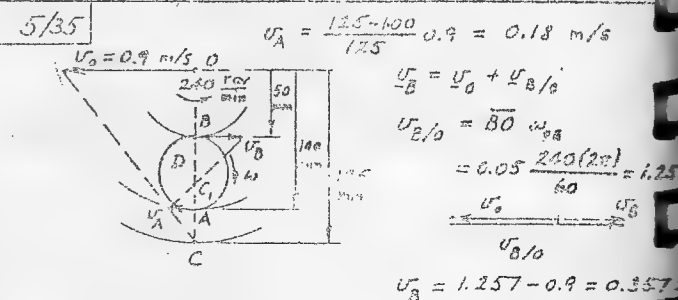
$$\overline{OC} \omega_B = \frac{3a}{2} \cdot 360 = 1620 \text{ mm/s}$$

$$v_A = \overline{OA} \omega_{CA} = 90a$$

$$\frac{v_A + v_D}{a} = \frac{v_E - v_A}{a}, \quad v_E = v_D + v_A = 300a$$

$$\omega_{DS} = \omega_B = \frac{v_E}{a/2} = \frac{300a}{a/2} = 600 \text{ rad/s}$$

5/35



$$v_A = \frac{125 - 100}{125} \cdot 0.9 = 0.18 \text{ m/s}$$

$$v_B = v_A + v_{B/A}$$

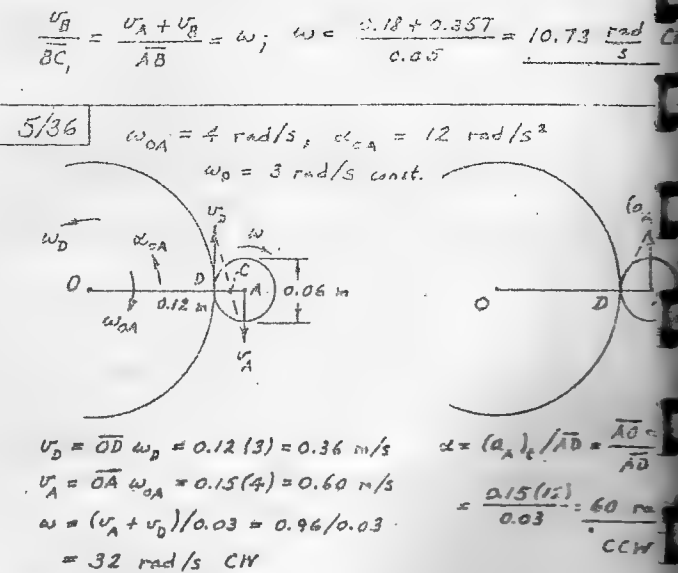
$$v_{B/A} = \overline{BO} \omega_{B/A}$$

$$= 0.05 \frac{240(2\pi)}{60} = 1.257$$

$$v_{B/A}$$

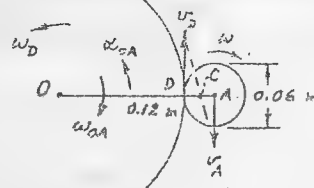
$$v_B = 1.257 - 0.9 = 0.357 \text{ m/s}$$

5/36



$$\omega_{0A} = 4 \text{ rad/s}, \quad \alpha_{0A} = 12 \text{ rad/s}^2$$

$$\omega_0 = 3 \text{ rad/s const.}$$



$$v_D = \overline{OD} \omega_D = 0.12(3) = 0.36 \text{ m/s}$$

$$v_A = \overline{OA} \omega_{0A} = 0.15(4) = 0.60 \text{ m/s}$$

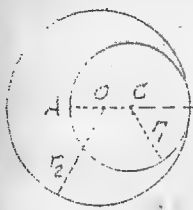
$$\omega = (v_A + v_D)/0.03 = 0.96/0.03$$

$$= 32 \text{ rad/s CCW}$$

$$\alpha = (a_A)_t / \overline{AD} = \frac{\overline{AD}}{\overline{AD}}$$

$$= \frac{0.15(12)}{0.03} = 60 \text{ rad/s CCW}$$

5/37 $v_C = (r_2 - r_1) \frac{2\pi}{T}$, $a_C = \frac{v_C^2}{r_2 - r_1} = (r_2 - r_1) \left(\frac{2\pi}{T} \right)^2$ C is O

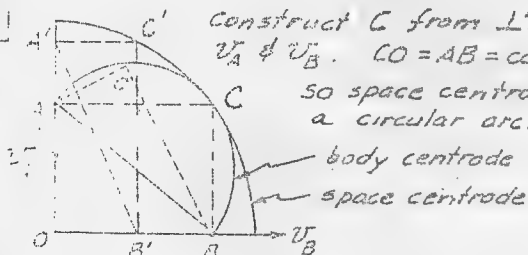


$a_{A/C} = AC \omega^2 = r_1 \left(\frac{v_C}{r_2 - r_1} \right)^2 = (r_2 - r_1) \left(\frac{2\pi}{T} \right)^2 \frac{r_1}{r_2}$ A to C

$a_A = a_C + a_{A/C}$

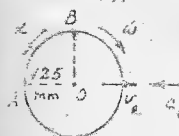
$a_A = (r_2 - r_1) \left(\frac{2\pi}{T} \right)^2 \left(1 - \frac{r_2 - r_1}{r_2} \right)$
 $= (r_2 - r_1) \left(2 - \frac{r_2}{r_1} \right) \left(\frac{2\pi}{T} \right)^2$

5/38 Construct C from \perp 's to v_A & v_B . $CO = AB = \text{const.}$ so space centre is a circular arc.



For position $A'B'$, locate C' & transfer to position AB of bar. Point C' falls on circular arc with AB as its diameter. This is body centre which is seen to roll on space centre.

5/39 $a_A = a_o + (a_{A/o})_n + (a_{A/o})_t$



$(a_{A/o})_n = AO \omega^2 = 0.125 \left(\frac{0.5}{0.125} \right)^2 = 2 \text{ m/s}^2$

$(a_{A/o})_t = AO \alpha = 0.125 \left(\frac{1.5}{0.125} \right) = 1.5 \text{ m/s}^2$

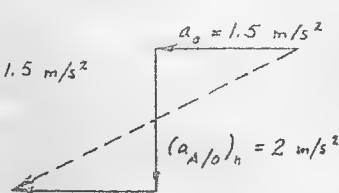
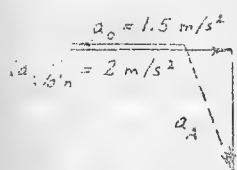
$a_B = a_o + (a_{B/o})_n + (a_{B/o})_t$

$v_o = 0.5 \text{ m/s}$

$(a_{B/o})_n = BO \omega^2 = 2 \text{ m/s}^2$

$v_o = 1.5 \text{ m/s}^2$

$(a_{B/o})_t = BO \alpha = 1.5 \text{ m/s}^2$

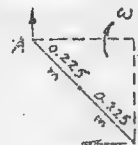


$a_A = \sqrt{(2 - 1.5)^2 + (1.5)^2}$
 $= 1.581 \text{ m/s}^2$

$(a_{B/o})_t = 1.5 \text{ m/s}^2$

$a_B = \sqrt{(1.5 + 1.5)^2 + 2^2} = 3.61 \text{ m/s}^2$

5/40 $\omega_{AB} = \omega_{BC} = \omega = \frac{v_B}{CB} = \frac{1.8}{0.225\sqrt{2}} = 5.66 \text{ rad/s}$



$a_A = a_B + (a_{A/B})_n + (a_{A/B})_t$

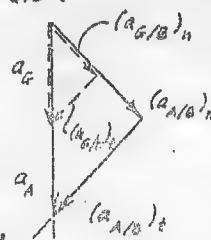
$a_B = 0$, $(a_{A/B})_n = AB \omega^2 = 0.45 \cdot 5.66^2$
 $= 14.4 \text{ m/s}^2$

$a_G = a_B + (a_{G/B})_n + (a_{G/B})_t$

$(a_{G/B})_n = GB \omega^2 = 7.2 \text{ m/s}^2$

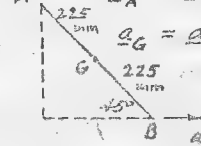
$(a_{G/B})_t = GB \alpha_{AB} = \frac{GB}{AB} (a_{A/B})_t$
 $= \frac{1}{2} (a_{A/B})_t$

From diag., $a_G = \frac{1}{2} a_A = 7.2\sqrt{2}$
 $= 10.18 \text{ m/s}^2$

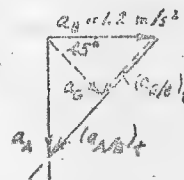


5/41 $v_B = 0$ so $v_A = 0$ & $\omega_{AB} = 0$

$a_A = a_B + (a_{A/B})_n + (a_{A/B})_t$, $(a_{A/B})_n = AB \omega^2 = 0$

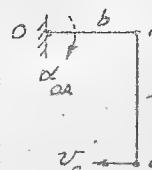


$a_G = a_B + (a_{G/B})_n + (a_{G/B})_t$
 $(a_{G/B})_t = GB \alpha = \frac{1}{2} (a_{A/B})_t$
 $a_B = 1.2 \text{ m/s}^2$



From diagram, $a_G = \frac{1.2}{\sqrt{2}} = 0.849 \text{ m/s}^2$

5/42 $a_A = a_B + (a_{A/B})_n + (a_{A/B})_t$; $\omega_{AB} = \frac{v_B}{l}$

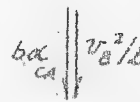


$(a_A)_n = 0$; $(a_A)_t = b \alpha_{OA}$, $a_B = 0$

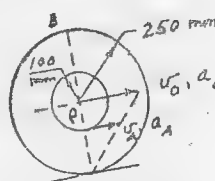
$(a_{A/B})_n = l \left(\frac{v_B}{l} \right)^2 = \frac{v_B^2}{l}$

So $b \alpha_{CA} = \frac{v_B^2}{l}$

$\alpha_{CA} = \frac{v_B^2}{bl}$ CW



5/43 $v_o^2 = 2a_o s$, $a_o = \frac{(1.2)^2}{2(2.4)} = 0.30 \text{ m/s}^2$

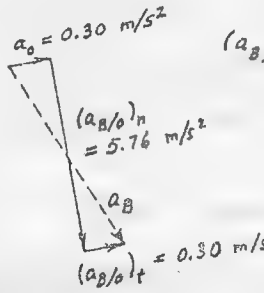


$a_A = \frac{250 - 100}{250} (0.30) = 0.18 \text{ m/s}^2$

$a_B = a_o + (a_{B/o})_n + (a_{B/o})_t$

$(a_{B/o})_n = BO \omega^2 = 0.25 \left(\frac{1.2}{0.25} \right)^2 = 5.76 \text{ m/s}^2$

$(a_{B/o})_t = BO \alpha = 0.25 \left(\frac{0.30}{0.25} \right) = 0.30 \text{ m/s}^2$



$a_B = \sqrt{(0.30 + 0.30)^2 + (5.76)^2}$
 $= 5.79 \text{ m/s}^2$

5/44

$$a_B = a_A + (a_{B/A})_n + (a_{B/A})_t$$

$$a_A = 0$$

$$(a_B)_n = \frac{v_B^2}{BO} = \frac{(80/\sqrt{2})^2}{80} = 80 \text{ mm/s}^2$$

$$(a_B)_t = 40 \text{ mm/s}^2$$

$$(a_{B/A})_n = \overline{AB} \omega_{AB}^2 = 150 \omega_{AB}^2$$

$$(a_{B/A})_t = \overline{AB} \omega_{AB} = 150 (0.377) = 56.55 \text{ mm/s}^2$$

Substitute & get

$$0 = 21.3 \text{ j} + 150 \omega_{AB} \text{ i} \text{ giving}$$

$$\omega_{AB} = \frac{-0}{150} = -0.267 \text{ rad/s}^2 \text{ CCW}$$

$$\omega_{AB} = 0.267 \text{ rad/s}^2 \text{ CW}$$

$$a_B = 21.3 \text{ j} + 150 (0.267)^2 \text{ i} = 45.5 \text{ mm/s}^2$$

5/45

$$\omega_A = 20 \text{ rad/s} = \omega_{BC}$$

$$a_A = 100 \text{ rad/s}^2 = a_{BC}$$

$$v_C = v_B + v_{C/B}$$

$$v = v_C = v_{C/B} / \sqrt{2}$$

$$a_C = \frac{\overline{BC} \omega_{BC}^2}{\sqrt{2}} = \frac{0.3(20)^2}{\sqrt{2}} = 4.24 \text{ m/s}^2$$

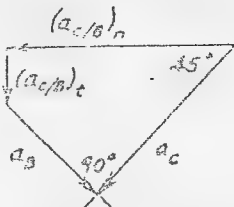
$$a_C = a_B + (a_{C/B})_n + (a_{C/B})_t$$

$$(a_{C/B})_n = \overline{CB} \omega_{CB}^2 = 0.3(20)^2 = 120 \text{ m/s}^2$$

$$(a_{C/B})_t = \overline{CB} \alpha_{CB} = 0.3(100) = 30 \text{ m/s}^2$$

$$a_C = 120/\sqrt{2} + 30/\sqrt{2}$$

$$a_C = \frac{150}{\sqrt{2}} = 106.1 \text{ m/s}^2$$



5/46

$$v_{C/O} = v_{C/B} = \overline{BC} \omega_{BC} = 0.15 \frac{150(2\pi)}{60} = 2.36 \text{ m/s}$$

$$\omega_{C/O} = v_{C/O} / \overline{CO} = 2.36 / 0.375 = 6.28 \text{ rad/s}$$

$$v_O = r\omega = |v_{C/O}| = 2.36 \text{ m/s}$$

$$r = \text{const. accel.}, v_O^2 = 2a_O s, a_O = \frac{(2.36)^2}{2(6)} = 0.463 \text{ m/s}^2$$

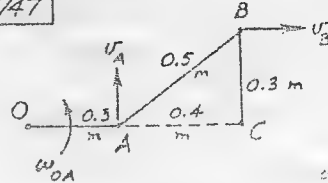
$$a_A = a_O + (a_{A/O})_n + (a_{A/O})_t, a_O = 0.463 \text{ i m/s}^2$$

$$(a_{A/O})_n = \overline{AO} \omega^2 (-\text{j}) = -0.375 (6.28)^2 \text{ j} = -14.80 \text{ j m/s}^2$$

$$(a_{A/O})_t = \overline{AO} \alpha_{AO} \text{ i} = a_O \text{ i} = 0.463 \text{ i m/s}^2$$

$$a_A = 0.925 \text{ i} - 14.80 \text{ j m/s}^2$$

5/47



C is instantaneous center of zero velocity of AB at instant shown.

$$\omega = \frac{v_B}{BC} = 3 \text{ rad/s}$$

$$0.3(3) = 0.9 \text{ m/s}$$

$$\text{so } v_A = 0.4$$

$$a_B = a_A + a_{B/A}$$

$$(a_B)_n = v_B^2 / \overline{BC} = (0.9)^2 / 0.3 = 2.7 \text{ m/s}^2$$

$$(a_B)_t = \overline{BC} \alpha_{BC}$$

$$(a_{B/A})_n = \overline{AB} \omega_{AB}^2 = 0.5(3)^2 = 4.5 \text{ m/s}^2$$

$$(a_{B/A})_t = \overline{AB} \alpha_{AB}$$

$$(a_A)_n = v_A^2 / \overline{OA} = (1.2)^2 / 0.3 = 4.8 \text{ m/s}^2$$

$$\alpha_{BC} = \frac{(a_B)_t}{\overline{BC}} = \frac{8.4}{0.3} = 28 \text{ rad/s}^2 \text{ CCW}$$

$$(a_A)_t = 8.4$$

5/48

$$v_C = 0.2 \text{ m/s constant}$$

$$\omega_{BC} = \frac{v_C}{\overline{BC}} = \frac{0.2}{0.2/\sqrt{2}} = 1.414 \text{ rad/s CCW}$$

$$a_B = a_C + (a_{B/C})_n + (a_{B/C})_t$$

$$a_C = 0$$

$$(a_{B/C})_n = \overline{BC} \omega_{BC}^2 = 0.2(1.414)^2 = 0.4 \text{ m/s}^2$$

$$a_B = a_{B/C} = 0.4\sqrt{2} = 0.566 \text{ m/s}^2$$

$$a_A = a_B + (a_{A/B})_n + (a_{A/B})_t, \omega_{AB} = 1.155 \text{ rad/s (Prob. 5)}$$

$$(a_{A/B})_n = \overline{AB} \omega_{AB}^2 = 0.2(1.155)^2 = 0.267 \text{ m/s}^2$$

$$(a_{A/B})_t = \frac{v_A}{\omega_{AB}} = \frac{0.566}{1.155} = 0.49 \text{ m/s}^2$$

$$(a_{A/B})_t = 0.507 \text{ m/s}^2$$

$$\alpha_{AB} = \frac{0.507}{0.2} = 2.535 \text{ rad/s}^2 \text{ CW}$$

5/49

C = instantaneous center of zero velocity of

$$\omega_{AB} = \frac{v_B}{\overline{CB}} = \frac{0}{\overline{CB}} = 0$$

$$= \frac{100}{439} \frac{60(2\pi)}{60} = 1.432 \text{ rad/s}$$

$$a_B = (a_B)_n = 100(2\pi)^2 = 3950 \text{ mm/s}^2$$

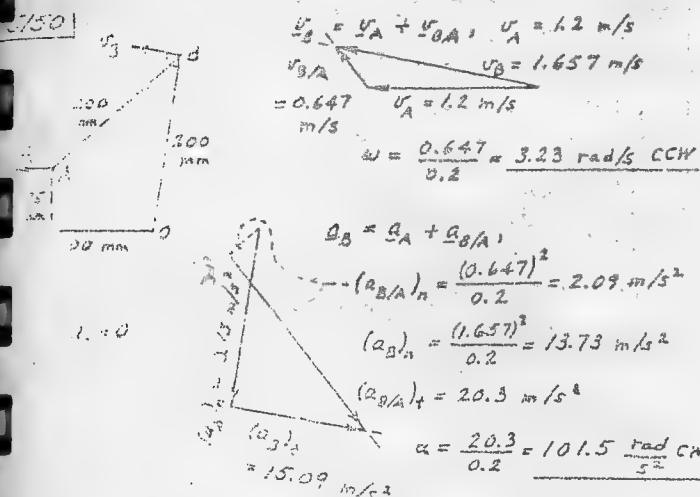
$$a_A = a_B + a_{A/B}; (a_{A/B})_n = 450(1.432)^2 = 923 \text{ mm/s}^2$$

$$a_B = 3950 \text{ mm/s}^2, (a_{A/B})_t = 923 \text{ mm/s}^2, (a_{A/B})_t = 210 \text{ mm/s}^2$$

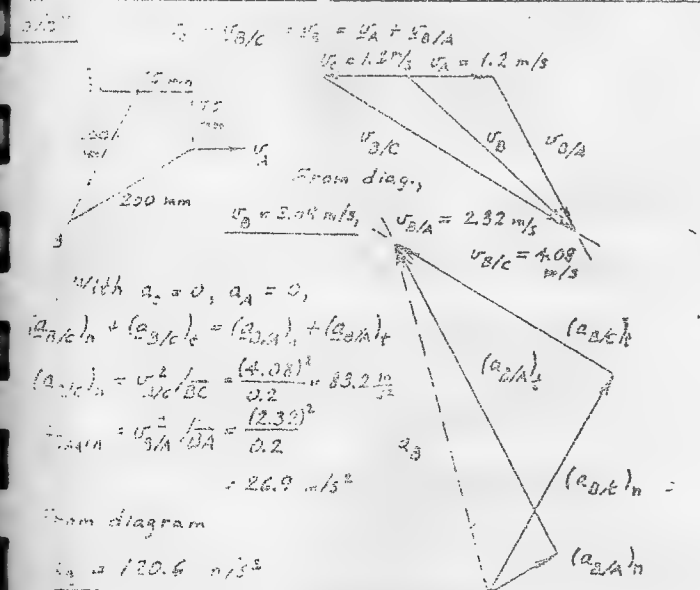
$$a_A = 4870 \text{ mm/s}^2$$

$$\alpha_{AB} = \frac{(a_{A/B})_t}{\overline{AB}} = \frac{210}{450} = 0.467 \text{ rad/s}^2 \text{ CCW}$$

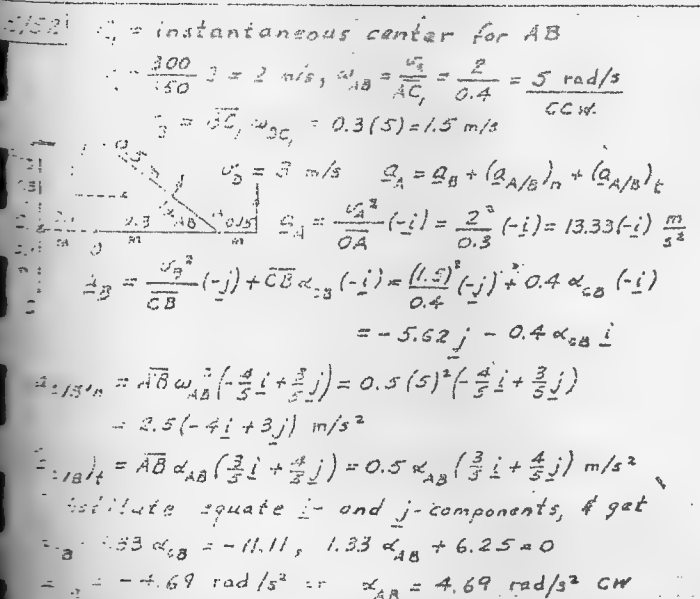
4/5/50



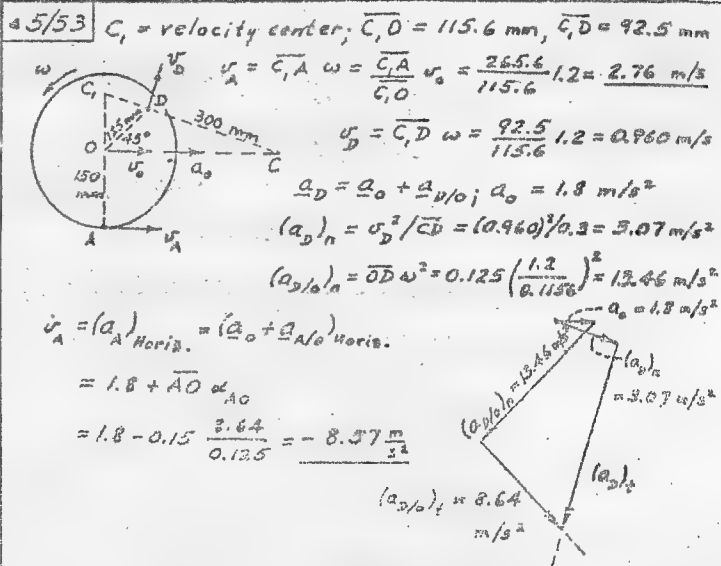
4/5/51



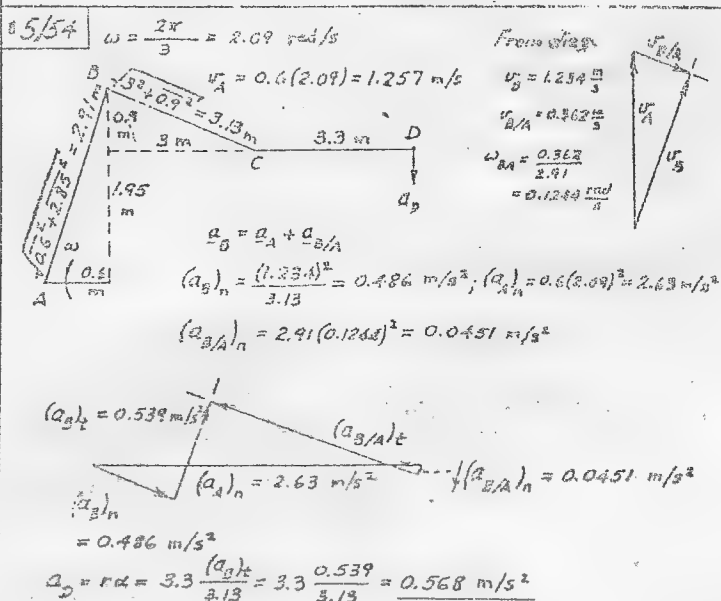
4/5/52



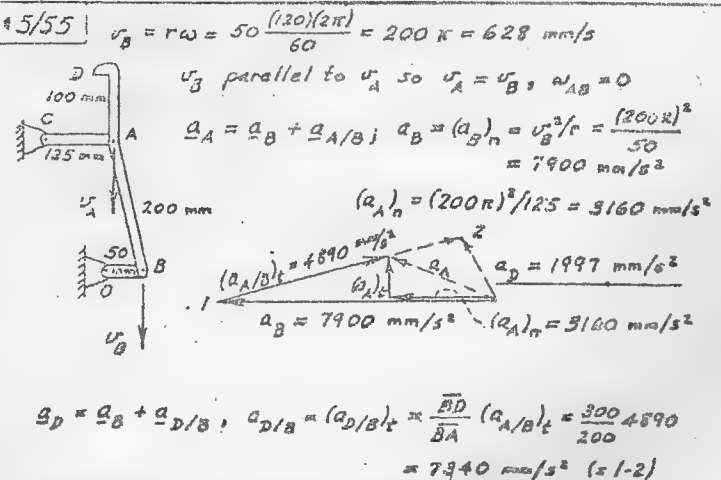
4/5/53



4/5/54



4/5/55



5/57 Before

$v_A = v_P + v_{A/P}$

$v_P = r\omega = \frac{200}{\sqrt{3}/2} \cdot 2 = 693 \text{ mm/s}$

$v_A = 800 \text{ mm/s}$

$v_{A/P} = 400 \text{ mm/s}$

$\omega = 2 \text{ rad/s}$

5/58 Point on OC coincident with point A.

$v_{A/P} = 400 \text{ mm/s}$ (Prob. 5/57)

$a_A = a_P + 2\omega \times v_{rel} + a_{rel}$

$a_P = (a_P)_n = 200 \frac{4}{3} = 1200 \frac{4}{3} \text{ mm/s}^2$

$|2\omega \times v_{rel}| = 2(2)(400) = 2400 \frac{4}{3} \text{ mm/s}^2$

$a_A = 2770 \text{ mm/s}^2$

5/59 $v = +50 \text{ mm/s}$

$\omega = \frac{450}{300} = 1.5 \text{ rad/s}$

$a_A = a_B + (a_{A/B})_n + (a_{A/B})_t$

$a_A = a_B + 2\omega \times v_{rel} + a_{rel}$

$a_B = 0; (a_{A/B})_n = 225 \frac{2}{3} (1.5)^2 = 585 \text{ mm/s}^2$

$(a_{A/B})_t = 2(1.5)v_{rel} = 3(150)(1.5) = 675 \text{ mm/s}^2$

Thus $(a_{A/B})_n + (a_{A/B})_t = 2\omega \times v_{rel} + a_{rel}$

$a_A = \sqrt{(585)^2 + (675)^2}$

$= \frac{675}{2} \sqrt{7} = 893 \text{ mm/s}^2$

$(a_{A/B})_n = 585 \text{ mm/s}^2$

5/60 Let P be a point on EBO coincident with

$v_A = v_P + v_{A/P}$

$v_A = 150(2) = 300 \text{ mm/s}$

$v_{A/P} = v_{rel}$

$\omega = 2 \text{ rad/s}$

$a_A = a_P + 2\omega \times v_{rel} + a_{rel}$

$a_A = (a_A)_n = 150(2)^2 = 600 \frac{4}{3} \text{ mm/s}^2$

$|2\omega \times v_{rel}| = 2(2)(300\sqrt{2}) = 1200\sqrt{2} \text{ mm/s}^2$

$(a_P)_n = 150(2)^2 = 600 \text{ mm/s}^2$

From diagram, $a_{rel} = 1200/150 = 8 \text{ rad/s}^2 \text{ CW}$

5/61 $v_A = v_P = v_{A/P}$

$v_A = 200(10) = 2000 \text{ mm/s}$

$v_{A/P} = v_{rel} = 2000(\frac{1}{2}) = 1000 \text{ mm/s}$

$v_P = 2000(\frac{1}{2}) = 1000 \text{ mm/s}$

$\omega_{PC} = \frac{v_P}{PC} = \frac{1000}{200} = 5 \text{ rad/s CW}$

$a_A = a_P + 2\omega \times v_{rel} + a_{rel}$

$(a_P)_n = v_P^2/PC = (1000)^2/(200\sqrt{3}/2) = 5000\sqrt{3} \text{ mm/s}^2$

$|2\omega \times v_{rel}| = 2(5)(1000) = 10000 \text{ mm/s}^2$

$a_A = (a_A)_n = v_A^2/OA = (2000)^2/200 = 20000 \text{ mm/s}^2$

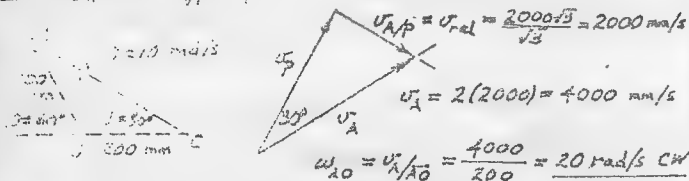
Since $d^2/dt^2(2\theta) = 0$, $\ddot{\theta} = \alpha_{PC} = 0$

$d(a_0)_t = 0$; resolve components

\perp to a_A & qat

$a_{rel}(0.5) = 10000\frac{\sqrt{3}}{2} - 5000\frac{\sqrt{3}}{2}$, $a_{rel} = 8460 \frac{mm}{s^2}$

5/62 $v_A = v_B = v_{A/B}$, $v_B = 2(200\sqrt{3}/2)/10 = 3460 \text{ mm/s}$



$\omega = v_B / AB = 3460 / 200 = 17.3 \text{ rad/s}$

$a_{A/B} = AB \omega^2 = 200(17.3)^2 = 59700 \text{ mm/s}^2$

$a_B = a_{A/B} = 59700 \text{ mm/s}^2$

$a_B = 59.7 \text{ m/s}^2$

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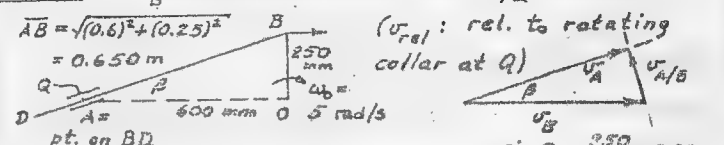
$a_B = 59.7 \text{ m/s}^2$

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$a_B = 59.7 \text{ m/s}^2$

5/64

$v_B = 0.25(5) = 1.25 \text{ m/s}$ $v_A = v_{rel} = v_B + v_{A/B}$



$v_A = v_{rel} = v_B \cos \beta = 1.25(0.9231) = 1.154 \text{ m/s}$

$\omega_{AB} = \frac{v_{A/B}}{AB} = \frac{1.25(0.3846)}{0.65} = 0.740 \text{ rad/s}$

$a_B + a_{A/B} = a_A = a_B + 2\omega \times v_{rel} + a_{rel}$

$a_B = (a_B)_n = 0.25(5)^2 = 6.25 \text{ m/s}^2$

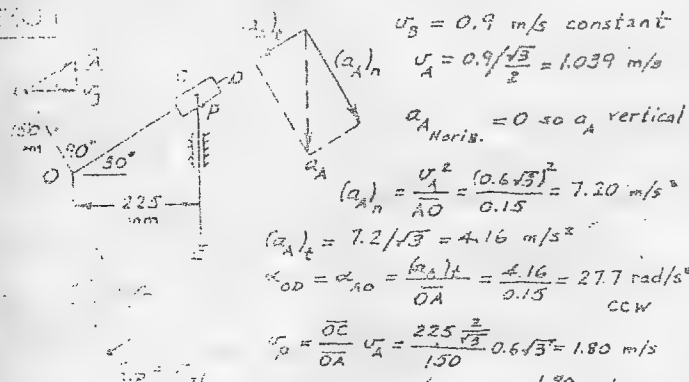
$(a_{A/B})_n = 0.65(0.740)^2 = 0.356 \text{ m/s}^2$

$|2\omega \times v_{rel}| = 2(0.740)(1.154) = 1.707 \text{ m/s}^2$

From diag. $(a_{A/B})_t = 6.25(0.9231) - 1.707 = 4.06 \text{ m/s}^2$

$a_{AB} = \frac{(a_{A/B})_t}{AB} = \frac{4.06}{0.65} = 6.25 \text{ rad/s}^2 \text{ CCW}$

5/63



$v_B = 0.9 \text{ m/s}$ constant

$v_A = 0.9/\sqrt{3} = 0.519 \text{ m/s}$

$a_{A, \text{horiz.}} = 0$ so a_A vertical

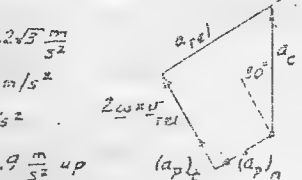
$(a_A)_n = \frac{v_A^2}{AO} = \frac{(0.519)^2}{0.15} = 1.78 \text{ m/s}^2$

$(a_A)_t = 7.2/\sqrt{3} = 4.16 \text{ m/s}^2$

$\omega_{OD} = \omega_{AO} = \frac{(a_A)_t}{AO} = \frac{4.16}{0.15} = 27.7 \text{ rad/s}^2 \text{ CCW}$

$v_P = \frac{OC}{OA} v_A = \frac{225}{150} \cdot 0.519 = 0.778 \text{ m/s}$

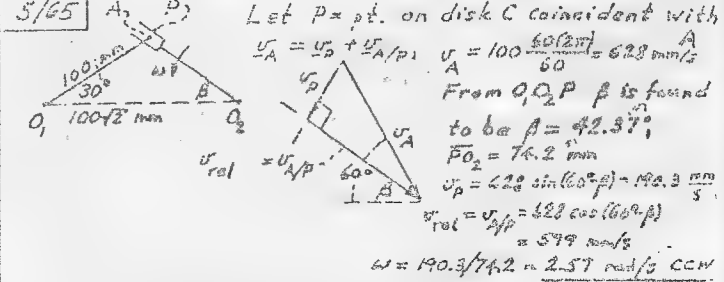
$v_{rel} = \frac{1.80}{\sqrt{3}} \text{ m/s}$



$(a_P)_t = 7.2 + 1.80 = 9.0 \text{ m/s}^2$

$(a_P)_t = 9.0 \text{ m/s}^2$

5/65



Let P = pt. on disk C coincident with

$v_A = v_P + v_{A/P}$ $v_A = 100 \frac{60(2\pi)}{60} = 628 \text{ mm/s}$

From O_1, O_2, P β is found

to be $\beta = 42.3^\circ$

$PO_2 = 74.2 \text{ mm}$

$v_P = 628 \sin(42.3^\circ) = 418 \text{ mm/s}$

$v_{rel} = v_P = 418 \text{ mm/s}$

$\omega = 418/74.2 = 5.64 \text{ rad/s CCW}$

$a_A = a_P + 2\omega \times v_{rel} + a_{rel}$

$a_A = 100 \left(\frac{60(2\pi)}{60} \right)^2 = 3950 \text{ mm/s}^2$

$(a_P)_n = 74.2(5.64)^2 = 2380 \text{ mm/s}^2$

$|2\omega \times v_{rel}| = 2(5.64)(418) = 4710 \text{ mm/s}^2$

$= 4710 \text{ mm/s}^2$

From diagram

$(a_P)_t = 3950 \cos 17.63^\circ + 4710$

$= 6134 \text{ mm/s}^2$

$\alpha = \frac{6134}{74.2} = 82.7 \text{ rad/s}^2$

CCW

$\alpha = 82.7 \text{ rad/s}^2$

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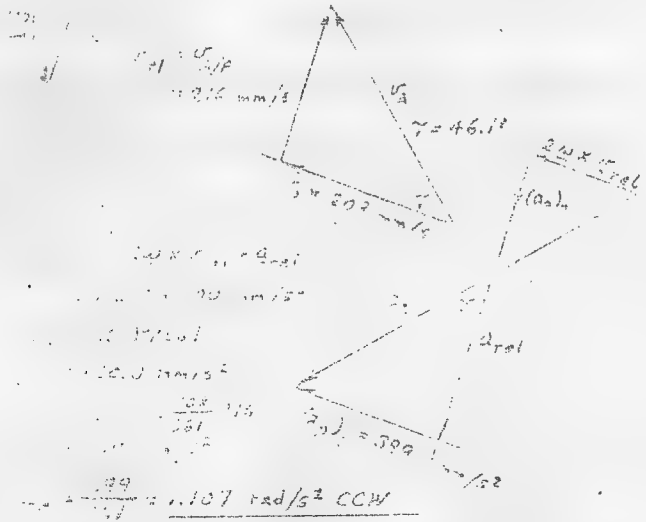
$\alpha = 82.7 \text{ rad/s}^2$

Prob. 15/68 in an BC coincident with A

from geometry of COA, $\overline{CA} = 31 \text{ mm}$

$$\overline{CA} = \overline{CB} \sin \theta = 31 \text{ mm} \quad \theta = 13.90^\circ$$

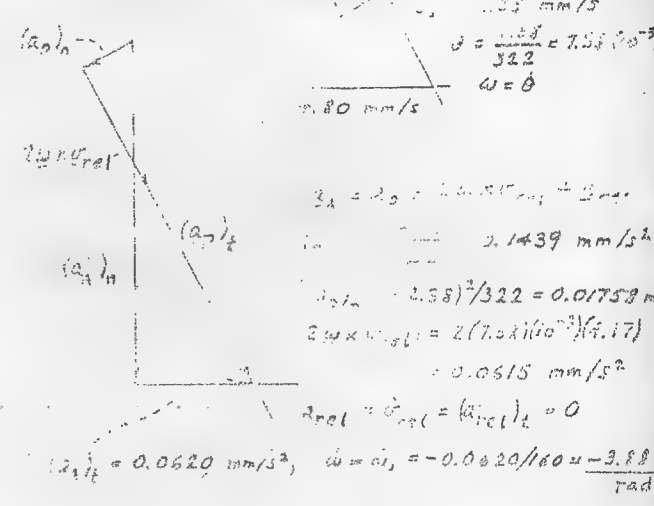
$$v_A = v_B \sin \theta = 300 \text{ mm/s} \quad v_B = v_A \frac{\overline{BC}}{\overline{AC}} = 238 \text{ mm/s}$$



15/68 P = pt. on screw it is coincident with A

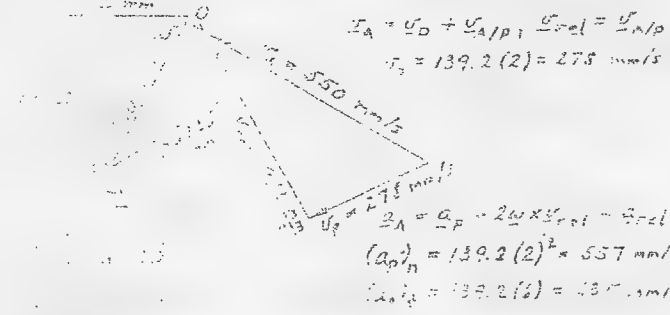
$$\theta = 23.74^\circ \quad \overline{CP} = 322 \text{ mm} \quad v_A = v_B + v_{A/P} \quad v_{A/P} = v_{rel}$$

$$v_{A/P} = 2.5 \frac{150}{50} = 4.17 \text{ mm/s}$$



$$\overline{CP} = \sqrt{3^2 + 150^2} - 2(125)(150) \cos 60^\circ = 18375 \text{ mm}^2$$

$$\overline{CP} = 139.2 \text{ mm}$$

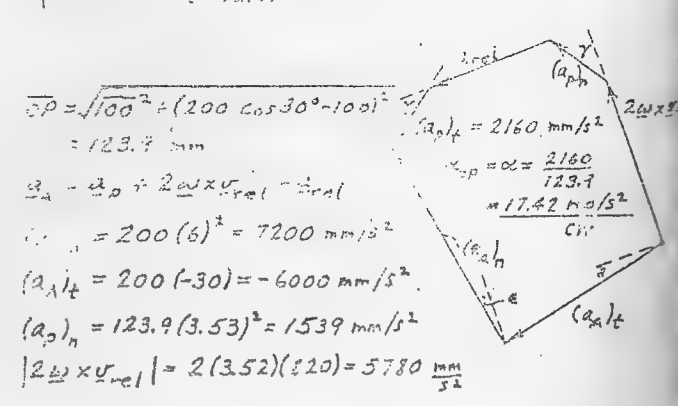


15/69 Let P be on slotted member & coincident with

$$\theta = 23.74^\circ \quad v_A = v_B + v_{A/P} \quad v_A = 200(6) = 1200 \text{ mm/s}$$

$$\text{From sol. to diagram, } v_B = 437 \text{ mm/s, } v_{A/P} = 820 \text{ mm/s}$$

$$\omega_{BP} = \omega = \frac{437}{123.9} = 3.53 \text{ rad/s CCW}$$



$$a_A = a_B + a_{A/P} \quad a_{A/P} = 2(2)(437) = 1732 \text{ mm/s}^2$$

$$a_{A/P} = \frac{(437)^2}{123.9} = 1507 \text{ mm/s}^2$$

$$a_{A/P} = 1507/150 = 2020 \text{ mm/s}^2$$

$$\alpha = \frac{1742}{150} = 9.96 \text{ rad/s}^2 \text{ CCW}$$

$$\overline{CP} = \sqrt{100^2 + (200 \cos 30^\circ - 100)^2} = 123.7 \text{ mm}$$

$$a_A = a_B + 2\omega \times v_{rel} - \omega^2 r$$

$$a_{A/P} = 200(6)^2 = 7200 \text{ mm/s}^2$$

$$(a_A)_t = 200(-30) = -6000 \text{ mm/s}^2$$

$$(a_A)_n = 123.9(3.53)^2 = 1539 \text{ mm/s}^2$$

$$|2\omega \times v_{rel}| = 2(3.52)(820) = 5780 \text{ mm/s}^2$$

CHAPTER SIX

LINE KINETICS OF RIGID BODIES

6/4 error $e = \frac{\frac{1}{2}m(\ell^2 + a^2/4) - \frac{1}{2}m\ell^2}{\frac{1}{2}m(\ell^2 + a^2/4)} = \frac{1}{1 + (\frac{2\ell}{a})^2}$

% error $e = \frac{100\%}{1 + (2\ell/a)^2} = \frac{100\%}{4.01} = 0.249\%$

6/5 error $e = \frac{(\frac{1}{2}mr^2 + md^2) - md^2}{\frac{1}{2}mr^2 + md^2} = \frac{1}{1 + 2(\frac{d}{r})^2}$

% error $e = \frac{100}{1 + 200} = 0.498\%$

% error $e = \frac{100}{1 + 8} = 11.1\%$

6/6 Complete sphere of mass $2m$, $I_{zz} = \frac{2}{5}(2m)r^2$
For hemisphere $I_{zz} = I_{xx} = \frac{2}{5}mr^2$

6/7 Cone has same radial distribution of mass as a circular disk of the same mass and radius. Thus $I_{zz} = \frac{1}{2}mr^2$

6/8 $\rho t = \text{mass per unit area}$
 $= 20.5 \text{ kg/m}^2$

$I_{zz} = \rho t I_z = \rho t (I_x + I_y) = 20.5(1.40 + 3.05)(10^{-8})(10^{-12})$
 $= 9.12(10^{-3}) \text{ kg} \cdot \text{m}^2$

Diagram: A rod of length $2b \sin \alpha$ is shown. The mass distribution is the same as for a rod of length $2b \sin \alpha$ normal to the x-axis.

Thus $I_{xx} = \frac{1}{12}m\ell^2 = \frac{1}{12}m(4b^2 \sin^2 \alpha)$
 $= \frac{1}{3}mb^2 \sin^2 \alpha$

6/10 From Sample Prob. 6/3,

Diagram: A rectangular plate of width 150 mm and height 200 mm, with a center of mass G at the center. The distance from the bottom edge to G is 75 mm.

$$I_G = \frac{1}{12}m(a^2 + b^2)$$

$$= \frac{1}{12}(0.15)(0.20)(0.25)(7.83)(0.15^2 + 0.20^2)$$

$$= \frac{1}{12}(58.7)(0.0225 + 0.0400) = 58.7 \frac{0.0625}{12}$$

$\overline{OG} = \sqrt{(150)^2 + (100)^2} = 180.3 \text{ mm} = 0.1803 \text{ m}$

$I_O = I_G + md^2$; $I_O = 58.7 \left[\frac{0.0625}{12} + (0.1803)^2 \right]$

so $I_{Oo} = 58.7(0.0377) = 2.21 \text{ kg} \cdot \text{m}^2$

6/11 $r = 0.2 \text{ m}$, $F = \frac{4r}{3\pi}$; $m = 45 \text{ kg}$

Diagram: A semi-circular arc of radius r and mass m . The center of mass G is at a distance F from the center O.

$$I_A = I_G + m(F^2 + r^2)$$

$$= I_O - mF^2 + mF^2 + mr^2 = I_O + mr^2$$

$$= \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$$

$$= \frac{3}{2}(45)(0.2)^2 = 2.70 \text{ kg} \cdot \text{m}^2$$

6/12 Total length of rod $= 0.25 + 0.1\pi = 0.564 \text{ m}$

Mass of part I $= m_I = \frac{0.250}{0.564} 0.6 = 0.266 \text{ kg}$

Diagram: A rod of length 0.25 m and mass 0.266 kg, bent into a semi-circular arc of radius 0.1 m.

$$m_{II} = \frac{0.314}{0.564} 0.6 = 0.334 \text{ kg}$$

(I): $I_O = \bar{I} + md^2 = m \left(\frac{\ell^2}{12} + d^2 \right)$
 $= 0.266 \left(\frac{(0.25)^2}{12} + \left[\frac{(0.25)^2}{2^2} + 0.1^2 \right] \right)$
 $= 0.266(0.0308) = 8.20(10^{-3}) \text{ kg} \cdot \text{m}^2$

(II): $I_O = mr^2 = 0.334(0.1)^2 = 3.34(10^{-3}) \text{ kg} \cdot \text{m}^2$

Total $I_{Oo} = 11.54(10^{-3}) \text{ kg} \cdot \text{m}^2$

6/13 $I = I_{2o} - I_{1o} = \frac{1}{2}m_2r_2^2 - \left[\frac{1}{2}m_1r_1^2 + m_1d^2 \right]$

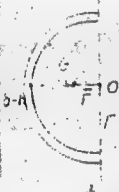
$= \rho \pi t \left[\frac{r_2^4}{2} - \frac{r_1^4}{2} - r_1^2 d^2 \right]$

$m = \rho \pi t [r_2^2 - r_1^2]$

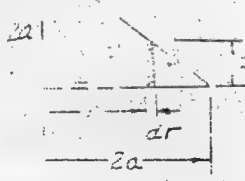
$k_o^2 = \frac{I}{m} = \frac{1}{2} \left(\frac{r_2^4}{r_2^2 - r_1^2} - \frac{r_1^4}{r_2^2 - r_1^2} - \frac{r_1^2 d^2}{r_2^2 - r_1^2} \right)$

$= \frac{1}{2} \left(\frac{0.3^4 + 0.15^4}{0.3^2 - 0.15^2} - \frac{0.15^2 \cdot 0.1^2}{0.3^2 - 0.15^2} \right) = 0.0529 \text{ m}^2$; $k_o = 0.230 \text{ m}$

6/14 For complete ring of mass $2m$,
 $I_{aa} = \frac{1}{2}(2m)r^2$
 So for half ring $I_{aa} = \frac{1}{2}mr^2$
 $I_o = I + (r-\bar{r})^2 m$
 $= I_o - m\bar{r}^2 + (r-\bar{r})^2 m = I_o + m(r^2 - 2r\bar{r})$
 $= 2mr^2 - 2mr \frac{2r}{\pi} = 2mr^2(1 - \frac{2}{\pi})$

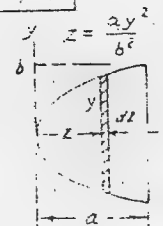


6/15 For elemental shell.
 $\frac{r}{2a} + \frac{z}{2a} = 1$
 $z = 2a - r$
 $dI_{zz} = r^2 dm = r^2 \rho (2\pi r) dz$
 $= 2\pi \rho r^3 (2a - r) dr$
 $I_{zz} = 2\pi \rho \int_a^{2a} r^3 (2a - r) dr$
 $= 2\pi \rho \left[\frac{ar^4}{2} - \frac{r^5}{5} \right]_a^{2a}$
 $= 2\pi \rho a^5 \frac{13}{10}$

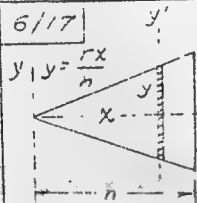


$m = \int dm = \int_a^{2a} 2\pi \rho r (2a - r) dr = 2\pi \rho \left[ar^2 - \frac{r^3}{3} \right]_a^{2a}$
 $= 2\pi \rho a^3 \frac{2}{3}$
 Thus $I_{zz} = \frac{39}{2a} ma^2$

6/16 $dI_{zz} = \frac{1}{2}(dm)y^2 = \frac{1}{2}(\pi y^2 \rho dz)y^2$
 $I_{zz} = \frac{\rho \pi b^4}{2a^2} \int_0^a z^2 dz = \frac{\rho \pi a b^4}{6}$
 $V = \int dV = \pi \int_0^a y^2 dz = \pi \frac{b^2}{a} \int_0^a z dz = \frac{\pi b^2 a}{2}$
 $m = \rho V = \frac{\rho \pi b^2 a}{2}$
 So $I_{zz} = \frac{1}{3} m b^2$

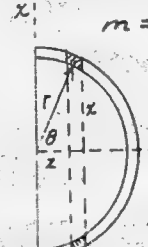


6/17 $dI_{xx} = \frac{1}{2}(dm)y^2 = \frac{1}{2}(\pi y^2 \rho dx)y^2$
 $I_{xx} = \frac{\pi \rho r^4}{2h^4} \int_0^h x^4 dx = \frac{\pi r^2 h \rho}{3} \frac{3r^2}{10}$
 $I_{xx} = \frac{3}{10} mr^2$
 $dI_{yy} = dI_{yy} + x^2 dm = \frac{1}{4} dm y^2 + x^2 dm = \frac{\rho \pi r^2}{h^2} \left[\frac{r^2 x^4}{4h^2} + x^4 \right] dx$
 $I_{yy} = \frac{\pi r^2}{h^2} \int_0^h \left(\frac{r^2 x^4}{4h^2} + x^4 \right) dx = \frac{\pi r^2}{h^2} \left(\frac{r^2 h^5}{20} + \frac{h^5}{5} \right)$
 $= \frac{\pi r^2 h \rho}{3} \left(\frac{3r^2}{20} + \frac{3h^2}{5} \right) = \frac{3}{5} m \left(\frac{r^2}{4} + h^2 \right)$

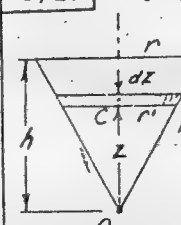


6/18 Spherical wedge $m = \frac{\theta}{2\pi} m'$ m' = mass of complete sphere
 (a) $I_{zz} = \frac{\theta}{2\pi} \frac{2}{5} m' a^2 = \frac{2}{5} m a^2$
 (b) Similarly for conical wedge $I_{zz} = \frac{3}{10} mr^2$

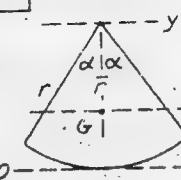
6/19 $dm = 2\pi \rho r \sin \theta \cdot r d\theta$
 $m = 2\pi \rho r^2 \int_0^{\pi/2} \sin \theta d\theta = 2\pi \rho r^2$
 $I_{zz} = \int x^2 dm = 2\pi \rho r^4 \int_0^{\pi/2} \sin^3 \theta d\theta$
 $= \frac{4}{3} \pi \rho r^4 = \frac{2}{3} mr^2$
 Also $I_{xx} = I_{zz} = \frac{2}{3} mr^2$ since each is half that for whole shell of mass



6/20 $dm = 2\pi \rho r' dl$ where ρ = mass/unit area
 $dI_{oo} = dI_{cc} + z^2 dm$
 $= \frac{1}{2}(dm)r'^2 + z^2 dm$
 $= \frac{1}{2}(2\pi \rho \frac{r}{h} z \frac{\sqrt{r^2 + h^2}}{h} dz) \frac{r^2}{h^2} z^2$
 $+ (2\pi \rho \frac{r}{h} z \frac{\sqrt{r^2 + h^2}}{h} dz) z^2$
 $r' = \frac{r}{h} z$ $I_{oo} = 2\pi \rho \frac{r}{h^2} \sqrt{r^2 + h^2} \left(\frac{r^2}{2h^2} + 1 \right) \int_0^h z^3 dz$
 $\frac{dl}{\sqrt{r^2 + h^2}} = \frac{dz}{h} = \frac{\pi \rho r \sqrt{r^2 + h^2}}{4} (r^2 + 2h^2)$
 But $m = \rho \frac{2\pi r}{2\pi l} \pi l^2 = \pi \rho r l = \pi \rho r \sqrt{r^2 + h^2}$
 $I_{oo} = \frac{1}{4} m (r^2 + 2h^2)$

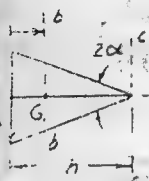


6/21 For small I_{yy} at I_y
 From Prob. 8/23 (STATICS)
 $I_y = \frac{r^4}{4} \left(\alpha + \frac{\sin 2\alpha}{2} \right)$
 Also $\rho t = m/A = \frac{m}{r^2 \alpha}$
 $I_{oo} = I_{yy} + (r-\bar{r})^2 m$
 $= I_{yy} - m\bar{r}^2 + (r-\bar{r})^2 m = I_{yy} + (r^2 - 2r\bar{r})m$
 $= \frac{m}{r^2 \alpha} \frac{r^4}{4} \left(\alpha + \frac{\sin 2\alpha}{2} \right) + \left(r^2 - 2r \frac{2}{3} \frac{r \sin \alpha}{\alpha} \right) m$
 $= mr^2 \left[\frac{5}{4} + \frac{\sin \alpha}{\alpha} \left(\frac{\cos \alpha}{4} - \frac{4}{3} \right) \right]$



122 $I_{aa} = \frac{3}{10} mr^2$; $I_{cc} = \frac{3}{20} mr^2 + \frac{3}{5} mn^2$

$I_{bb} = I_{cc} - m(h-h)^2$



$a = \frac{3}{20} mr^2 + \frac{3}{5} mh^2 = \frac{9}{16} mn^2$

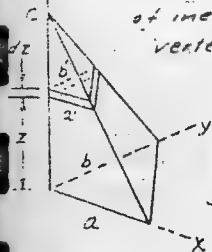
$\frac{3}{20} mr^2 + \frac{3}{80} mn^2$

For $I_{aa} = I_{bb}$, $\frac{3}{10} mr^2 = \frac{3}{20} mr^2 + \frac{3}{80} mn^2$

So $(\frac{r}{h})^2 = \frac{20}{80}$, $\frac{r}{h} = \frac{1}{2}$ & $\tan \alpha = \frac{1}{2}$, $\alpha = 26^\circ 34'$

123 $a' = a \frac{c-z}{c}$ Area of elemental triangle
 $A = \frac{1}{2} a' b' = \frac{ab}{2c^2} (c-z)^2$

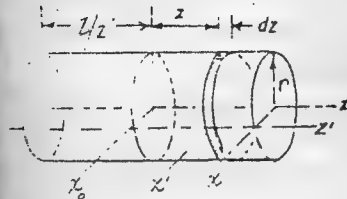
$dI_{zz} = \rho dz (J_z)$ where J_z = polar moment of inertia of triangular area about vertex; $J_z = \frac{1}{12} a' b'^3 + \frac{1}{12} b' a'^3 = \frac{1}{12} a' b' (a'^2 + b'^2)$



$\frac{1}{12} \frac{ab}{c^2} (a^2 + b^2) (c-z)^4$
 So $I_{zz} = \frac{\rho ab(a^2 + b^2)}{12c^2} \int_0^c (c-z)^3 dz$
 $= \frac{\rho ab(a^2 + b^2)}{12c^2} \left(-\frac{(c-z)^4}{4} \right)_0^c$
 $= \frac{\rho abc(a^2 + b^2)}{60}$

But $m = \frac{1}{3} \frac{ab}{c} \rho$, so $I_{zz} = \frac{1}{10} m(a^2 + b^2)$

124 $I_{zz'} = I_{zz} + mr^2 = \frac{1}{2} mr^2 + mr^2 = \frac{3}{2} mr^2$



$dI_{x'x'} = \frac{1}{4} (dm) r^2 = \frac{\rho \pi}{4} r^4 dz$

$dI_{x_0x_0} = dI_{x'x'} + z^2 dm$
 $= \frac{\rho \pi}{4} r^4 dz + \rho \pi r^2 z^2 dz$

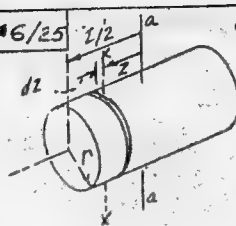
So $I_{x_0x_0} = \rho \pi r^2 \int_{-L/2}^{L/2} \left(\frac{r^2}{4} + z^2 \right) dz = \rho \pi r^2 \left(\frac{r^2 L}{4} + \frac{L^3}{12} \right)$

$I_{x_0x_0} = \frac{m}{12} (3r^2 + L^2)$

$I_{xx} = I_{x_0x_0} + m\left(\frac{L}{2}\right)^2 = \frac{m}{12} (3r^2 + L^2) + \frac{m}{4} L^2$

$I_{xx} = \frac{m}{12} (3r^2 + 4L^2)$

16/25 Consider complete cylindrical shell with mass ρ per unit area:



$dI_{xx} = \frac{1}{2} dm r^2$

$dI_{aa} = dI_{xx} + dm z^2$

$= \frac{1}{2} (2\pi r dz \rho) r^2 + (2\pi r dz \rho) z^2$

$= \pi \rho r (r^2 + 2z^2) dz$

$I_{aa} = \pi \rho r \int_{-L/2}^{L/2} (r^2 + 2z^2) dz = \pi \rho r \left[r^2 z + \frac{2z^3}{3} \right]_{-L/2}^{L/2}$

$= 2\pi \rho r L \left(\frac{r^2}{2} + \frac{L^2}{12} \right)$

For half-shell of mass $m = \pi \rho r L$

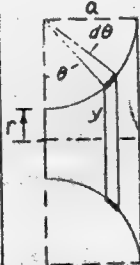
$I_{aa} = \frac{m}{2} \left(r^2 + \frac{L^2}{6} \right)$

16/26 $dm = \rho (2\pi y a d\theta)$ where ρ = mass/unit area

$dI_{zz} = y^2 dm = 2\pi \rho a y^3 d\theta$

But $y = r + a(1 - \cos \theta) = (r+a) - a \cos \theta$

& $y^3 = (r+a)^3 - 3a(r+a)^2 \cos \theta + 3a^2(r+a) \cos^2 \theta - a^3 \cos^3 \theta$



$\int_0^{\pi/2} \cos \theta d\theta = \sin \theta \Big|_0^{\pi/2} = 1$

$\int_0^{\pi/2} \cos^2 \theta d\theta = \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\pi/2} = \frac{\pi}{4}$

$\int_0^{\pi/2} \cos^3 \theta d\theta = \left[\frac{\sin \theta}{3} (2 + \cos^2 \theta) \right]_0^{\pi/2} = \frac{2}{3}$

So $I_{zz} = 2\pi \rho a \left[(r+a)^3 \frac{\pi}{2} - 3a(r+a)^2 (1) + 3a^2(r+a) \frac{\pi}{4} - a^3 \frac{2}{3} \right]$

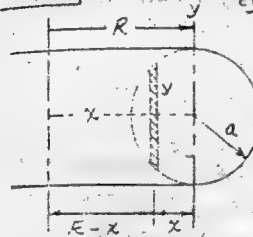
& for $r = a/3$, $I_{zz} = 2\pi \rho a^4 \left(\frac{59}{27} \pi - 6 \right) = 2\pi \rho a^4 (0.865)$

$m = 2\pi \rho a \int_0^{\pi/2} [(r+a) - a \cos \theta] d\theta = 2\pi \rho a^2 \left(\frac{2\pi}{3} - 1 \right)$

$k_z^2 = \frac{I_{zz}}{m} = \frac{0.865}{1.0944} a^2$, $k_z = 0.890 a$

46/27

Cross section of elemental cylindrical shell



$$dm = 2\pi(R-x)(2\rho y) dx$$

$$dI = (R-x)^2 dm$$

$$= 4\pi\rho(R^3 - 3R^2x + 3Rx^2 - x^3)\sqrt{a^2 - x^2} dx$$

$$I = 4\pi\rho[\textcircled{1} - \textcircled{2} + \textcircled{3} - \textcircled{4}]$$

$$\textcircled{1} = R^3 \int_{-a}^a \sqrt{a^2 - x^2} dx = \frac{\pi a^3 R^3}{2}$$

$$\textcircled{2} = 3R^2 \int_{-a}^a x \sqrt{a^2 - x^2} dx = 0$$

$$\textcircled{3} = 3R \int_{-a}^a x^2 \sqrt{a^2 - x^2} dx = \frac{3Ra^4\pi}{8}$$

$$\textcircled{4} = \int_{-a}^a x^3 \sqrt{a^2 - x^2} dx = 0$$

$$m = \rho V$$

$$= \rho(\pi R^2)(\pi a^2)$$

$$= 2\pi^2 a^2 R$$

$$\text{Thus } I = 2\pi^2 a^2 R(R^2 + \frac{3}{4}a^2) = m(R^2 + \frac{3}{4}a^2)$$

46/28

$$\text{Hub; } I = \frac{1}{2}m(r_1^2 + r_2^2) = \frac{1}{2}\rho\pi t(r_2^4 - r_1^4)$$

$$= \frac{1}{2}(7830)\pi(0.075)(0.025^4 - 0.0125^4) = 0.338(10^{-3})\text{ kg}\cdot\text{m}^2$$

$$\text{Spokes; } I = 6\left[\frac{m\ell^2}{12} + md^2\right] = 6(7830)(250)(10^{-6})(0.125)\left[\frac{1}{12} + 0.0515^2\right]$$

$$= 0.01315\text{ kg}\cdot\text{m}^2$$

$$\text{Rim; } I = m(R^2 + \frac{3}{4}a^2) \text{ (see Prob. 6/27)}$$

$$= 2\pi(0.175)(\pi(0.025^2)(7830)(0.175^2 + \frac{3}{4}(0.025^2)))$$

$$= 0.526\text{ kg}\cdot\text{m}^2$$

$$\text{Total } I = 0.00338 + 0.01315 + 0.526$$

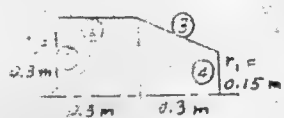
$$= 0.539\text{ kg}\cdot\text{m}^2$$

46/29

$$\textcircled{1} I = \frac{1}{2}mr^2 = \frac{1}{2}(17.5)\pi(0.3)^2(0.3)^2 = 0.223\text{ kg}\cdot\text{m}^2$$

$$\textcircled{2} I = mr^2 = 17.5(2\pi)(0.3)(0.3)(0.3)^2$$

$$= 0.891\text{ kg}\cdot\text{m}^2$$



$$\textcircled{3} I = \frac{1}{2}m_2 r_2^2 - \frac{1}{2}m_1 r_1^2 = \frac{1}{2}\rho(\pi r_2 \ell_2) r_2^2 - \frac{1}{2}\rho(\pi r_1 \ell_1) r_1^2$$

where ρ = mass/unit area, ℓ = slant height,
 & cone area = $\frac{2\pi r}{2\pi \ell} \pi \ell^2 = \pi r \ell$.

$$\text{So } I = \frac{1}{2}\rho\pi(r_2^3 \ell_2 - r_1^3 \ell_1); \ell_2 = 0.3\sqrt{5} = 0.671\text{ m}, \ell_1 = 0.335\text{ m}$$

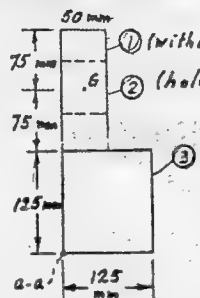
$$I = \frac{1}{2}(17.5)\pi(0.3^3(0.671) - 0.15^3(0.335)) = 0.467\text{ kg}\cdot\text{m}^2$$

$$\textcircled{4} I = \frac{1}{2}mr^2 = \frac{1}{2}17.5\pi(0.15)^2(0.15)^2 = 0.014\text{ kg}\cdot\text{m}^2$$

$$\text{Total } I = 0.223 + 0.891 + 0.467 + 0.014$$

$$= 1.594\text{ kg}\cdot\text{m}^2$$

46/30



$$\textcircled{1} I_{aa} = \bar{I} + md^2$$

$$I_{aa} = 7830(0.05)(0.15)(0.125)\left[\frac{0.05^2 + 0.15^2}{12} + 0.025^2 + 0.12^2\right]$$

$$= 0.314\text{ kg}\cdot\text{m}^2$$

$$\textcircled{2} I_{aa} = \bar{I} + md^2; \text{ from Prob. 6/24}$$

$$\bar{I} = \frac{m}{12}(3r^2 + \ell^2)$$

$$\text{so } I_{aa} = 7830\pi(0.075)^2(0.05)\left[\frac{3(0.0375)^2 + (0.05)^2}{12} + (0.025)^2 + (0.12)^2\right] = 0.0712\text{ kg}\cdot\text{m}^2$$

(negative)

$$\textcircled{3} \text{ By symmetry } I_{aa} = \frac{1}{2} \text{ of that for complete cylinder}$$

$$\text{so from Prob. 6/24 } I_{aa} = \frac{1}{2} \frac{(2\pi)(3r^2 + \ell^2)}{12}$$

$$= \frac{1}{2} 7830 \frac{0.125\pi(0.125)^2}{12} [3(0.125)^2 + 4(0.125)^2] = 0.219\text{ kg}\cdot\text{m}^2$$

$$\text{Total } I_{aa} = 0.314 + 0.0712 - 0.219 = 0.166\text{ kg}\cdot\text{m}^2$$

46/31

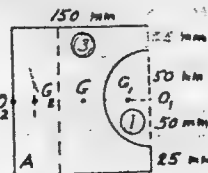
$$\text{Groove } \textcircled{1} I_{aa} = \bar{I} + m(\bar{G}_1 A)^2$$

$$= I_{G_1 O_1} - m(\bar{G}_1 O_1)^2 + m(\bar{G}_1 A)^2$$

$$= m\left(\frac{1}{2}r^2 - \bar{G}_1 O_1^2 + \bar{G}_1 A^2\right)$$

$$= (11370)\frac{\pi(0.05)^2(0.15)}{2}\left[\frac{0.05^2}{2} - 0.0212^2 + 0.15^2\right]$$

$$= 0.1571\text{ kg}\cdot\text{m}^2 \text{ (negative)}$$



$$\bar{G}_1 O_1 = \bar{G}_2 O_1 = \frac{4(0.05)}{3\pi}$$

$$= 0.0212\text{ m}$$

$$\bar{G}_1 A^2 = (0.15 - 0.0212)^2 + (0.075)^2$$

$$= 0.0222\text{ m}^2$$

$$\text{Groove } \textcircled{2} I_{aa} = \frac{1}{2} \text{ that for complete cyl. by symmetry}$$

$$\text{From Prob. 6/24, } I_{aa} = \frac{1}{2} \frac{(2\pi)(3r^2 + \ell^2)}{12}$$

$$I_{aa} = \frac{(11370)\pi(0.05)^2(0.15)}{2}\left[\frac{0.05^2}{2} + 4(0.15)^2\right]$$

$$= 0.0544\text{ kg}\cdot\text{m}^2 \text{ (negative)}$$

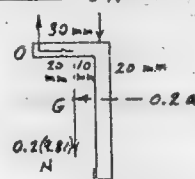
$$\textcircled{3} I_{aa} = \frac{1}{2}m(a^2 + a^2) + m\left(\frac{a^2}{4} + \frac{a^2}{4}\right) = \frac{3}{2}ma^2$$

$$= \frac{3}{2}(11370)(0.15)^2(0.15)^2 = 0.576\text{ kg}\cdot\text{m}^2$$

$$\text{Total } I_{aa} = 0.576 - 0.0544 - 0.1571 = 0.367\text{ kg}\cdot\text{m}^2$$

6/36

$$\sum M_O = mad$$



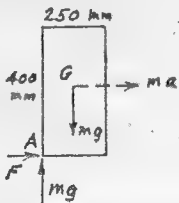
$$8(30) + 0.2(9.81)(20) = 0.2a(20)$$

$$a = 69.8\text{ m/s}^2 = 7.12g$$

6/37

$$v = 1.2 + 0.9t^2\text{ m/s}$$

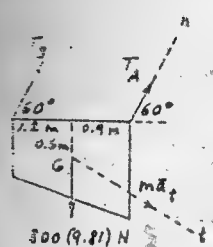
$$a = \dot{v} = 1.8t\text{ m/s}^2$$



$$\sum M_A = mad; \text{ i.e. } \frac{250}{2} = m(1.8t)(200)$$

$$t = \frac{250.9}{720} = \frac{250(9.81)}{720} = 3.41\text{ s}$$

6/38



$$\Sigma F_x = 0; T_A + T_B - 300(9.81)(0.866) = 0$$

$$\Sigma M_G = 0; (T_A + T_B)(0.5)(0.6) + 0.866(1.2T_B - 0.9T_A) = 0$$

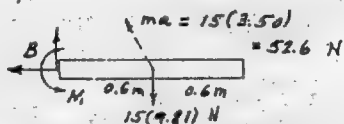
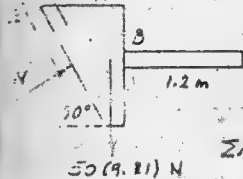
Solve for T_B & get

$$T_B = 1792 \text{ N}$$

6/39

$$\Sigma F_x = m\ddot{a}_x; 600 - 50(9.81)\frac{\sqrt{3}}{2} = 50a$$

$$a = 3.50 \text{ m/s}^2$$

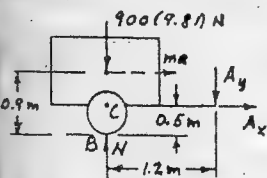


$$\Sigma M_B = m\ddot{a}_d; M - 15(9.81)(0.6) = 52.6\frac{\sqrt{3}}{2}(0.6)$$

$$M = 115.6 \text{ N}\cdot\text{m}$$

6/40

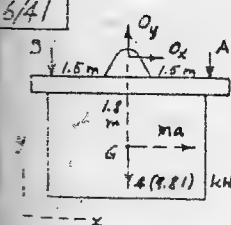
$$v^2 = 2as, a = \frac{v^2}{2s} = \frac{(60/3.6)^2}{2(30)} = 4.63 \text{ m/s}^2$$



$$\Sigma M_C = m\ddot{a}_d; 12A_y = 900(4.63)(0.9 - 0.5)$$

$$A_y = 1389 \text{ N}$$

6/41



$$A = 400\pi(0.05)^2 = 3.14 \text{ kN}$$

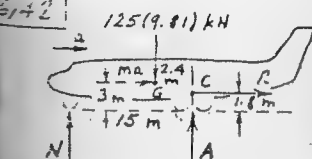
$$B = 500\pi(0.05)^2 = 3.93 \text{ kN}$$

$$\Sigma M_O = m\ddot{a}_d;$$

$$3.93(1.5) - 3.14(1.5) = 4a(1.8)$$

$$a = 0.1636 \text{ m/s}^2$$

6/42



$$v^2 = v_o^2 + 2as$$

$$a = \frac{1}{2(450)}(200^2 - 50^2)/(3.6)^2$$

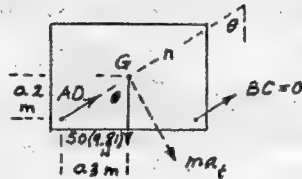
$$= 3.22 \text{ m/s}^2$$

$$\Sigma M_C = m\ddot{a}_d; 15N - 2.4(125)(9.81) = 125(3.22)(3 - 1.8)$$

$$N = 228 \text{ kN}$$

6/43

$$\Sigma M_G = 0; \text{ For } \theta = \tan^{-1} \frac{0.3}{0.2} = 56.31^\circ \text{ \& } BC = 0$$

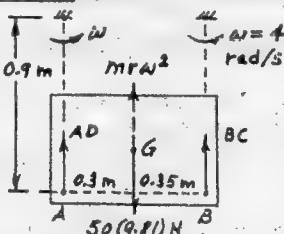


$$\Sigma F_n = 0; AD - 50(9.81) \cos 56.31^\circ = 0$$

$$AD = 272 \text{ N}$$

6/44

$$mr\omega^2 = 50(0.9)(4)^2 = 720 \text{ N}$$



$$\Sigma M_B = m\ddot{a}_d;$$

$$AD(0.3 + 0.35) - 50(9.81)(0.35) = 720(0.35)$$

$$AD = 652 \text{ N}$$

6/45

$$\Sigma F_x = m\ddot{a}_x; fN_2 = ma$$

$$\Sigma M_N = m\ddot{a}_d; -mg\frac{b}{2} + N_2b = mah$$

Solve & get

$$-mg\frac{b}{2} + \frac{ma}{f}b = mah, a = \frac{1}{2} \frac{bf}{b - hf}$$

6/46

$$P > fmg; \Sigma F = ma; P - fmg = ma$$

(a) Tipping about B

$$\Sigma M_B = m\ddot{a}_d; Ph + mg\frac{c}{2} = ma\frac{b}{2}$$

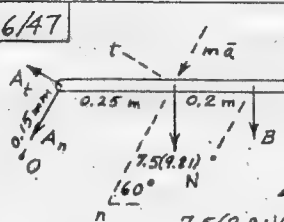
$$h = h_{\min} = \frac{1}{2} \left[b - \frac{mg(c + fb)}{P} \right]$$

(b) Tipping about A

$$\Sigma M_A = m\ddot{a}_d; Ph - mg\frac{c}{2} = ma\frac{b}{2}$$

$$h = h_{\max} = \frac{1}{2} \left[b + \frac{mg(c - fb)}{P} \right]$$

6/47



$$\ddot{a} = \ddot{a}_n = \ddot{a}_o + (\ddot{a}_{A/o})_n$$

where $\ddot{a}_o = 0$

$$\ddot{a} = (\ddot{a}_{A/o})_n = 0.15 \left(\frac{2.4}{0.175} \right)^2 = 28.2 \frac{\text{m}}{\text{s}^2}$$

$$\Sigma M_A = m\ddot{a}_d;$$

$$7.5(9.81)(0.25) + B(0.45) = 7.5(28.2)(0.25) \sin 60^\circ$$

$$B = 60.9 \text{ N}$$

$$\Sigma F_x = 0; A_x = (7.5(9.81) + 60.9) \cos 60^\circ = 67.3 \text{ N}$$

$$\Sigma F_n = m\ddot{a}_n; A_n + (7.5(9.81) + 60.9) \sin 60^\circ = 7.5(28.2)$$

$$A_n = 95.1 \text{ N}$$

$$A = \sqrt{(67.3)^2 + (95.1)^2} = 116.5 \text{ N}$$

6/48 $\sum \vec{r}_i \times \vec{F}_i = \vec{M}_O = \sum m_i \vec{r}_i \times \vec{a}_i = \alpha \sum m_i r_i^2$
 $\sum M_O = \sum m_i r_i^2 \alpha = \alpha \sum m_i r_i^2$
 $\sum M_O = I_O \alpha$
 where $\sum M_O$ includes moments of external forces only.

6/49 (a) $\sum M_O = I_O \alpha$; $T(0.25) = 100(0.375)^2 \alpha$
 $\sum F = ma$; $20(9.81) - T = 20(0.25 \alpha)$
 Combine & solve to get $\alpha = 3.20 \text{ rad/s}^2$
 Drum: $m = 100(9.81) = 981 \text{ N}$
 $k = 0.375 \text{ m}$
 (b) $\sum M_O = I_O \alpha$; $20(9.81)(0.25) = 100(0.375)^2 \alpha$
 $\alpha = 3.49 \text{ rad/s}^2$

6/50 Drum & cable $I_O = 110(0.546)^2 + 105(0.863)(0.6)^2$
 $= 32.8 + 32.6 = 65.4 \text{ kg} \cdot \text{m}^2$
 $\sum M_O = I_O \alpha$; $0.6T = 65.4 \alpha$
 15-m section: $\sum F_x = ma_x$
 $127.0 - T = 15(0.863)(0.6 \alpha)$
 Combine & get $\alpha = 1.087 \text{ rad/s}^2$
 $15(0.863)(9.81) = 127.0 \text{ N}$

6/51 Accelerating force on rear wheels is $F = ma = 2.8(0.6)(9.81) = 16.48 \text{ kN}$
 $\alpha_{\text{drum}} = \frac{a}{r} = \frac{0.6(9.81)}{0.9} = 6.54 \text{ rad/s}^2$
 $\sum M_O = I_O \alpha$; $I_O = \frac{16.48(0.9)}{6.54} = 2.27 \text{ t} \cdot \text{m}^2$

6/52 (a) Pin in place; $\sum M_O = I_O \alpha$
 $30 = 43 \left(\frac{1}{2}(0.2)^2 + (0.25)^2 \right) \alpha$, $\alpha = 8.46 \frac{\text{rad}}{\text{s}^2}$
 (b) Pin removed: $F_t = M/r = 30/0.25 = 120 \text{ N}$
 Rotor: $\sum F_t = ma_t$
 $120 = 43(0.25 \alpha)$
 $\alpha = 11.16 \text{ rad/s}^2$

6/53 $I_O = \frac{1}{12} 10(0.6)^2 + 10(0.1)^2$, $k^2 = 0.04 \text{ m}^2$
 $q = k^2/r = 0.04/0.1 = 0.4 \text{ m}$
 $\sum M_Q = 0$; $0.4 \theta - 0.2(150) = 0$

6/54 $I_O = \frac{1}{3} m \ell^2 = \frac{1}{3} (20)(1.5)^2 = 15 \text{ kg} \cdot \text{m}^2$
 $\sum M_O = I_O \alpha$
 $20(9.81)(0.75) - 6 = 15 \alpha$
 $\alpha = 9.41 \text{ rad/s}^2$
 $\sum F_t = m \bar{a}_t$; $20(9.81) - 0 = 20(0.75) \alpha$
 $Q = 55.0 \text{ N}$

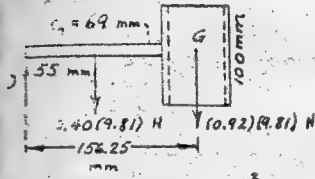
6/55 $I_O = \frac{1}{4} m r^2 + \frac{1}{3} m h^2$ (Appendix C)
 $q = \frac{k^2}{r} = \frac{r^2/4 + h^2/3}{h/2} = \frac{0.15^2/4 + 0.2^2/3}{0.1} = 0.1$
 $\sum M_Q = 0$; $100(0.1896) - P(0.1896 - 0.1) = 0$
 $P = 212 \text{ N}$

6/56 Configuration shown is impossible since neither $\sum \vec{M} = \vec{I} \alpha$ nor $\sum M_Q = 0$ can be satisfied. Thus T cannot pass through G .

6/57 $\bar{GO} = 125 \text{ mm}$, $m \bar{GO} \omega^2 = 4(0.125)(10^2) = 50 \text{ N}$
 $\sum M_O = 0$ so A is directed toward O
 $\sum F_x = m \bar{a}_x$; $T = 50 \frac{3}{5} = 30 \text{ N}$
 $\omega = 10 \text{ rad/s}$

6/58 $\sum M_O = I_O \alpha$; $M = \frac{1}{3} m \ell^2 \alpha$, $\alpha = \frac{3M}{m \ell^2}$
 $\sum F_t = m \bar{a}_t$; $O_t = m \frac{\ell}{2} \alpha = \frac{3M}{2 \ell}$
 $\sum F_n = m \bar{a}_n$; $O_n - mg = m \frac{\ell}{2} \omega^2$
 $O_n = mg \left(1 + \frac{\ell \omega^2}{2g} \right)$

6/59 For tube (see Appendix C)



$$I_G = \frac{1}{2} m(r^2 + \frac{h^2}{4})$$

$$= \frac{1}{2} (0.92)(31.25^2 + \frac{100^2}{4})(10^{-6})$$

$$= 0.001216 \text{ kg}\cdot\text{m}^2$$

$$I_O = I_G + m\bar{r}^2$$

$$= 0.001216 + 0.92(0.15625)^2$$

$$= 0.0237 \text{ kg}\cdot\text{m}^2$$

Link $I_O = 0.40(0.069)^2$

$$= 0.001904 \text{ kg}\cdot\text{m}^2$$

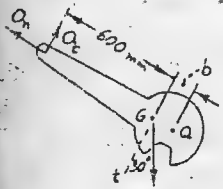
$$\Sigma M_O = I_O \alpha; 0.92(9.81)(0.15625) + 0.40(9.81)(0.055) = (0.0237 + 0.001904)\alpha$$

$$\alpha = 63.6 \text{ rad/s}^2$$

$$\Sigma F_x = m\bar{a}_x; (0.40 + 0.92)(9.81) - O = (0.40[0.055] + 0.92[0.15625])\alpha$$

$$O = 12.95 - 10.54 = \underline{2.41 \text{ N}}$$

6/60 If impact occurs at center of percussion, bearing reaction will have vertical component only & hence will be minimum. So $q = 600 + b = k_0^2/F = 650^2/600$



$$b = 104.2 \text{ mm}$$

$$\Sigma F_n = m\bar{a}_n = 0;$$

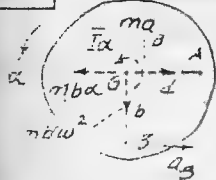
$$O_n = 35(9.81)(0.5) = 171.7 \text{ N}$$

$$\Sigma M_O = 0; (600 + 104.2)O_n - 35(9.81)(0.866)(104.2) = 0$$

$$O_n = 44.0 \text{ N}$$

$$O = \sqrt{(44.0)^2 + (171.7)^2} = \underline{177.2 \text{ N}}$$

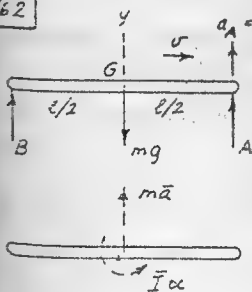
6/61 In counterclockwise sense,



$$\Sigma M_A = \bar{I}\alpha + mbd\omega^2$$

$$\Sigma M_G = \bar{I}\alpha + mb^2\alpha - m\bar{a}_G b$$

6/62



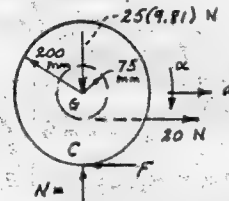
$$\Sigma M_B = \bar{I}\alpha + m\bar{a}_d$$

$$A\ell - mg\frac{\ell}{2} = \frac{1}{12}m\ell^2\alpha + m\frac{v^2}{2r}\frac{\ell}{2}$$

$$\alpha = \frac{a_A}{\ell} = \frac{v^2}{r\ell}$$

$$A = mg\left(\frac{1}{2} + \frac{1}{3}\frac{v^2}{gr}\right)$$

6/63 Assume no slipping: $a = r\alpha$



$$\Sigma M_C = I_C \alpha; 20(0.20 - 0.075)$$

$$= 25(0.175^2 + 0.20^2)\alpha; \alpha = 1.416 \text{ rad/s}^2$$

$$\text{Check: } \Sigma F = m\bar{a};$$

$$20 - F = 25(0.20)(1.416)$$

$$F = 20 - 7.08 = 12.92 \text{ N}$$

$$F_{\max} = fN = 0.10(25)(9.81) = 24.5 \text{ N} > 12.92 \text{ N, so}$$

original assumption OK, $\alpha = 1.416 \text{ rad/s}^2$

6/64

Counterweight $\Sigma F = ma;$

$$10(9.81) - T_2 = 10(2a) \text{ --- (1)}$$

Cylinder $\Sigma M_C = I_C \alpha;$

$$I_C = \frac{1}{2}mr^2 + mr^2$$

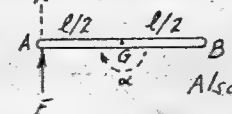
$$= \frac{3}{2}16(0.2)^2 = 0.96 \text{ kg}\cdot\text{m}^2$$

$$0.4 T_2 - 16(9.81)(0.2) = 0.96 \frac{a}{0.2} \text{ --- (2)}$$

Combine (1) & (2) & get $a = 0.613 \text{ m/s}^2$

6/65

$$\Sigma \bar{M} = \bar{I}\alpha; F\frac{\ell}{2} = \frac{1}{12}m\ell^2\alpha, F = \frac{1}{6}m\ell\alpha$$



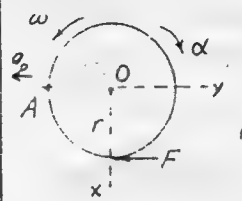
$$\Sigma F_x = m\bar{a}_x; F = m\bar{a}$$

$$\text{Also } a_B = \bar{a} + \frac{a}{2}\ell; a_B = \bar{a} - \frac{1}{2}\ell\alpha = \frac{F}{m} - \frac{1}{2}\frac{6F}{m\ell}\ell$$

$$a_B = -\frac{2F}{m} \text{ (-x-dir.)}$$

6/66

$$\Sigma M_O = I_O \alpha; Fr = mk^2\alpha, \alpha = \frac{Fr}{mk^2}$$



$$\Sigma F_y = m\bar{a}_y; F = m\bar{a}_y$$

$$\bar{a}_A = \bar{a}_O + \bar{a}_{A/O}$$

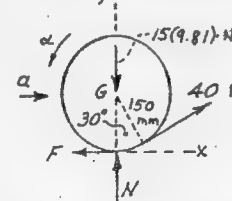
$$(\bar{a}_{A/O})_n = r\omega^2 \underline{j}, (\bar{a}_{A/O})_t = -r\alpha \underline{i} = -\frac{Fr}{mk^2} \underline{i}$$

$$\bar{a}_O = -\frac{F}{m} \underline{j}$$

$$\text{So } \bar{a}_A = -\frac{Fr^2}{mk^2} \underline{i} - \left(\frac{F}{m} - r\omega^2\right) \underline{j}$$

6/67

Assume roll slips, $F = 0.2 \text{ N}$



$$\Sigma \bar{M} = \bar{I}\alpha; (40 - 0.2N)(0.15) = \frac{1}{2}15(0.15)^2\alpha$$

$$\Sigma F_y = 0; N + 40(0.5) - 15(9.81) = 0,$$

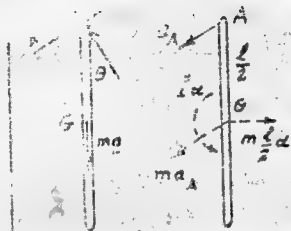
$$N = 127.2 \text{ N}$$

Substitute N & get

$$\alpha = 12.95 \text{ rad/s}^2 \text{ CCW}$$

$$\Sigma F_x = m\bar{a}_x; 40\frac{\sqrt{3}}{2} - 0.2(127.2) = 15\bar{a}, \bar{a} = 0.614 \text{ m/s}^2$$

6/68



$$\Sigma M_A = \bar{I}\alpha + m \frac{l}{2} \frac{l}{2} - m a_A \frac{l}{2} \cos \theta$$

$$0 = \frac{1}{12} m l^2 \alpha + \frac{1}{4} m l^2 \alpha - m a_A \frac{l}{2} \cos \theta$$

$$2l\alpha = 3a_A \cos \theta$$

$$\Sigma F_x = m\bar{a}_x; mg \sin \theta = m(a_A - \frac{l}{2} \alpha \cos \theta)$$

$$g \sin \theta = a_A - \frac{l}{2} \alpha \cos \theta$$

Eliminate α & get

$$a_A = \frac{g \sin \theta}{1 - \frac{3}{4} \cos^2 \theta}$$

6/69

$$\Sigma M_A = \bar{I}\alpha + m\bar{a}d;$$

$$-M = -\frac{1}{12} p b l^2 (\ddot{\theta}) + p b l (\frac{l}{2}) \ddot{\theta}$$

$$-p b l a \frac{l}{2} \sin \theta$$

$$M = p b l^2 (\frac{a}{2} \sin \theta - \frac{1}{3} \ddot{\theta})$$

6/70

$$\Sigma M_D = \bar{I}\alpha + m\bar{a}d;$$

$$500(9.81)(1.25) = \frac{1}{12} 500(5)^2 \alpha + 500(2.5)(1.25) \alpha$$

$$\alpha = 2.35 \text{ rad/s}^2$$

$$\Sigma F_n = m\bar{a}_n$$

$$T - 500(9.81)/\sqrt{2} = -500(2.5)(2.35)/\sqrt{2}$$

$$T = 1387 \text{ N}$$

6/71

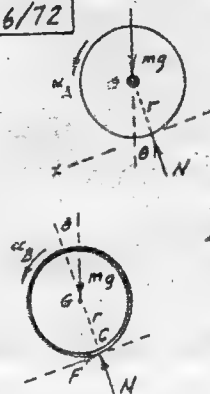
$$F = r/2 \quad \Sigma M_C = \bar{I}\alpha + m\bar{a}d;$$

$$I_C = \frac{2}{3} m r^2 \quad mg \frac{r}{2} \sin \theta = (\frac{2}{3} m r^2 - m [\frac{r}{2}]^2) \alpha + m r \alpha (r - \frac{r}{2} \cos \theta)$$

$$- m \frac{r}{2} \alpha (r \cos \theta - \frac{r}{2})$$

$$\text{Solve & get } \alpha = \frac{3g \sin \theta}{2r(5 - 3 \cos \theta)}$$

6/72



$$\Sigma \bar{M} = \bar{I}\alpha, \text{ but } \bar{I} = 0 \text{ so } \Sigma \bar{M} = 0.$$

Hence no friction force & $f_A = 0$

$$\Sigma F_x = m\bar{a}_x; mg \sin \theta = m r \alpha$$

$$\alpha = \frac{g \sin \theta}{r}$$

$$\Sigma M_C = I_C \alpha; mgr \sin \theta = m(2r)^2 \alpha$$

$$\alpha_B = \frac{g \sin \theta}{2r}$$

$$\Sigma \bar{M} = \bar{I}\alpha; Fr = m r^2 \frac{g \sin \theta}{2r}$$

$$F = \frac{1}{2} m g \sin \theta$$

$$f_B = \frac{F}{N} = \frac{1}{2} mg \sin \theta / mg \cos \theta$$

$$f_B = \frac{1}{2} \tan \theta$$

6/73

$$x = x_0 \sin pt, \dot{x} = x_0 p \cos pt, a_x = \ddot{x} = -x_0 p^2 \sin pt$$

$$|a_x|_{\max} = x_0 p^2 = 10(40 \times 2\pi)^2 \text{ m/s}^2 = 64 \pi^2 \text{ m/s}^2$$

$$m|a_x|_{\max} = 4(0.4 - r)64 \pi^2 = 2530(0.4 - r)$$

$$\Sigma M_V = m\bar{a}_x d; M = 2530(0.4 - r) \frac{0.4 - r}{2} = 1263(0.4 - r) \text{ in-mats}$$

6/74

Let p = mass per unit length of tape

$$\alpha = a/r$$

$$\bar{I} = (L-x)p r^2$$



$$\Sigma M_C = m\bar{a}r + \bar{I}\alpha; M = (L-x)par + (L-x)pr^2 \frac{a}{r}$$

$$= 2(L-x)par$$

$$\text{so } a = \frac{M}{2(L-x)pr}$$

$$\Sigma F = m\bar{a}; P = (L-x)p \frac{M}{2(L-x)pr} = \frac{M}{2r} \text{ constant}$$

6/75

(Appendix C) For hemispherical shell

$$\text{mass } m, I_O = \frac{2}{3} m r^2 + m r^2 = \frac{5}{3} m r^2$$

$$\text{so } I_O = \frac{5}{3} 1.7(1.2)^2 = 4.08 \text{ t}\cdot\text{m}^2$$

$$\Sigma M_O = I_O \alpha; 25(0.45) - 1.7(9.81)(0.6) = 4.08 \alpha$$

$$\alpha = 0.305 \text{ rad/s}^2$$

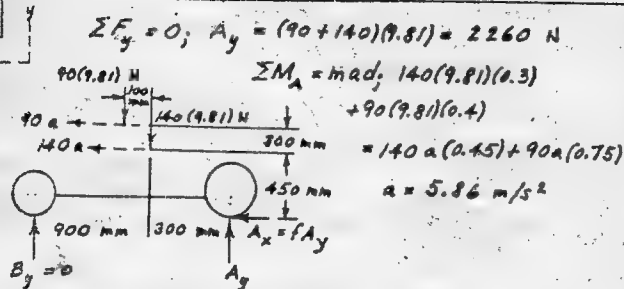
$$\bar{a}_x = \bar{a}_t \cos \theta = F \cos \theta = r \alpha = 1.2(0.305)$$

$$= 0.366 \text{ m/s}^2$$

$$(\bar{a}_n = \bar{r}\omega^2 = 0)$$

$$\bar{x} = r/2 = 600 \text{ mm} \quad \Sigma F_x = m\bar{a}_x; O_x - 25 = 1.7(0.366), O_x = 2.22 \text{ kN}$$

6/76



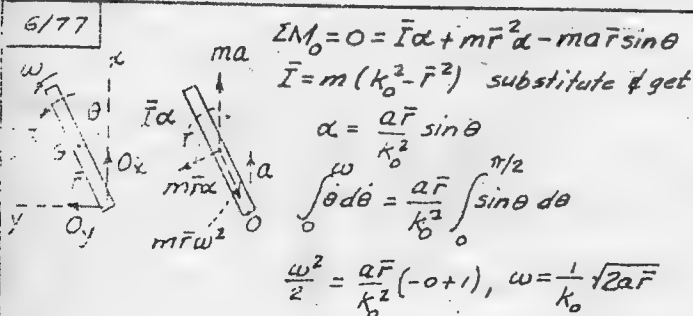
$$\Sigma F_y = 0; A_y = (90 + 140)(9.81) = 2260 \text{ N}$$

$$\Sigma M_A = m\ddot{a}d; 140(9.81)(0.3) + 90(9.81)(0.4) = 140a(0.45) + 90a(0.75)$$

$$a = 5.86 \text{ m/s}^2$$

$$\Sigma F_x = m\ddot{a}_x; 2260f = (40 + 140)5.86, f = \frac{5.86}{9.81} = 0.598$$

6/77



$$\Sigma M_O = 0 = \bar{I}\alpha + m\bar{r}^2\alpha - m\bar{r}\sin\theta$$

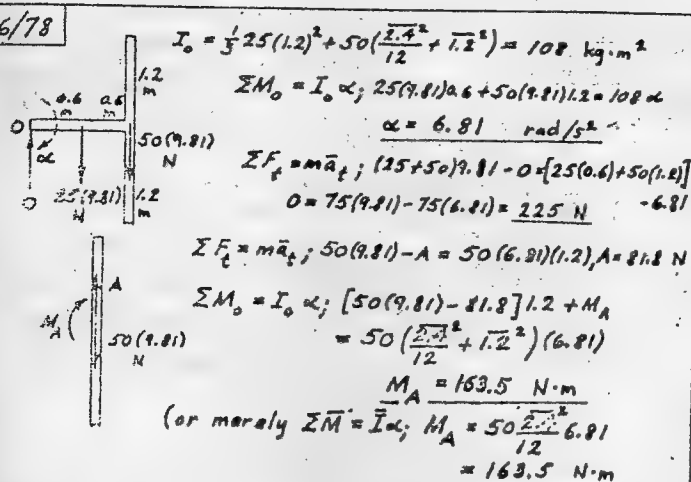
$$\bar{I} = m(k_o^2 - \bar{r}^2) \text{ substitute \& get}$$

$$\alpha = \frac{a\bar{r}}{k_o^2} \sin\theta$$

$$\int \theta d\theta = \frac{a\bar{r}}{k_o^2} \int \sin\theta d\theta$$

$$\frac{\omega^2}{2} = \frac{a\bar{r}}{k_o^2} (-\cos\theta + 1), \omega = \frac{1}{k_o} \sqrt{2a\bar{r}}$$

6/78



$$I_o = \frac{1}{12} 25(1.2)^2 + 50(\frac{2.4^2}{12} + 1.2^2) = 108 \text{ kg}\cdot\text{m}^2$$

$$\Sigma M_o = I_o\alpha; 25(9.81)0.6 + 50(9.81)1.2 = 108\alpha$$

$$\alpha = 6.81 \text{ rad/s}^2$$

$$\Sigma F_y = m\ddot{a}_y; (25 + 50)9.81 - 0 = 25(0.6) + 50(1.2)$$

$$0 = 75(9.81) - 75(6.81) = 225 \text{ N}$$

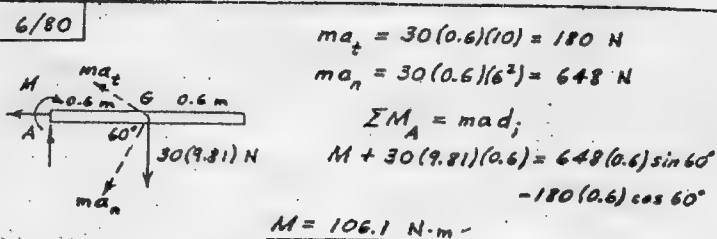
$$\Sigma F_x = m\ddot{a}_x; 50(9.81) - A = 50(6.81)(1.2), A = 81.8 \text{ N}$$

$$\Sigma M_o = I_o\alpha; [50(9.81) - 81.8]1.2 + M_A = 50(\frac{2.4^2}{12} + 1.2^2)(6.81)$$

$$M_A = 163.5 \text{ N}\cdot\text{m}$$

$$\text{(or merely } \Sigma \bar{M} = \bar{I}\alpha; M_A = 50\frac{\bar{I}}{12}6.81 = 163.5 \text{ N}\cdot\text{m)}$$

6/80



$$m\ddot{a}_t = 30(0.6)(10) = 180 \text{ N}$$

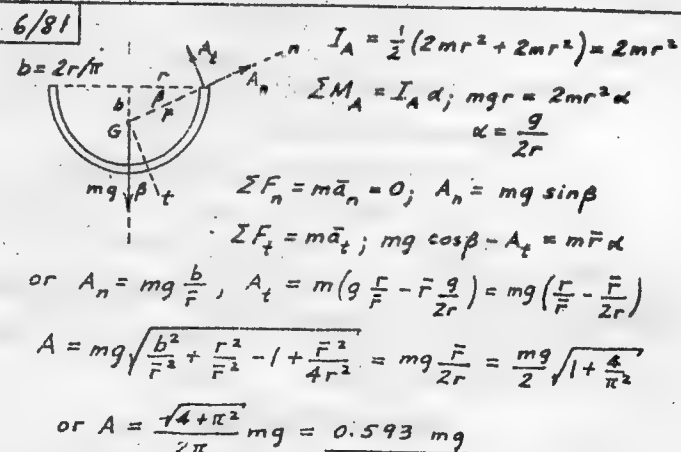
$$m\ddot{a}_n = 30(0.6)(6^2) = 648 \text{ N}$$

$$\Sigma M_A = m\ddot{a}d;$$

$$M + 30(9.81)(0.6) = 648(0.6)\sin 60^\circ - 180(0.6)\cos 60^\circ$$

$$M = 106.1 \text{ N}\cdot\text{m}$$

6/81



$$I_A = \frac{1}{2}(2mr^2 + 2mr^2) = 2mr^2$$

$$\Sigma M_A = I_A\alpha; mgr = 2mr^2\alpha$$

$$\alpha = \frac{g}{2r}$$

$$\Sigma F_n = m\ddot{a}_n = 0; A_n = mg \sin\beta$$

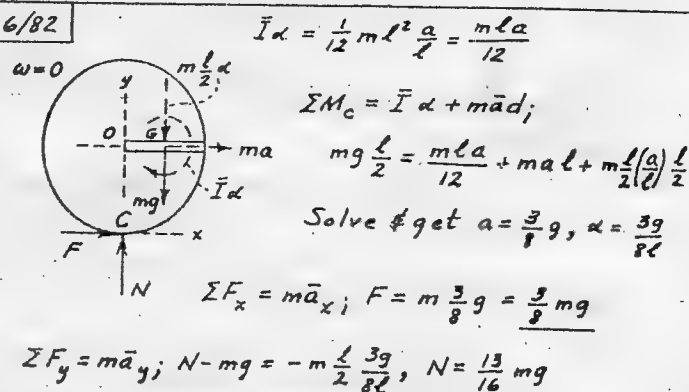
$$\Sigma F_t = m\ddot{a}_t; mg \cos\beta - A_t = m\bar{r}\alpha$$

$$\text{or } A_n = mg \frac{b}{r}, A_t = m(g \frac{r}{r} - \bar{r} \frac{g}{2r}) = mg(\frac{r}{r} - \frac{\bar{r}}{2r})$$

$$A = mg\sqrt{\frac{b^2}{r^2} + \frac{r^2}{r^2} - 1 + \frac{\bar{r}^2}{4r^2}} = mg\sqrt{\frac{\bar{r}^2}{4r^2}} = \frac{mg}{2}\sqrt{1 + \frac{4}{\pi^2}}$$

$$\text{or } A = \frac{\sqrt{4 + \pi^2}}{2\pi} mg = 0.593 mg$$

6/82



$$\bar{I}\alpha = \frac{1}{12} m l^2 \frac{a}{l} = \frac{m l a}{12}$$

$$\Sigma M_c = \bar{I}\alpha + m\ddot{a}d;$$

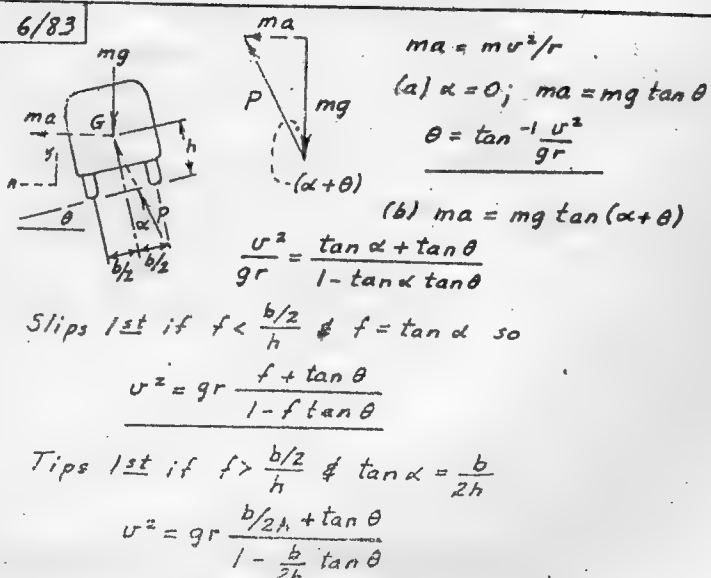
$$mg \frac{l}{2} = \frac{m l a}{12} + m a l + m \frac{l}{2} \left(\frac{a}{l} \right) \frac{l}{2}$$

$$\text{Solve \& get } a = \frac{3}{8} g, \alpha = \frac{3g}{8l}$$

$$\Sigma F_x = m\ddot{a}_x; F = m \frac{3}{8} g = \frac{3}{8} mg$$

$$\Sigma F_y = m\ddot{a}_y; N - mg = -m \frac{1}{2} \frac{3g}{8l}, N = \frac{13}{16} mg$$

6/83



$$ma = m v^2 / r$$

$$\text{(a) } \alpha = 0; ma = mg \tan\theta$$

$$\theta = \tan^{-1} \frac{v^2}{gr}$$

$$\text{(b) } ma = mg \tan(\alpha + \theta)$$

$$\frac{v^2}{gr} = \frac{\tan\alpha + \tan\theta}{1 - \tan\alpha \tan\theta}$$

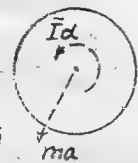
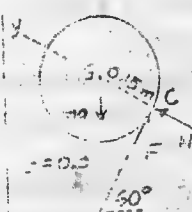
$$\text{Slips 1st if } f < \frac{b/2}{h} \& f = \tan\alpha \text{ so}$$

$$v^2 = gr \frac{f + \tan\theta}{1 - f \tan\theta}$$

$$\text{Tips 1st if } f > \frac{b/2}{h} \& \tan\alpha = \frac{b}{2h}$$

$$v^2 = gr \frac{b/2h + \tan\theta}{1 - \frac{b}{2h} \tan\theta}$$

6/84 Assume cylinder slips so $F = 0.3N$
 $\sum F_x = 0; N - 0.5mg = 0, N = 0.5mg, = 0.3(0.5mg) = 0.15mg$



$$I\ddot{\theta} = \ddot{\theta} \cdot 0.15 = gr = \frac{1}{2}mr^2\ddot{\theta}$$

$$\ddot{\theta} = \frac{0.15(2g)}{r} = \frac{0.15(19.62)}{0.15}$$

$$= 19.62 \text{ rad/s}^2$$

$$\sum F_x = m\ddot{x}; mg \frac{1}{2} - 0.15mg = ma$$

$$a = g \left(\frac{1}{2} - 0.15 \right) = 7.02 \text{ m/s}^2$$

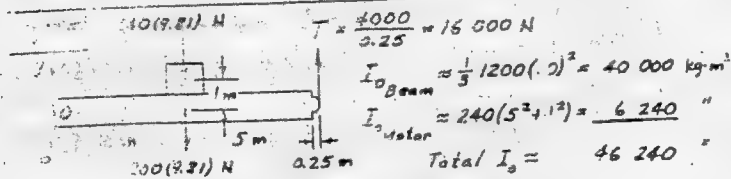
$$(For no slipping \ddot{\theta} = a/r = 7.02/0.15 = 46.8 \text{ rad/s}^2)$$

$$46.8 > 19.62 \text{ so assumption of slipping OK}$$

$$v = \sqrt{2(7.02)(3)} = 6.49 \text{ m/s}$$

$$-a t^2; t = \sqrt{2(3)/7.02} = 0.924 \text{ s}$$

$$v = at; \omega = 19.62(0.924) = 18.13 \text{ rad/s}$$



$$T = \frac{4000}{0.25} = 16000 \text{ N}$$

$$I_{beam} = \frac{1}{3} 1200(5)^2 = 40000 \text{ kg-m}^2$$

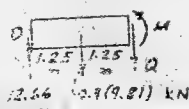
$$I_{water} = 240(5^2 + 1^2) = 6240$$

$$Total I_o = 46240$$

$$\sum M_o = I_o \ddot{\alpha}; 16000(0.25) - 1440(9.81)(5) = 46240 \ddot{\alpha}$$

$$\ddot{\alpha} = 2.02 \text{ rad/s}^2$$

$$\sum F_y = m\ddot{y}; 16000 - 1440(9.81) + P = 1440(5)(2.02); P = 12660 \text{ N}$$



$$\sum F_y = m\ddot{y}; 12.66 - 0.3(9.81) - Q = 0.3(1.25)(2.02)$$

$$Q = 8.96 \text{ kN}$$

$$\sum M_o = I_o \ddot{\alpha}; M - 8.96(2.5) - 0.3(9.81)(1.25) = \frac{1}{3} 0.3(2.5)^2 (2.02)$$

$$M = 27.4 \text{ kN-m}$$



$$\phi = \tan^{-1} 0.4$$

Moment arms measure

$$d_1 = 0.486l, d_2 = 0.118l$$

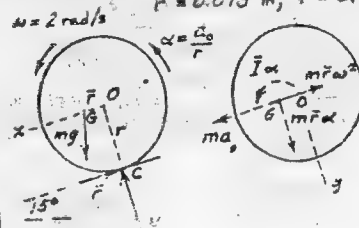
$$\sum M_o = mad;$$

$$mg d_2 = mad,$$

$$a = g \frac{d_2}{d_1} = 9.81 \frac{0.118l}{0.486l}$$

$$= 2.38 \text{ m/s}^2$$

6/87 $mg = 30(9.81) = 294 \text{ N}$
 $\bar{r} = 0.075 \text{ m}, r = 0.225 \text{ m}, \bar{k} = 0.2 \text{ m}, \bar{I} = 30(0.2)^2 = 1.20 \text{ kg-m}^2$



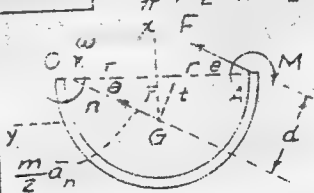
$$\sum M_o = \bar{I}\ddot{\alpha} + m(\bar{a}_o - \bar{r}\omega^2)r + m\bar{r}^2\ddot{\alpha}$$

$$294(0.225 \sin 15^\circ + 0.075 \cos 15^\circ) = 1.20 \frac{a_o}{0.225} + 30(0.2 - 0.075(27))$$

$$a_o = 3.15 \text{ m/s}^2$$

$$\sum F_y = m\ddot{y}; 294 \cos 15^\circ - N = 30(0.075) \frac{3.15}{0.225}; N = 253 \text{ N}$$

6/88 $\bar{r} = \frac{2r}{\pi}; \frac{m}{2} \bar{a}_n = \frac{m}{2} \bar{OG} \omega^2; \sum M_A = \frac{m}{2} \bar{a}_n d$



$$d = 2r \sin \theta$$

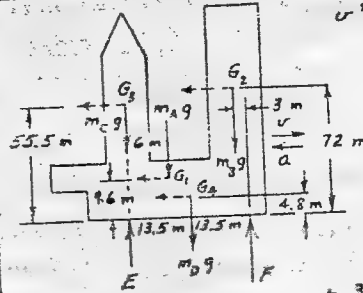
$$M = \frac{m}{2} \frac{r}{\cos \theta} \omega^2 (2r \sin \theta)$$

$$= mr^2 \omega^2 \tan \theta$$

$$= mr^2 \omega^2 \frac{2}{\pi} = \frac{2}{\pi} mr^2 \omega^2$$

$$\sum F_y = m\ddot{a}_y; V = F \cos \theta = \frac{m}{2} \bar{a}_n \cos \theta = \frac{m}{2} \bar{OG} \omega^2 \frac{r}{OG} = \frac{m r \omega^2}{2}$$

6/89 $m_A = 3 \text{ kt}, m_B = 3.3 \text{ kt}, m_C = 0.23 \text{ kt}, m_D = 3 \text{ kt}$
 $v^2 = 2as; a = \frac{v^2}{2s} = \frac{(1.5/3.6)^2}{2(0.1)} = 0.868 \text{ m/s}^2$

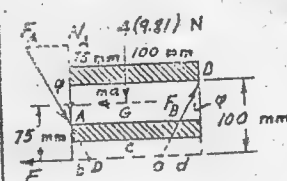


$$\sum M_E = mad;$$

$$(-3[6] - 3[13.5] - 3.3[24]) + F(27) = (3[72] + 0.23[55.5] + 3[9.6] + 3[4.8])(0.868)$$

$$F = 59.5 \text{ kN}$$

6/90 $d = 100(0.5) = 50 \text{ mm}$ $v^2 = 2as, a = 9^2/2(1.2) = 33.8 \text{ m/s}^2$
 $b = 50(0.5) = 25 \text{ mm}$
 $\phi = \tan^{-1} 0.5$ $ma = 4(33.8) = 135.0 \text{ N}$

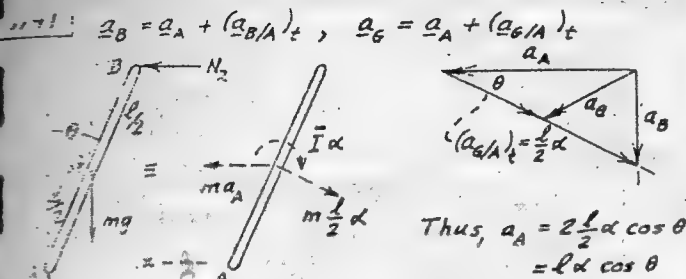


$$\sum M_o = mad;$$

(move F_A to D & get)

$$N_A(175 - 50 - 25) + 4(9.81)(100) = 135.0(75)$$

$$N_A = 81.6 \text{ N}$$



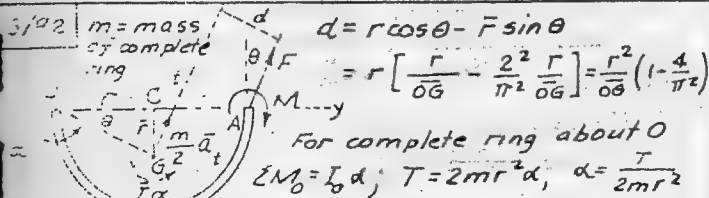
$$\Sigma M_A = \bar{I} \alpha + m \bar{a}_d$$

$$mg \frac{l}{2} \sin \theta - N_2 l \cos \theta = \frac{1}{12} m l^2 \alpha + m \frac{l}{2} \alpha \frac{l}{2} - m l \alpha \cos \theta \frac{l}{2} \cos \theta \quad (1)$$

$$= m \bar{a}_x; N_2 = m \left(l \alpha \cos \theta - \frac{l}{2} \alpha \cos \theta \right) \quad (2)$$

Substitute N_2 & get

$$\alpha = \frac{3g}{2l} \sin \theta$$



For split ring, $\Sigma M_A = \bar{I} \alpha - \frac{m}{2} \bar{a}_t d$

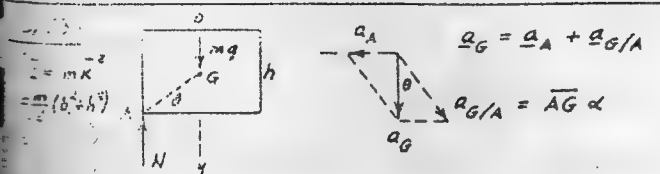
where $\bar{I} = I_C - \frac{m}{2} r^2 = \frac{m}{2} (r^2 - \frac{4r^2}{\pi^2}) = \frac{mr^2}{2} (1 - \frac{4}{\pi^2})$

$$\Sigma M_A = \frac{mr^2}{2} (1 - \frac{4}{\pi^2}) \frac{T}{2mr^2} - \frac{m}{2} \left(\frac{OG}{2mr^2} \right) \frac{r^2}{OG} (1 - \frac{4}{\pi^2})$$

$$= \frac{T}{4} (1 - \frac{4}{\pi^2}) - \frac{T}{4} (1 - \frac{4}{\pi^2}) = 0, \quad M = 0$$

$$F = m \bar{a}_t; v = F \sin \theta = \frac{m}{2} \bar{a}_t \sin \theta = \frac{m}{2} \left(\frac{OG}{2mr^2} \right) \frac{r^2}{OG} \frac{2r}{\pi}$$

$$v = \frac{T}{2\pi r}$$



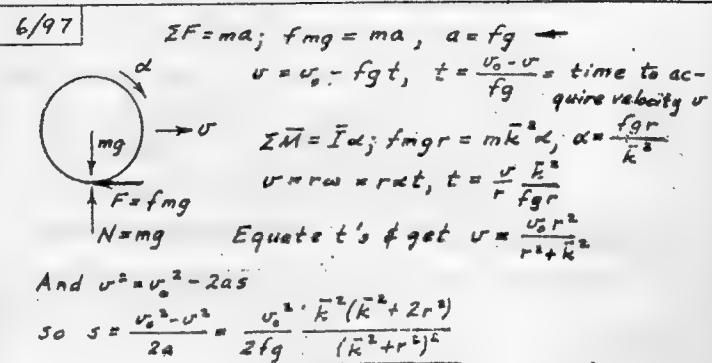
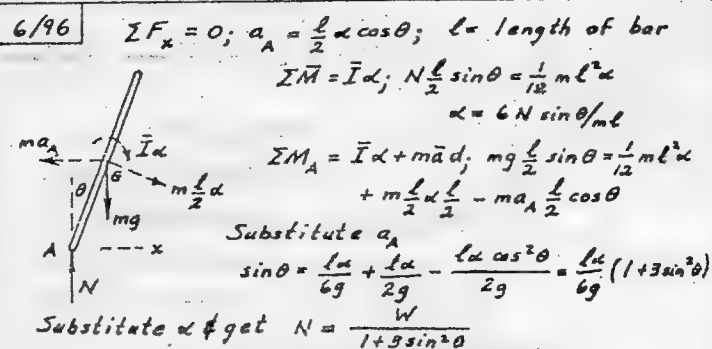
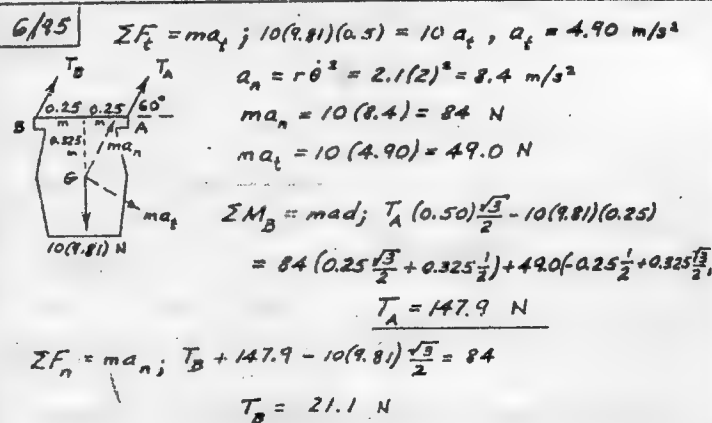
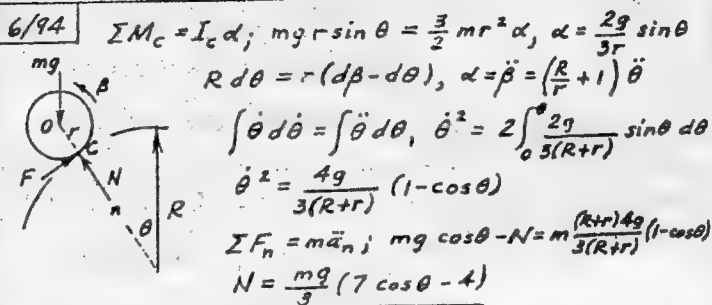
$$\Sigma F_y = m \bar{a}_y; mg - N = m a_G \text{ where } a_G = \bar{AG} \alpha \cos \theta = b \alpha / 2$$

$$\Sigma M = \bar{I} \alpha; N \frac{b}{2} = m \bar{k}^2 \alpha$$

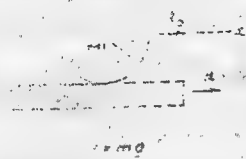
Combine & get $\alpha = \frac{2bg}{b^2 + 4k^2} = \frac{6bg}{4b^2 + h^2}$

$$a_G = \bar{AG} \alpha \sin \theta = \bar{AG} \alpha \frac{h/2}{\bar{AG}} = \frac{3bhg}{4b^2 + h^2}$$

or $a_A = \frac{3g}{4 \frac{b}{h} + \frac{h}{b}}$



16/74



$$\alpha = (a_0/b)_r = \frac{a-a_0}{r}$$

$$\sum \tau = 0 \Rightarrow F r = m a_0 r \Rightarrow F = m a_0$$

$$\sum F_x = 0 \Rightarrow F \cos \theta = m a_0 \Rightarrow a_0 = \frac{F \cos \theta}{m}$$

$$\sum F_y = 0 \Rightarrow F \sin \theta = m a_0 \Rightarrow a_0 = \frac{F \sin \theta}{m}$$



$$\begin{aligned} \sum \tau &= 0 \Rightarrow F r = m a_0 r \Rightarrow F = m a_0 \\ \sum F_x &= 0 \Rightarrow F \cos \theta = m a_0 \Rightarrow a_0 = \frac{F \cos \theta}{m} \\ \sum F_y &= 0 \Rightarrow F \sin \theta = m a_0 \Rightarrow a_0 = \frac{F \sin \theta}{m} \end{aligned}$$

$$\alpha = \frac{a \cos \theta - \frac{1}{2} a \cos \theta}{r} = \frac{\frac{1}{2} a \cos \theta}{r}$$

$$\sum \tau = 0 \Rightarrow F r = m a_0 r \Rightarrow F = m a_0$$

$$\sum F_x = 0 \Rightarrow F \cos \theta = m a_0 \Rightarrow a_0 = \frac{F \cos \theta}{m}$$

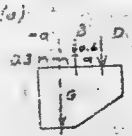
$$\sum F_y = 0 \Rightarrow F \sin \theta = m a_0 \Rightarrow a_0 = \frac{F \sin \theta}{m}$$

$$\alpha = \frac{a \cos \theta - \frac{1}{2} a \cos \theta}{r} = \frac{\frac{1}{2} a \cos \theta}{r}$$

$$\alpha = \frac{a \cos \theta - \frac{1}{2} a \cos \theta}{r} = \frac{\frac{1}{2} a \cos \theta}{r}$$

$$\alpha = \frac{a \cos \theta - \frac{1}{2} a \cos \theta}{r} = \frac{\frac{1}{2} a \cos \theta}{r}$$

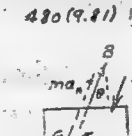
16/101



$$m a = 480 \frac{1.5^2}{0.6} = 1800 \text{ N}$$

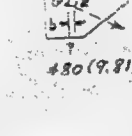
$$\sum M_B = m a d; 0.6 D - 0.3(480)(9.81) = 0.3(1800)$$

$$D = 3250 \text{ N}$$



$$(b) \sum F_x = m a_x; 480(9.81) \sin \theta = 480(0.6) \sin \theta$$

$$\theta = -16.35^\circ$$



$$\int \omega d\omega = \int (-16.35 \sin \theta) d\theta$$

$$\frac{\omega^2}{2} \Big|_0^\theta = 16.35 \cos \theta \Big|_0^\theta, \cos \theta - 1 = -\frac{2.35}{0.72(16.35)}$$

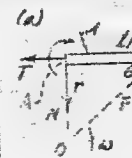
$$\theta = 36.01^\circ$$

$$\sum M_B = 0 \text{ with } a_n = r\omega^2 = 0$$

$$0(0.6 \cos 36.01^\circ) = 480(9.81)b \text{ where } b = \frac{0.3-0.9 \tan \theta}{\sec \theta} = 0.053$$

$$D = \frac{480(9.81)(0.053)}{0.6(0.8029)} = 520 \text{ N}$$

16/102



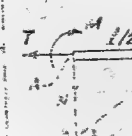
$$R = m a_n, \sum M_A = m a d; M = m r \omega^2 \frac{1}{2} \sin \theta$$

$$M = \frac{m r \omega^2}{2}$$

$$\sum M_B = 0; T r - M = 0, T = \frac{m r \omega^2}{2}$$

$$\sum M = 0; N \frac{1}{2} - M = 0, N = m r \omega^2$$

(b)



$$\sum M_A = \bar{I} \alpha + m a d;$$

$$-M = \frac{1}{2} m l^2 \alpha + m r \alpha \frac{1}{2} \cos \theta$$

$$M = -\frac{m l^2 \alpha}{3}$$

$$\sum F_x = m a_x; N = m r \alpha \cos \theta, N = -\frac{1}{2} m l \alpha$$

$$\sum F_y = m a_y; T = m r \alpha \sin \theta, T = m l \alpha$$

16/103



$$(a) \alpha = 0; \sum M_C = 0; O_y = 0$$

$$\sum M_B = m a d; A r = m \frac{2\sqrt{2}}{\pi} r \omega^2 \frac{r}{\sqrt{2}}$$

$$A = \frac{2}{\pi} m r \omega^2$$

$$\text{By symm, } \sum M_A = m a d, \text{ or } \sum F_n = 0$$

$$0 = -O_x = A = \frac{2}{\pi} m r \omega^2$$

$$(b) \omega = 0; \sum M_C = \bar{I}_C \alpha; O_y r = m r^2 \alpha, O_y = m r \alpha$$

$$\sum F_x = m a_x; O_x = m \frac{2\sqrt{2}}{\pi} r \alpha \frac{1}{\sqrt{2}} = \frac{2}{\pi} m r \alpha$$

$$\sum F_y = m a_y; O_y - B = m \frac{2\sqrt{2}}{\pi} r \alpha \frac{1}{\sqrt{2}}, B = m r \alpha - \frac{2}{\pi} m r \alpha$$

$$= m r \alpha \left(1 - \frac{2}{\pi}\right)$$

$$\text{Thus } O = \sqrt{O_x^2 + O_y^2} = m r \alpha \sqrt{1 + \frac{4}{\pi^2}}$$

6/104 $\Sigma F_r = m\ddot{a}_r$; $2T \sin \frac{d\theta}{2} + dT \sin \frac{d\theta}{2} + N \cos \frac{d\theta}{2} - (N+dN) \cos \frac{d\theta}{2} = \rho r d\theta r \omega^2$

Simplify & get $T - \rho r^2 \omega^2 = \frac{dN}{d\theta} \dots (1)$
 $\Sigma F_t = m\ddot{a}_t = 0$; $T \cos \frac{d\theta}{2} - (T+dT) \cos \frac{d\theta}{2} - N \sin \frac{d\theta}{2} - (N+dN) \sin \frac{d\theta}{2} = 0$
 Simplify & get $N = -\frac{dT}{d\theta} \dots (2)$

Combine (1) & (2) & get $\frac{d^2 N}{d\theta^2} + N = 0$.
 Sol. is $N = A \sin \theta + B \cos \theta$.
 By symm. $N = 0$ for $\theta = 0$ so $B = 0$ & $N = A \sin \theta$
 From (1) $T = \rho r^2 \omega^2 + A \cos \theta$. $T = 0$ for $\theta = \pi$ so $A = \rho r^2 \omega^2$
 Thus $N = \rho r^2 \omega^2 \sin \theta$ & $T = \rho r^2 \omega^2 (1 + \cos \theta)$

$\Sigma M_c = m\ddot{a}_c$; $M_c = m\bar{r}\omega^2 r$
 $= \rho \pi r \frac{2r}{\pi} \omega^2 r$
 $M_c = 2\rho r^3 \omega^2$
 $\bar{r} = \frac{2r}{\pi}$

6/105 $\Sigma M_o = I_o \alpha$; $mg \frac{l}{2} \cos \theta = \frac{1}{3} m l^2 \alpha$, $\alpha = \frac{3g}{2l} \cos \theta$
 $\Sigma F_t = m\ddot{a}_t$; $-Q_t + mg \cos \theta = m \frac{l}{2} \frac{3g}{2l} \cos \theta$, $Q_t = \frac{mg}{4} \cos \theta$

$\Sigma F_c = m\ddot{a}_c$; $\frac{x}{l} mg \cos \theta - \frac{1}{4} mg \cos \theta - Q = \frac{x}{l} m \frac{3g}{2} \frac{x}{l} \cos \theta$
 $-Q = mg \cos \theta \left[\frac{1}{4} - \frac{x}{l} + \frac{3}{4} \left(\frac{x}{l} \right)^2 \right]$
 $\Sigma M_o = I_o \alpha$; $\frac{x}{l} mg \cos \theta \frac{x}{2} - M + mg \cos \theta \left[\frac{1}{4} - \frac{x}{l} + \frac{3}{4} \left(\frac{x}{l} \right)^2 \right] x = \frac{1}{3} \frac{x}{l} m x \frac{3g}{2} \frac{x}{l} \cos \theta$

Solve for M & get $M = mg \frac{x}{4} \left(1 - \frac{x}{l} \right)^2 \cos \theta$
 $\frac{dM}{dx} = \frac{mg}{4} \cos \theta \left(1 - \frac{4x}{l} + \frac{3x^2}{l^2} \right) = 0$ gives $x = \frac{l}{3}$ (max.)
 $x = l$ (min.)

$M_{\max} = \frac{mg l}{27} \cos \theta$

6/106 Entire bar: $\Sigma F_y = m\ddot{a}_y$; $T \sin \theta = m\ddot{a}_y$, $\ddot{a}_y = \frac{T \sin \theta}{m}$

$\Sigma \bar{M} = \bar{I} \alpha$;
 $T \frac{l}{2} \sin \theta = \frac{1}{12} m l^2 \alpha$, $\alpha = \frac{6T \sin \theta}{m l}$
 $(a_G)_y = \ddot{a}_y - \frac{x}{2} \alpha = \frac{T \sin \theta}{m} \left(1 - \frac{3x}{l} \right)$

For section
 $\Sigma M_{cut} = \bar{I} \alpha - m(a_G)_y d$
 so $M = \frac{1}{12} \frac{l-x}{l} m (l-x)^2 \frac{6T \sin \theta}{m l}$
 $- \frac{l-x}{l} m \frac{T \sin \theta}{m} \left(1 - \frac{3x}{l} \right) \frac{l-x}{2}$

Simplify & get $M = \left(\frac{l-x}{l} \right)^2 x T \sin \theta$

6/111 $\Delta V_g + \Delta T = 0$; $-6mg + \frac{1}{2} m (v^2 - 1.8^2) = 0$
 where $g = \text{lunar gravity } 1.62 \text{ m/s}^2$
 $v^2 = 2(6)(1.62) + (1.8)^2 = 22.7$, $v = 4.76 \text{ m/s}$

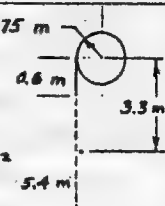
6/112 $\Delta V_g + \Delta T = 0$; $-2mg \frac{l}{2} \sin \theta + \frac{1}{2} I_c \omega^2 + \frac{1}{2} I_A \omega^2 = 0$
 $I_A = I_c = \frac{1}{3} m l^2$; thus $mg l \sin \theta = \frac{1}{3} m l^2 \left(\frac{v}{l} \right)^2$
 $v = \sqrt{3gl \sin \theta}$

6/113 $\Delta V_g + \Delta T = 0$; $\Delta V_g = -mg(9 - 2.5) = -6.5 mg$
 $\Delta T = \frac{1}{2} m v^2$
 $-6.5 mg + \frac{1}{2} m v^2 = 0$; $v^2 = 13g$, $v = 11.29 \text{ m/s}$

6/114 $\Delta V_g + \Delta T = 0$; Let $m = \text{mass of AB}$
 so $2m = \text{ " " OA}$
 $-2mg(0.9/2) - mg(0.9/2) + \frac{1}{2} \frac{1}{3} 2m(0.9)^2 \left(\frac{v}{0.9} \right)^2 + \frac{1}{2} m v^2 = 0$
 $v^2 = 15.89$, $v = 3.99 \text{ m/s}$

6/115 $\Delta V_g + \Delta V_c + \Delta T = 0$
 $-30(9.81)(1.2) + \frac{1}{2} 3000(2.4 - 1.2\sqrt{2})^2 - \frac{1}{2} \frac{1}{3} 30(2.4)^2 \omega^2 = 0$
 $\omega^2 = 13.47$, $\omega = 3.67 \text{ rad/s}$

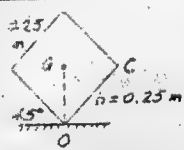
6/116 $\Delta V_g + \Delta T = 0$
 $\Delta V_g = -5.4(3.08)(9.81)(3.3) = -538 \text{ J}$
 $\Delta T = \frac{1}{2} 6.0(3.08)(0.375 \omega)^2$
 $+ \frac{1}{2} [41(0.30)^2 + (3.08)(18-6)(0.375)^2] \omega^2$
 $\Delta T = 1.299 \omega^2 + 4.44 \omega^2 = 5.74 \omega^2$
 Thus $-538 + 5.74 \omega^2 = 0$, $\omega^2 = 93.8$,
 $\omega = 9.68 \text{ rad/s}$



6/117 $\Delta V_g + \Delta V_c + \Delta T = 0$
 $\Delta V_g = -7.5(9.81)(0.6) = 44.1 \text{ J}$
 $\Delta V_c = \frac{1}{2} 45(\sqrt{1.8^2 + 1.2^2} - 0.6)^2 = 55.0 \text{ J}$
 $\Delta T = 0 - \frac{1}{2} \frac{1}{8} 7.5(1.2)^2 \omega^2 = -1.800 \omega^2$
 $-44.1 + 55.0 - 1.800 \omega^2 = 0$, $\omega^2 = 6.03$, $\omega = 2.45 \text{ rad/s}$

6/118 $\Delta V_g = -mg(2\pi l) \sin \theta$
 $\Delta T = \frac{1}{2} I_c \omega^2 = \frac{1}{2} m \left(\frac{l}{12} + \left[\frac{3l}{2} \right]^2 \right) \omega^2 = \frac{7}{6} m l^2 \omega^2$
 $= \frac{7}{6} m v^2$
 $U = 0 = \Delta V_g + \Delta T$; $0 = -2\pi l mg \sin \theta + \frac{7}{6} m v^2$
 $v = 2 \sqrt{\frac{3\pi l g \sin \theta}{7}}$

6/119



$$\Delta V_g + \Delta T = 0; -mg\left(\frac{b}{\sqrt{2}} - \frac{b}{2}\right) + \frac{1}{2} I_o \omega^2 = 0$$

$$\omega = v_c/b, I_o = I + mr^2 = \frac{1}{2} mb^2 + m\left(\frac{b}{\sqrt{2}}\right)^2 = \frac{3}{4} mb^2$$

$$\text{so } \frac{mgb}{2}(\sqrt{2}-1) = \frac{1}{2} \cdot \frac{3}{4} mb^2 \frac{v_c^2}{b^2}, v_c^2 = \frac{3(\sqrt{2}-1)}{2} gb$$

$$v_c = \sqrt{\frac{3(\sqrt{2}-1)(9.81)(0.25)}{2}} = \sqrt{1.524} = 1.234 \text{ m/s}$$

6/120

$$\Delta V_g + \Delta V_e + \Delta T = 0$$

$$\Delta V_g = -15(9.81)(0.3) = -44.1 \text{ J}, \Delta V_e = \frac{1}{2} 20(10^3) h^2 = 10^4 h^2 \text{ J}$$

$$\Delta T = \frac{1}{2} \cdot \frac{1}{3} 15(0.6)^2 (4)^2 = 14.40 \text{ J}$$

$$-44.1 - 10^4 h^2 + 14.40 = 0, h^2 = 29.7(10^{-4}), h = 54.5 \text{ mm}$$

thus has no effect on the energy change but does influence the bearing force at O.

6/121

$$U = \Delta T + \Delta V_g + \Delta V_e$$

$$U = M\theta = 880 \pi/2 = 1382 \text{ J}$$

$$\Delta V_e = \frac{1}{2} 450(1.2)^2 = 324 \text{ J}$$

$$\Delta V_g = 100(9.81)(0.6) = 589 \text{ J}$$

$$\Delta T = \frac{1}{2} I_o \omega^2 = \frac{1}{2} \left[\frac{1}{12} 100(1.2)^2 + 100(0.6)^2 \right] \omega^2 = 24.0 \omega^2$$

$$1382 = 24.0 \omega^2 + 324 + 589, \omega^2 = 19.54, \omega = 4.42 \text{ rad/s}$$

6/122

Note: the wheel has no motion in initial & final positions, so $\Delta T_{\text{wheel}} = 0$.

$$U = \Delta V_g + \Delta T, U = Fb \sin \theta$$

$$\Delta V_g = -2m_0 g \frac{b}{2} \sin \theta$$

$$\Delta T = 2\left(\frac{1}{2} I_c \omega^2\right) = \frac{1}{3} m_0 b^2 \omega^2$$

$$\text{Thus } Fb \sin \theta = -m_0 g b \sin \theta + \frac{1}{3} m_0 b^2 \omega^2$$

$$\omega = \sqrt{\frac{3(F + m_0 g) \sin \theta}{m_0 b}}$$

6/123

Let ℓ_0 = unstretched length of spring

$$\Delta V_g = 0 \text{ (horizontal plane)}, U = 0$$

$$\Delta V_e = \frac{1}{2} 1400 ([\overline{BC} - \ell_0]^2 - [0.10 - \ell_0]^2)$$

$$\overline{BC}^2 = [2(0.05) \cos 15^\circ]^2 = 0.00933 \text{ m}^2$$

$$\overline{BC} = 0.0966 \text{ m}$$

$$v_A = 0.25 \text{ m/s}$$

$$\Delta T = \frac{1}{2} I_o \omega^2 = \frac{1}{2} 1.5(0.055)^2 (0.25/0.060)^2 = 0.0394 \text{ J}$$

$$U = \Delta V_g + \Delta V_e + \Delta T$$

$$0 = 0 + 700([0.0966 - \ell_0]^2 - [0.10 - \ell_0]^2) + 0.0394$$

$$0 = 0 + 700(0.00933 - 0.1932\ell_0 - 0.0100 + 0.20\ell_0) + 0.0394$$

$$+7.7\ell_0 = 0.430, \ell_0 = 0.0559 \text{ m} = 55.9 \text{ mm}$$

6/124

$$U = \Delta T + \Delta V_g + \Delta V_e; U = M\theta = 12 \frac{\pi}{4} = 9.42 \text{ J}$$

$$\Delta T = \frac{1}{2} \left(\frac{1}{12} 6(0.50)^2 \right) \omega^2 + \frac{1}{2} \left(\frac{1}{3} 3(0.25)^2 \right) \omega^2$$

$$= 0.0625 \omega^2 + 0.0312 \omega^2 = 0.0938 \omega^2 \text{ J}$$

$$\Delta V_g = 3(9.81) \left([0.25 + \frac{0.25}{2}] - [0.25 + \frac{0.25}{2}] / \sqrt{2} \right) = 3.23 \text{ J}$$

$$\Delta V_e = \frac{1}{2} 140(x_2^2 - x_1^2) \text{ where } x_2 = 0.15 + 0.50(1 - \frac{1}{\sqrt{2}}) = 0.296 \text{ m}$$

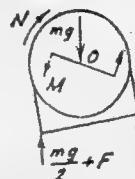
$$= 70(0.296^2 - 0.15^2) = 7.15 \text{ J}$$

$$\text{Thus } 9.42 = 0.0938 \omega^2 + 3.23 + 7.15$$

$$\omega^2 = 17.23, \omega = 4.15 \text{ rad/s}$$

6/125

$$P = 2\pi MN/60, M = \frac{4500(60)}{2\pi(1725)} = 24.9 \text{ N}$$



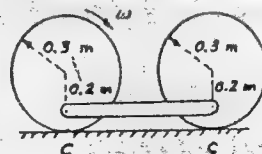
$$\Sigma M_o = 0; 2F(0.4) - 24.9 = 0, F = 31.1 \text{ N}$$

$$F = kx; x = \frac{31.1}{10} = 3.11 \text{ mm}$$

$$\delta = \tan^{-1} \frac{3.11}{200} = 0.892^\circ, \text{ CW}$$

6/126

$$v_{\text{bar}} = (0.3 - 0.2)\omega = 0.1\omega \text{ in lowest position}$$



$$\Delta V_g = -24(9.81)(0.4) = -94.2 \text{ J}$$

$$\Delta T_{\text{bar}} = \frac{1}{2} 24(0.1\omega)^2 = 0.12\omega^2 \text{ J}$$

$$\Delta T_{\text{wheels}} = 2\left(\frac{1}{2} I_c \omega^2\right) = \frac{3}{2} 16(0.3)^2 \omega^2 = 2.16\omega^2 \text{ J}$$

$$\Delta V_g + \Delta T = 0$$

$$\text{Thus } -94.2 + 0.12\omega^2 + 2.16\omega^2 = 0$$

$$\omega^2 = 41.3, \omega = 6.43 \text{ rad/s}$$

6/127

$$0 = \Delta V_g + \Delta T; \Delta V_g = -100(9.81)[0.3 \sin 30^\circ + 0.45(1 - \cos 30^\circ)] = -206 \text{ J}$$

$$\Delta T = \frac{1}{2} I_c (\omega_2^2 - \omega_1^2)$$

$$I_c = 100(0.7^2 + 0.15^2) = 3.25 \text{ kg}\cdot\text{m}^2$$

$$\omega_1 = \frac{0.6}{0.15} = 4 \text{ rad/s}, \omega_2 = \frac{v}{0.15}$$

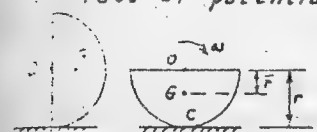
$$\text{so } \Delta T = \frac{1}{2} 3.25 (v^2/0.0225 - 4^2) = 72.2 v^2 - 26 \text{ J}$$

$$\text{Thus } 0 = -206 + 72.2 v^2 - 26$$

$$v^2 = 3.22, v = 1.793 \text{ m/s}$$

$$\Sigma F_n = m\ddot{a}_n; N - 100(9.81) = 100 \frac{3.22}{0.45}, N = 1696 \text{ N}$$

6/129 Max. kinetic energy occurs when loss of potential energy is greatest



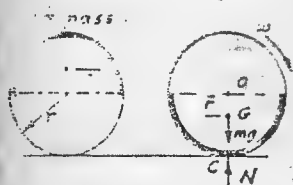
$$\Delta V_g + \Delta T = 0; \\ -mg \frac{4r}{3\pi} + \frac{1}{2} I_c \omega^2 = 0$$

Let $m = \text{mass}$; $\bar{r} = 4r/3\pi$

$$I_c = I_g + m(r - \bar{r})^2 = I_g - m\bar{r}^2 + m(r - \bar{r})^2 \\ = \frac{1}{2}mr^2 + mr^2 - 2m\bar{r}r = mr^2\left(\frac{3}{2} - \frac{8}{9\pi}\right)$$

$$\text{Thus } mg \frac{4r}{3\pi} = \frac{1}{2}mr^2\left(\frac{3}{2} - \frac{8}{9\pi}\right)\omega^2, \quad \omega = 4\sqrt{\frac{g/r}{9\pi - 16}}$$

6/129 $\Delta V_g + \Delta T = 0; -mg(2\bar{r}) + \frac{1}{2}I_c \omega^2 = 0$



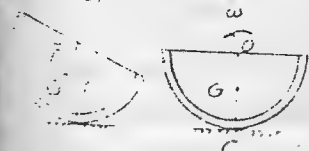
$$I_c = I_g + m(r - \bar{r})^2 \\ = I_g - m\bar{r}^2 + m(r - \bar{r})^2 \\ = mr^2 + mr^2 - 2m\bar{r}r \\ = 2mr^2\left(1 - \frac{2}{\pi}\right)$$

$$\text{so } \frac{2}{2}mr^2\left(1 - \frac{2}{\pi}\right)\omega^2 = 2\frac{2r}{\pi}mg, \quad \omega^2 = \frac{4g}{(\pi - 2)r}$$

$$\sum F_n = m\bar{a}_n; N - mg = m(\bar{r}\omega^2), \quad N = mg\left(1 + \frac{2r}{\pi g r(\pi - 2)}\right)$$

$$\text{or } N = mg\left(1 + \frac{8}{\pi(\pi - 2)}\right)$$

$$6/130 \quad \bar{r} = \frac{2}{3}mr^2, \quad \bar{r} = r/2; \quad \bar{I} = \frac{2}{3}mr^2 - m\left(\frac{r}{2}\right)^2 \\ I_c = \frac{2}{3}mr^2 - m\frac{r^2}{4} + m\frac{r^2}{4} = \frac{2}{3}mr^2$$



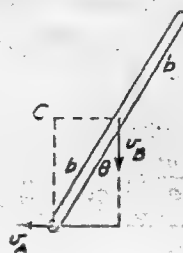
$$\Delta T = \frac{1}{2}I_c \omega^2 = \frac{1}{3}mr^2 \omega^2$$

$$\Delta V_g = -mg\bar{r}(1 - \cos\theta) \\ = -\frac{mg r}{2}(1 - \cos\theta)$$

$$\text{Thus } \frac{1}{3}mr^2 \omega^2 = \frac{mg r}{2}(1 - \cos\theta)$$

$$\omega = \sqrt{\frac{3g}{2r}(1 - \cos\theta)}$$

6/131 $C = \text{instantaneous center}$



$$\Delta V_g + \Delta T = 0$$

$$\Delta V_g = mg(b \cos\theta - b) = -mgb(1 - \cos\theta)$$

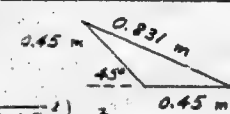
$$\Delta T = \frac{1}{2}I_c \omega^2$$

$$\omega = \frac{v_A}{b \cos\theta}, \quad I_c = \frac{1}{12}m(2b)^2 + m(b \sin\theta)^2 \\ = mb^2\left(\frac{1}{3} + \sin^2\theta\right)$$

$$\text{Thus } -mgb(1 - \cos\theta) + \frac{mb^2}{2}\left(\frac{1}{3} + \sin^2\theta\right)\left(\frac{v_A}{b \cos\theta}\right)^2 = 0$$

$$\text{Solve for } v_A \text{ \& get } v_A = \cos\theta \sqrt{\frac{6gb(1 - \cos\theta)}{1 + 3 \sin^2\theta}}$$

6/132 $U = \Delta T + \Delta V_g + \Delta V_c$



$$U = M\theta = 18 \frac{\pi}{4} = 14.14 \text{ J}$$

$$\Delta T = \frac{1}{2}15(0.45\omega)^2 + \frac{1}{2}\left(\frac{1}{3}3(0.45)^2\right)\omega^2 \\ = 1.519\omega^2 + 0.405\omega^2 = 1.924\omega^2 \text{ J}$$

$$\Delta V_g = -15(9.81)(0.45)(1 - 0.707) - 4(3)(9.81)\frac{0.45}{2}(1 - 0.707) = -27.2 \text{ J}$$

$$\Delta V_c = 2\frac{1}{2}700(0.831 - 0.45\sqrt{2})^2 = 26.6 \text{ J}$$

$$\text{so } 14.14 = 1.924\omega^2 - 27.2 + 26.6, \quad \omega^2 = 7.67$$

$$\omega = 2.77 \text{ rad/s}$$

6/133 Let $\rho = \text{mass per unit length of bar}$

$$m = 4\rho c + 4\rho(2b) = 4\rho(c + 2b)$$

$$U = mgb \cos \frac{\theta}{2} = 4\rho(c + 2b)gb \cos \frac{\theta}{2}$$

By symmetry A & B move vertically, so each leg is rotating about its bottom end as the upper ends approach the floor.

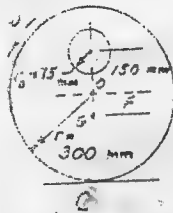
$$\text{so } \Delta T_{\text{horiz. members}} = 2\frac{1}{2}\rho c v^2 = \rho c v^2$$

$$\Delta T_{\text{legs}} = 4\frac{1}{2}I_{\text{end}} \omega^2 = 4\frac{1}{2}\rho(2b)(2b)\left(\frac{v}{2b}\right)^2 = \frac{4}{3}\rho b v^2$$

$$U = \Delta T; 4\rho g b(c + 2b) \cos \frac{\theta}{2} = \rho c v^2 + \frac{4}{3}\rho b v^2$$

$$v = \sqrt{\frac{12gb(c + 2b) \cos \frac{\theta}{2}}{3c + 4b}}$$

6/134

 ω is greatest when loss of V_g is greatestLet m = mass of disk without hole m_0 = mass removed for hole

$$I_c = \frac{3}{2} m r^2 - \left(\frac{1}{2} m_0 r_0^2 + m_0 d^2 \right)$$

$$\text{But } m - m_0 = 75 \text{ kg}, \frac{m_0}{m} = \left(\frac{75}{300} \right)^2 = \frac{1}{16}$$

$$16 m_0 - m_0 = 75,$$

$$m_0 = 5 \text{ kg}, m = 80 \text{ kg}$$

$$75 \bar{r} = 80(0) - 5(-150), \bar{r} = 10 \text{ mm}$$

$$\text{Thus } I_c = \frac{3}{2} 80(0.3)^2 - \left(\frac{1}{2} 5[0.075]^2 + 5[0.45]^2 \right)$$

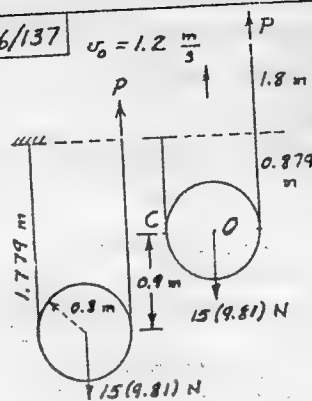
$$= 10.80 - 1.027 = 9.77 \text{ kg} \cdot \text{m}^2$$

$$\Delta V_g + \Delta T = 0; -75(9.81)(0.010) + \frac{1}{2} 9.77 \omega^2 = 0$$

$$\omega = 1.506, \omega = 1.227 \text{ rad/s}$$

46/137

$$v_0 = 1.2 \frac{\text{m}}{\text{s}}$$



$$\Delta T_{\text{pulley}} = \frac{1}{2} I_c \omega^2$$

$$= \frac{1}{2} 15 \left(0.25^2 + 0.30^2 \right) \left(\frac{1.2}{0.3} \right)^2 = 18.30 \text{ J}$$

$$\Delta T_{\text{cable}} = \frac{1}{2} m v^2 + \frac{1}{2} I_c \omega^2$$

$$= \frac{1}{2} 3(2.68)^2 + \frac{1}{2} \left[\frac{1}{2} 0.6 \pi 3 \left(\frac{1.2}{0.3} \right)^2 \right]$$

$$= 23.1 + 4.07 = 27.2 \text{ J}$$

$$\Delta V_g = 15(9.81)(0.9) + 3(9.81)(0.3\pi)(0.9) + 3(9.81) \left[(0.9)(1.779) + 0.1(2.67) \right]$$

$$= 275 \text{ J}$$

$$U = \Delta T + \Delta V_g; 1.8 P = 27.2 + 18.30 + 275$$

$$P = 178.3 \text{ N}$$

6/135

$$U = \Delta T$$

$$\text{For hoops } T = 2(T_{\text{hoops}} + T_{\text{top section}}), T_{\text{bottom section}} = 0$$

$$T_{\text{hoops}} = \frac{1}{2} I_c \omega^2 = \frac{1}{2} [2\pi r \rho (r^2 + r^2) \frac{v^2}{r^2}] = 2\pi \rho r v^2$$

$$T_{\text{top section}} = \frac{1}{2} m (2v)^2 = \frac{1}{2} \rho b 4v^2 = 2\rho b v^2$$

$$U = M\theta = M \frac{s}{r}$$

$$\text{Thus } M \frac{s}{r} = 2[2\pi \rho r v^2 + 2\rho b v^2], M = 4\rho \frac{r}{s} v^2 (\pi r + b)$$

6/136

$$a_0 = a_c - r\alpha = a - r\alpha$$

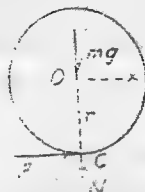
$$\sum F_x = ma_0; F = m(a - r\alpha)$$

$$\sum \bar{M} = \bar{I}\alpha; Fr = \frac{1}{2} m r^2 \alpha$$

$$\text{so } \frac{1}{2} m r \alpha = m(a - r\alpha), \alpha = \frac{2a}{3r}$$

$$\text{Time } t \text{ to turn through } 2\pi \text{ is}$$

$$\theta = \frac{1}{2} \alpha t^2; t^2 = 2(2\pi) \frac{3r}{2a} = \frac{6\pi r}{a}$$



Distance moved by C is

$$s = \frac{1}{2} a t^2 = \frac{1}{2} a \frac{6\pi r}{a} = 3\pi r$$

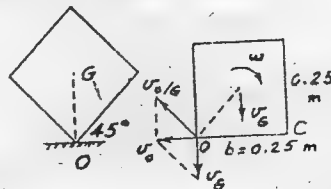
$$\text{Work done by } F \text{ is } U = Fs = 3\pi r \left(\frac{m r}{2} \frac{2a}{3r} \right)$$

$$U = \pi r m a$$

46/138

$$\Delta V_g + \Delta T = 0; -mg \left(\frac{b}{\sqrt{2}} - \frac{b}{2} \right) + \frac{1}{2} \bar{I} \omega^2 + \frac{1}{2} m \bar{v}^2 = 0$$

$$v_0 = v_G + v_{G/G}, v_{G/G} = \frac{b}{\sqrt{2}} \omega$$



From diagram

$$\left(\frac{b}{\sqrt{2}} \omega \right) \frac{1}{\sqrt{2}} = v_G (= \bar{v})$$

$$b\omega = 2v_G$$

$$\bar{I} = \frac{1}{6} m b^2$$

$$\text{Thus } \frac{mgb}{2} (\sqrt{2} - 1) = \frac{1}{2} \frac{1}{6} m b^2 \omega^2 + \frac{1}{2} m \left(\frac{b\omega}{2} \right)^2$$

$$\omega^2 = \frac{12g}{5b} (\sqrt{2} - 1) = \frac{12(9.81)}{5(0.25)} (\sqrt{2} - 1) = 39.0, \omega = 6.25 \text{ rad/s}$$

$$v_0 = |v_G| = \frac{b\omega}{2} = \frac{0.25(6.25)}{2} = 0.781 \text{ m/s}$$

46/139

Replace P by force P at E and couple

$$M = P(2b); dU = P \cos \theta d(-2b \sin \theta) + M(-d\theta)$$

$$= -2Pb \cos^2 \theta - 2Pb d\theta$$

$$U = -Pb \int_{\pi/2}^0 (2 \cos^2 \theta + 2) d\theta$$

$$M = P(2b) = Pb \left[\theta + \frac{\sin 2\theta}{2} + 2\theta \right]_0^{\pi/2} = 3Pb\pi$$

$$\bar{v} = v_E = -\frac{d}{dt} (2b \sin \theta) = -2b \dot{\theta} \cos \theta$$

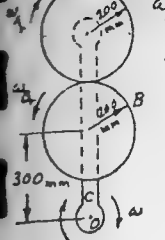
$$\omega = -\dot{\theta} \text{ so } \bar{v} = 2b\omega \cos \theta$$

$$\Delta T = \frac{1}{2} \bar{I} \omega^2 + \frac{1}{2} m \bar{v}^2 = \frac{1}{2} \frac{1}{12} m (4b)^2 \omega^2 + \frac{1}{2} m (2b\omega \cos \theta)^2$$

$$\Delta T_{\theta=0} = \frac{2}{3} m b^2 \omega^2 + 2m b^2 \omega^2 = \frac{8}{3} m b^2 \omega^2$$

$$U = \Delta T; \frac{3Pb\pi}{2} = \frac{8}{3} m b^2 \omega^2, \omega = \frac{3}{4} \sqrt{\frac{P\pi}{mb}}$$

6/171 $\omega_A =$ angular velocity of A relative to C } = 1720
 $\omega_B =$ " " " B " " " rev/min



$$\Sigma M_o = \dot{H}_o = 0 \text{ so } H_o = \text{const.}, H_{o, \text{before}} = H_{o, \text{after}}$$

$$I_{A_0} = 18(0.085)^2 + 18(0.70)^2 = 0.1301 + 8.82$$

$$= 8.95 \text{ kg} \cdot \text{m}^2$$

$$I_{B_0} = 5(0.140)^2 + 5(0.300)^2 = 0.0980 + 0.450$$
$$= 0.548 \text{ kg}\cdot\text{m}^2$$

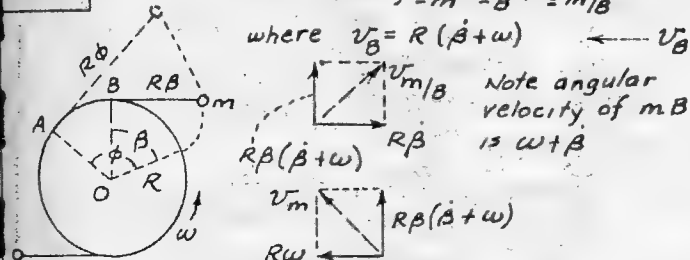
$$\omega_0 = 30 \text{ rev/min} \quad I_{C_0} = 24(0.450)^2 = 4.86 \text{ kg}\cdot\text{m}^2$$

$$H_{\text{before}} = (8.95 + 0.548 + 4.86) \frac{30(2\pi)}{60} = 431 \frac{2\pi}{60} \text{ kg}\cdot\text{m}^2/\text{s}$$

$$H_{o, \text{after}} = [0.1301(\omega + 1720) + 3.82 \omega] \frac{2\pi}{60} + [0.0980(\omega - 1720) + 0.450 \omega] \frac{2\pi}{60} + 4.86 \omega \left(\frac{2\pi}{60} \right)$$

Equate, solve for ω , & get $\omega = 26.2 \text{ rev/min}$
(same direction as ω_0)

6/172 Let $B = \text{tangent point}$; $\underline{v}_m = \underline{v}_B + \underline{v}_{m/B}$



$$\Delta H_0 = 0 \text{ so } (I + 2mR^2)\omega_0 = I\omega + 2mR^2\omega + 2mR^2\beta^2(\omega + \beta) \\ \text{or } (I + 2mR^2)(\omega - \omega) = 2mR^2\beta^2(\omega + \beta) \text{ ----- (a)}$$

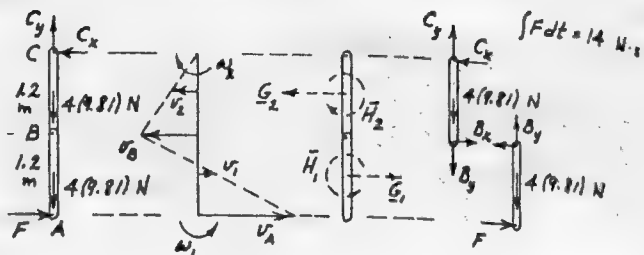
$$\Delta T = 0 \text{ SO } \frac{1}{2}(I + 2mR^2)\omega_0^2 = \frac{1}{2}I\omega^2 + 2\left(\frac{1}{2}\right)m(R^2\dot{\omega}^2 + R^2\dot{\beta}^2 + [\dot{\beta} + \dot{\omega}]^2)$$

$$\text{or } (I + 2mR^2)(\omega_0^2 - \omega^2) = 2mR^2\dot{\beta}^2(\dot{\beta} + \dot{\omega})^2 \dots\dots (b)$$

(b) \div (a); $\omega_0 + \omega = \beta + \omega$, $\beta = \omega_0$ constant

From (a), for $\beta = \phi$, $\omega_{\phi} = \frac{I + 2mR^2(1 - \phi^2)}{I + 2mR^2(1 + \phi^2)} \omega_0$

6/173 $\bar{I} = \frac{1}{12} m l^2 = \frac{1}{12} 4 (1.2)^2 = 0.48 \text{ kg} \cdot \text{m}^2$



$$\omega_2 = v_2/0.6, \omega_1 = (v_1 + 2v_2)/0.6, m = 4 \text{ kg}$$

System; $\int \Sigma \dot{M}_C dt = \Sigma \Delta H_C$; $14(2.4) = 4v_1(1.8) + 0.48w_1$
 $- 4v_2(0.6) - 0.48w_2$ [a]

$$AB; \int \dot{M}_C dt = \Delta H_C; 1/4(2.4) - \int 1.2 \dot{B}_x dt = 4u; (1.8) + 0.48u, \quad (1)$$

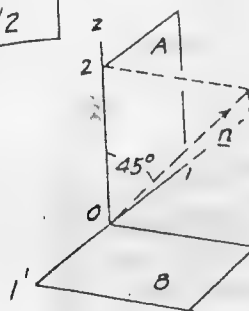
$$\int \Sigma F_x dt = \Delta G_x; 14 - \int B_x dt = 4 v_1$$

[b], [c], & w, give $2v_1 + v_2 = 10.5$; [a], w, & a_2 give $5v_1 - v_2 = 21$
Combine & get $v_1 = 4.5 \text{ m/s}$, $v_2 = 1.5 \text{ m/s}$

$$\omega_2 = 2.50 \text{ rad/s}$$

CHAPTER SEVEN
SPACE KINEMATICS OF RIGID BODIES

7/2



O-1 rotates to O-1' so axis must lie in plane (X-Z) which is perpendicular bisector of 1-1'

O-2 rotates to O-2' so axis must lie in plane which is perpendicular bisector of line 2-2'

Intersection of planes is line OC defined by

$$\text{unit vector } \underline{n} = \frac{\underline{i}}{\sqrt{2}} + \frac{\underline{k}}{\sqrt{2}} = \frac{1}{\sqrt{2}}(\underline{i} + \underline{k})$$

Angle through which revolution takes place is $\Delta\theta = \pi$

7/3

From Eq. (139) inverted

$$\{v_{xyz}\} = [T_\psi]^{-1} [T_\theta]^{-1} \{v_{xyz}\}$$

where $v_x = 75 \text{ mm/s}$, $v_y = 100 \text{ mm/s}$, $v_z = 0$, $\psi = 60^\circ$, $\theta = 30^\circ$

$$\{v_{xyz}\} = \begin{bmatrix} \cos \psi & -\cos \theta \sin \psi & \sin \theta \sin \psi \\ \sin \psi & \cos \theta \cos \psi & -\sin \theta \cos \psi \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \{v_{xyz}\}$$

$$v_x = \frac{1}{2}(75) - \frac{\sqrt{3}}{2} \frac{\sqrt{3}}{2}(100) + \frac{1}{2} \frac{\sqrt{3}}{2}(0) = -75/2 \text{ mm/s}$$

$$v_y = \frac{\sqrt{3}}{2}(75) + \frac{\sqrt{3}}{2} \frac{1}{2}(100) - \frac{1}{2} \frac{1}{2}(0) = 125\sqrt{3}/2 \text{ mm/s}$$

$$v_z = 0(75) + \frac{1}{2}(100) + \frac{\sqrt{3}}{2}(0) = 50 \text{ mm/s}$$

7/4 $\{v_{x'y'z'}\} = [T_\phi] \{v_{xyz}\}$ where $[T_\phi] = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Also $\{v_{xyz}\} = [T_\theta][T_\psi]\{v_{XYZ}\}$ from Eq. 139

Thus $\{v_{x'y'z'}\} = [T_\phi][T_\theta][T_\psi]\{v_{XYZ}\}$

where $[T_\phi][T_\theta][T_\psi] = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$

with

$$\begin{aligned} a_1 &= (\cos \phi \cos \psi - \sin \phi \cos \theta \sin \psi) \\ a_2 &= (\cos \phi \sin \psi + \sin \phi \cos \theta \cos \psi) \\ a_3 &= \sin \phi \sin \theta \\ a_4 &= (-\sin \phi \cos \psi - \cos \phi \cos \theta \sin \psi) \\ a_5 &= (-\sin \phi \sin \psi + \cos \phi \cos \theta \cos \psi) \\ a_6 &= \cos \phi \sin \theta \\ a_7 &= \sin \theta \sin \psi \\ a_8 &= -\sin \theta \cos \psi \\ a_9 &= \cos \theta \end{aligned}$$

7/5

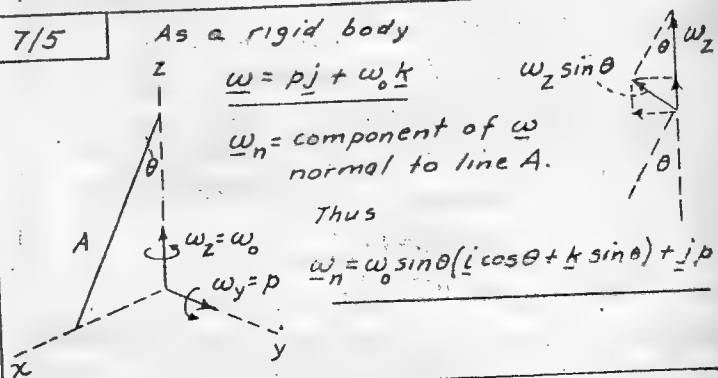
As a rigid body

$$\underline{\omega} = p\underline{j} + \omega_0\underline{k}$$

$\underline{\omega}_n$ = component of $\underline{\omega}$ normal to line A.

Thus

$$\underline{\omega}_n = \omega_0 \sin \theta (\underline{i} \cos \theta + \underline{k} \sin \theta) + \underline{j} p$$



7/6

$v_D = \frac{3b}{2} \frac{60(2\pi)}{60} = 3\pi b$; $v_E = 0$ (Gear C fixed)

$$\omega_y = \frac{v_D}{b} = 3\pi \text{ rad/s}$$

$$\omega_x = -\frac{60(2\pi)}{60} = -2\pi \text{ rad/s}$$

$$\underline{\omega} = -2\pi \underline{i} + 3\pi \underline{j} = \pi(-2\underline{i} + 3\underline{j}) \text{ rad/s}$$

$$\underline{\alpha} = \dot{\underline{\omega}}; \underline{i} = 0, \underline{j} = \omega \times \underline{j} = -2\pi \underline{k}$$

$$\text{so } \underline{\alpha} = 0 + 3\pi(-2\pi \underline{k}) = -6\pi^2 \underline{k} \text{ rad/s}^2$$

Body cone is the pitch cone of gear B

Space " " " " " " " " C

7/7

$v_E = \frac{3b}{2} \frac{20(2\pi)}{60} (-\underline{k}) = -b\pi \underline{k}$

$v_D = \frac{3b}{2} \frac{60(2\pi)}{60} (-\underline{k}) = -3\pi b \underline{k}$

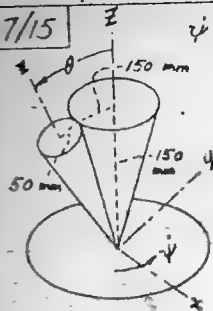
$$\underline{\omega}_y = \frac{v_D - v_E}{b} \underline{j} = \frac{3\pi - \pi}{b} b \underline{j} = 2\pi \underline{j}$$

$$\underline{\omega}_x = -\frac{60(2\pi)}{60} \underline{i} = -2\pi \underline{i} \text{ rad/s}$$

$$\underline{\omega} = -2\pi \underline{i} + 2\pi \underline{j} = 2\pi(-\underline{i} + \underline{j}) \text{ rad/s}$$

$$\underline{\alpha} = \dot{\underline{\omega}} = 0 + 2\pi \underline{j} = 2\pi(-2\pi \underline{k}) = -4\pi^2 \underline{k} \text{ rad/s}^2$$

7/15



$$\psi = \frac{2\pi}{4}, \theta = \tan^{-1} \frac{150}{50} + \tan^{-1} \frac{50}{\sqrt{150^2 + 150^2 - 50^2}}$$

$$= 45^\circ + 13.63^\circ = 58.63^\circ$$

$$\underline{\omega} = \dot{\theta} \underline{i} + \dot{\psi} \sin \theta \underline{j} + \dot{\phi} (\underline{j} \sin \theta + \underline{k} \cos \theta), \dot{\theta} = 0$$

$$\underline{\alpha} = \dot{\underline{\omega}} = \ddot{\psi} \sin \theta \underline{j} + \dot{\psi} \dot{\phi} \cos \theta \underline{j} + \dot{\psi} \dot{\phi} \sin \theta \underline{k}, \ddot{\psi} = \ddot{\phi} = 0$$

$$\underline{j} = \underline{\Omega} \times \underline{j} = \dot{\psi} (\underline{j} \sin \theta + \underline{k} \cos \theta) \times \underline{j} = -\dot{\psi} \cos \theta \underline{k}$$

$$\underline{k} = \underline{\Omega} \times \underline{k} = \dot{\psi} (\underline{j} \sin \theta + \underline{k} \cos \theta) \times \underline{k} = \dot{\psi} \sin \theta \underline{j}$$

$$\text{so } \underline{\alpha} = -\dot{\psi} \cos \theta (\dot{\psi} \sin \theta) \underline{j} + \dot{\psi} \sin \theta (\dot{\phi} + \dot{\psi} \cos \theta) \underline{k}$$

$$= \dot{\psi} \dot{\phi} \sin \theta$$

$$\alpha = \frac{\pi}{2} 3 \frac{\pi}{2} \sin 58.63^\circ = 6.32 \text{ rad/s}^2$$

$$150 \psi = 50 \phi$$

$$\dot{\phi} = 3 \dot{\psi}$$

7/20

$\underline{\omega}_{AB}$ = component of body angular velocity normal to $\underline{r}_{A/B}$

$$\underline{\omega}_{AB} = \frac{\omega_0}{2} (-\underline{i} + \underline{k})$$

$$\underline{\alpha}_{AB} = \dot{\underline{\omega}}_{AB} = \frac{\omega_0}{2} (-\dot{\underline{i}} + \dot{\underline{k}})$$

$$\dot{\underline{k}} = 0; \dot{\underline{i}} = \underline{\omega}_0 \times \underline{i} = \omega_0 \underline{j}$$

$$\text{so } \underline{\alpha}_{AB} = -\frac{\omega_0^2}{2} \underline{j}$$

7/21

Angular velocity of rotor is

$$\underline{\omega} = p \underline{k} - q \underline{i}, \underline{\alpha} = \dot{\underline{\omega}} = p \dot{\underline{k}} - q \dot{\underline{i}} = \underline{\Omega} \times (p \underline{k} - q \underline{i})$$

where $\underline{\Omega}$ = angular velocity of axes = $-q \underline{i}$

$$\text{Thus } \underline{\alpha} = -q \underline{i} \times p \underline{k} = pq \underline{j}$$

$$\text{Or from Eq. 142, } \underline{\alpha} = \left(\frac{d\omega}{dt} \right)_{XYZ} = 0 + \underline{\Omega} \times \underline{\omega}$$

$$= -q \underline{i} \times (p \underline{k} - q \underline{i}) = pq \underline{j}$$

7/16

Substitute γ for $90^\circ - \theta$ so Eq. 138 becomes

$$\underline{\omega} = -\dot{\gamma} \underline{i} + \dot{\psi} \cos \gamma \underline{j} + \dot{\phi} (\underline{j} \sin \gamma + \underline{k} \cos \gamma)$$

$$\underline{\alpha} = -\dot{\gamma} \underline{i} + \dot{\psi} \cos \gamma \underline{j} - \dot{\psi} \dot{\gamma} \sin \gamma \underline{j} + \dot{\phi} (\dot{\gamma} \sin \gamma + \dot{\psi} \cos \gamma) \underline{j} + \dot{\phi} (0 + \dot{\psi} \cos \gamma) \underline{k}, \ddot{\gamma} = \ddot{\psi} = \ddot{\phi} = 0$$

$$= -\underline{\Omega} \times \underline{i} \dot{\gamma} + \underline{\Omega} \times \underline{j} \dot{\psi} \cos \gamma - \dot{\psi} \dot{\gamma} \sin \gamma \underline{j} + \underline{\Omega} \times \underline{k} (\dot{\phi} + \dot{\psi} \cos \gamma)$$

$$+ \dot{\phi} \dot{\psi} \cos \gamma$$

But angular velocity of axes is

$$\underline{\Omega} = -\dot{\gamma} \underline{i} + \dot{\psi} \cos \gamma \underline{j} + \dot{\phi} \sin \gamma \underline{k}$$

Substitute, simplify, & get

$$\underline{\alpha} = \dot{\gamma} (\dot{\phi} - \dot{\psi} \sin \gamma) \underline{j} + \dot{\psi} \dot{\phi} \cos \gamma \underline{j} + \dot{\phi} \dot{\psi} \cos \gamma \underline{k}$$

$$= 4(10.47 - 2.094 \left[\frac{1}{2} \right]) \underline{j} + \dot{\psi} (2.094)(10.47) \frac{\sqrt{3}}{2} \underline{k}$$

$$+ (2.094)(4) \frac{\sqrt{3}}{2} \underline{k}$$

$$= 18.99 \underline{i} + 37.70 \underline{j} + 7.26 \underline{k}, \alpha = \sqrt{18.99^2 + 37.70^2 + 7.26^2} = 42.8 \frac{\text{rad}}{\text{s}^2}$$

7/19

$$\overline{AB} = 300 \text{ mm}$$

$$\omega = 40 \text{ rad/s}$$



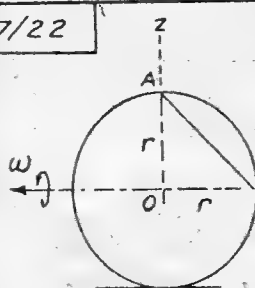
$$\underline{v}_{A/B} = \underline{\omega} \times \underline{r}_{A/B} = \underline{\omega}_2 \times \underline{r}_{A/B}$$

$$= 20 \underline{k} \times 300 \underline{j}$$

$$\underline{v}_{A/B} = -6000 \underline{i} \text{ mm/s}$$

$$\text{Spin } p = \omega_y = \frac{40\sqrt{3}}{2} = 34.6 \text{ rad/s}$$

7/22



Angular velocity of line AB is

$$\underline{\omega}_n = \frac{\omega}{\sqrt{2}} \left(-\frac{\underline{i}}{\sqrt{2}} - \frac{\underline{k}}{\sqrt{2}} \right)$$

where $\omega = v/r$

$$\text{so } \underline{\omega}_n = -\frac{v}{2r} (\underline{i} + \underline{k})$$

$$\& \quad |\underline{\omega}_n| = \frac{v}{r\sqrt{2}}$$

7/23

Let $\underline{\Omega}$ be the angular velocity of axes $x'-y'-z'$ attached to sphere. From

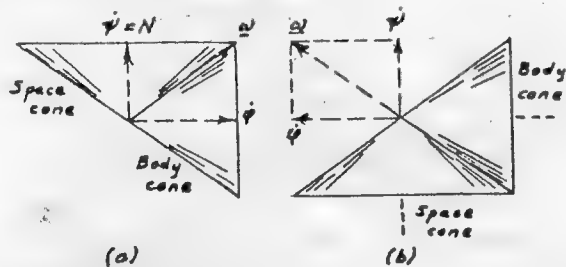
Prob. 7/22 angular velocity of line AB is

$$\underline{\omega}_n = -\frac{v}{2r} (\underline{i} + \underline{k}) \& \left(\frac{d\omega}{dt} \right)_{x'-y'-z'} = 0. \text{ Thus from}$$

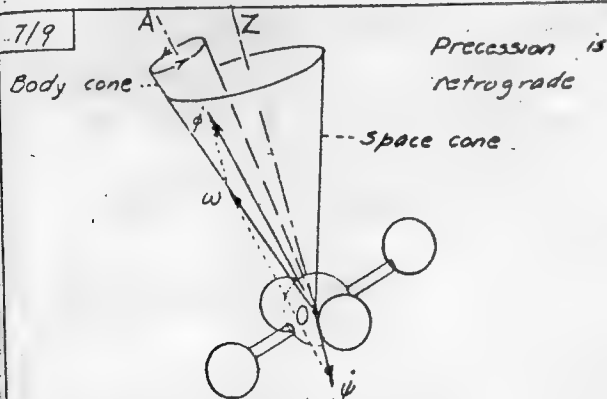
$$\text{Eq. 142, } \underline{\alpha}_l = \left(\frac{d\omega_n}{dt} \right)_{XYZ} = 0 + \underline{\Omega} \times \underline{\omega}_n = -\frac{v}{r} \underline{i} \times \frac{v}{2r} (\underline{i} + \underline{k})$$

$$\underline{\alpha}_l = -\frac{v^2}{2r^2} \underline{j}$$

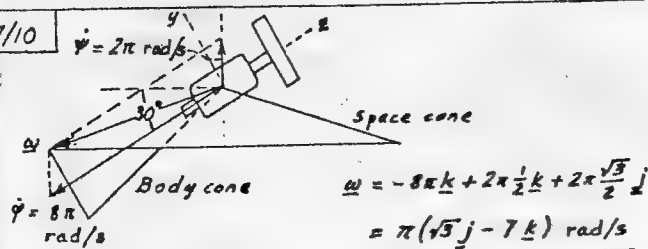
7/8 $\dot{\psi} = N = 10 \text{ rad/s}$, $\dot{\phi} = 15 \text{ rad/s}$



7/9



7/10



Body cone lies inside space cone, so retrograde precession

7/11

$\underline{a} = \frac{d}{dt}(\underline{\omega} \times \underline{r}) = \underline{\omega} \times (\underline{\omega} \times \underline{r})$; $\dot{\omega} = 0$
 where $\underline{r} = \underline{r}_2 - \underline{r}_1$
 so $\underline{r} = (75\hat{i} + 75\hat{j} + 150\hat{k}) - 50\hat{j}$
 $= 75\hat{i} + 25\hat{j} + 150\hat{k}$
 $\underline{\omega} \times \underline{r} = 3(3\hat{i} + 2\hat{j} + 6\hat{k}) \times (75\hat{i} + 25\hat{j} + 150\hat{k})$
 $= 225(2\hat{i} - \hat{k}) \text{ mm/s}$
 $\underline{\omega} \times (\underline{\omega} \times \underline{r}) = 3(3\hat{i} + 2\hat{j} + 6\hat{k}) \times 225(2\hat{i} - \hat{k})$
 $= 675(-2\hat{i} + 15\hat{j} - 4\hat{k}) \text{ mm/s}^2$
 $|\underline{a}_B| = a_{B_n} = 675\sqrt{2^2 + 15^2 + 4^2} = 675\sqrt{245} = 10570 \frac{\text{mm}}{\text{s}^2}$
 $= 10.57 \text{ m/s}^2$

7/12

$\underline{v}_B = -\underline{j}v$
 (a) $\underline{\omega} = \frac{v}{r \cos \gamma} \hat{i}$
 But $\cos \gamma = \frac{h}{\sqrt{h^2 + r^2}}$ so
 $\underline{\omega} = v \sqrt{\frac{1}{r^2} + \frac{1}{h^2}} \hat{i}$
 (b) $|\underline{\omega}_{OB}| = \omega \sin \gamma = \frac{v}{r} \tan \gamma$
 $= \frac{v}{h}$
 so $\underline{\omega}_{OB} = \frac{v}{h}(\hat{i} \sin \gamma - \hat{k} \cos \gamma)$
 $= \frac{v}{h \sqrt{r^2 + h^2}}(\hat{i}r - \hat{k}h)$

(c) $\underline{\omega}_{OA} = 0$ since $\underline{v}_A = 0$

(d) $\underline{\alpha} = \dot{\underline{\omega}} = \frac{v}{r \cos \gamma} \dot{\hat{i}}$ but $\dot{\hat{i}} = -\frac{v}{h \cos \gamma} \hat{j}$ so
 $\underline{\alpha} = -\frac{v^2}{h^2}(\frac{r}{h} + \frac{h}{r})\hat{j}$

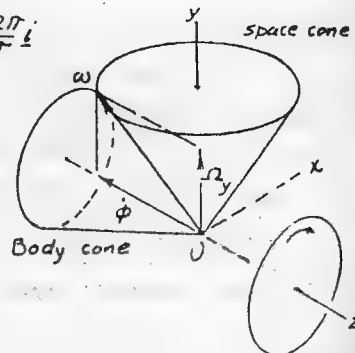
7/13

Axes revolve about y-axis

$v = R\Omega_y = R \frac{2\pi}{T}$; $\dot{\phi} = \frac{v}{r} = \frac{R}{r} \frac{2\pi}{T}$
 $\underline{\omega} = \text{angular velocity of disk}$
 $= \frac{2\pi}{T} \hat{j} - \frac{R}{r} \frac{2\pi}{T} \hat{k} = \frac{2\pi}{T}(\hat{j} - \frac{R}{r} \hat{k})$
 $\underline{\alpha} = \dot{\underline{\omega}} = \frac{2\pi}{T}(\dot{\hat{j}} - \frac{R}{r} \dot{\hat{k}}) = \frac{2\pi}{T}(0 - \frac{R}{r} \frac{2\pi}{T} \hat{i})$

where $\dot{\hat{k}} = \underline{\Omega} \times \hat{k} = \Omega_y \hat{j} \times \hat{k} = \frac{2\pi}{T} \hat{i}$

Thus $\underline{\alpha} = -(\frac{2\pi}{T})^2 \frac{R}{r} \hat{i}$



7/14

From Sample Prob. 7/1, angular velocity $\underline{\omega}$ of rotor is

$\underline{\omega} = -\hat{i}\dot{\gamma} + \hat{j}\dot{\psi} \cos \gamma + \hat{k}(\dot{\phi} + \dot{\psi} \sin \gamma)$, $\dot{\gamma} = -\dot{\theta}$
 $\underline{\alpha} = \dot{\underline{\omega}} = -\hat{i}\ddot{\gamma} + 0 + \hat{k}(\ddot{\phi} + 0) + \hat{k}(\ddot{\psi} + 0)$
 where $\dot{\hat{i}} = 0$, $\dot{\psi} = 0$
 Thus $\underline{\alpha} = -15\hat{i} + \underline{\Omega} \times \hat{k}(\frac{30[2\pi]}{60}) + \hat{k}(10 + 0)$
 $= -15\hat{i} + (-\dot{\gamma}\hat{i} \times \hat{k})(\pi) + 10\hat{k}$; $\underline{\Omega} = -\dot{\gamma}\hat{i}$
 $= -15\hat{i} + (-12\pi)(-\hat{j}) + 10\hat{k}$
 $= -15\hat{i} + 12\pi\hat{j} + 10\hat{k} \text{ rad/s}^2$
 so $\alpha = \sqrt{15^2 + (12\pi)^2 + 10^2} = 41.8 \text{ rad/s}^2$

7/24

$v_B = 50 \text{ mm/s}$
 $v_A = 125 \text{ mm/s}$
 $(v_{A/B})_{\text{normal to AB}} = 75 \cos \theta$
 $= \frac{75 \sqrt{75^2 + 150^2}}{\sqrt{75^2 + 150^2 + 50^2}}$
 $= \frac{225\sqrt{5}}{7} \text{ mm/s}$
 $\omega_n = \frac{225\sqrt{5}}{7} / 175 = \frac{9\sqrt{5}}{49} \text{ rad/s}$
 so $\omega_n = \frac{9\sqrt{5}}{49} (\underline{i} \cos \theta + \underline{j} \sin \theta)$
 $= \frac{9\sqrt{5}}{49} (\frac{2}{\sqrt{5}} \underline{i} + \frac{1}{\sqrt{5}} \underline{j}) = \frac{9}{49} (2\underline{i} + \underline{j}) \text{ rad/s}$

7/25 Angular velocity of axes is $\underline{\Omega} \times \underline{p}$

$$\underline{\alpha} = \underline{\dot{\omega}} = \underline{\dot{\Omega}} - \underline{\beta} \underline{\dot{i}} - \underline{\beta} \underline{\Omega} \times \underline{i}$$

$$= \underline{0} - \underline{\beta} \underline{\dot{i}} - \underline{\beta} \rho \underline{j}$$

(a) before; $\dot{\beta} d\beta = \ddot{\beta} d\beta$, $\ddot{\beta} = \dot{\beta} \frac{d\dot{\beta}}{d\beta} = \left(2 \frac{2\pi}{360}\right) \frac{2}{18}$
 $= 0.00388 \text{ rad/s}^2$

$$\underline{\alpha} = -0.00388 \underline{i} - \frac{\pi}{900} \underline{j} = -3.88 \underline{i} - 3.49 \underline{j} \text{ mrad/s}^2$$

(b) after; $\ddot{\beta} = 0$, $\underline{\alpha} = -3.49 \text{ j rad/s}^2$

7/26 Angular velocity $\underline{\omega}$ of A may have a component along \underline{x} -axis and along \underline{z} -axis but none about \underline{y} -axis. Thus $\underline{\omega} \cdot \underline{j} = 0$ But a vector along \underline{j} -direction is $\underline{h} \times \underline{n}$ or $\underline{h} \times (\underline{r} \times \underline{h})$; Magnitude is immaterial. Hence $\underline{\omega} \cdot \underline{h} \times (\underline{r} \times \underline{h}) = 0$ is the provision that $\underline{\omega}$ will not have a component along \underline{y} .

7/27 $\underline{\Omega}$ = angular velocity of axes x-y-z
 $\underline{\omega}$ = " " " simulator = $\underline{\Omega} + \underline{p}$

Let $N =$ angular velocity of frame $= 0.2 \text{ rad/sec}$
 $p = 0.9 \text{ rad/s const.}$, $\beta = 0.15 \text{ rad/s const.}$

$$\underline{Q} = j\beta + jN\cos\beta - kN\sin\beta ; \underline{P} = P_k$$

From Eq. 14.2 $\underline{\alpha} = \left(\frac{d\underline{\omega}}{dt} \right)_{XYZ} = \left(\frac{d\underline{\omega}}{dt} \right)_{xyz} + \underline{\Omega} \times \underline{\omega}$

$$\underline{\alpha} = (0 - jN\dot{\beta} \sin \beta - L N \dot{\beta} \cos \beta + 0) + \underline{\Omega} \times (\underline{\Omega} + \underline{p})$$

where $\underline{\Omega} \times (\underline{\Omega} + \underline{\rho}) = \underline{\Omega} \times \underline{\rho} = (\underline{i}\beta + jN\cos\beta - kN\sin\beta) \times p\hbar$
 $= jNp\cos\beta - j\rho\beta$

$$\begin{aligned} \alpha_{\beta=0} &= \dot{i} N p - j p \dot{\beta} - k N \dot{\beta} \\ &= 0.2(0.9)\dot{i} - 0.9(0.15)\dot{j} - 0.2(0.15)\dot{k} \text{ rad/s}^2 \\ &= 0.18\dot{i} - 0.135\dot{j} - 0.03\dot{k} \text{ rad/s}^2 \end{aligned}$$

7/28 Angular velocity of drum is $\underline{\omega} = (-p \cos \theta) \underline{i} + \theta \underline{j} + (p \sin \theta + \Omega) \underline{k}$; From Eq. 142

$$\underline{\alpha} = \dot{\underline{\omega}} = (\rho \dot{\theta} \sin \theta) \underline{i} + (\rho \dot{\theta} \cos \theta) \underline{k} + \underline{\Omega} \times \underline{\omega}$$

But angular velocity of axes is $\underline{\Omega} = \Omega \underline{k}$, so

$$\underline{\alpha} = (\rho \dot{\theta} \sin \theta) \underline{i} + (\rho \dot{\theta} \cos \theta) \underline{k} - (\rho \Omega \cos \theta) \underline{j} - \Omega \dot{\theta} \underline{i}$$

$$= \dot{\theta} (\rho \sin \theta - \Omega) \underline{i} - (\rho \Omega \cos \theta) \underline{j} + (\rho \dot{\theta} \cos \theta) \underline{k}$$

7/29 $\overline{OB} = \sqrt{7^2 - 2^2 - 3^2} = 6 \text{ m}; v_x = -3i \text{ m/s}$

$$\underline{v}_B = \underline{v}_A + \underline{\omega} \times \underline{r}_{B/A}, \quad \underline{r}_{B/A} = -2\hat{i} - 3\hat{j} + 6\hat{k} \text{ m}$$

So $\underline{v_B} = -3\underline{i} + \underline{j} + \underline{k}$; equate coefficients of $\underline{i}, \underline{j}, \underline{k}$ & get

$1 = 2\omega_y + \omega_z, \omega_z = -3\omega_x, v_B = -3\omega_x + 2\omega_y$
 Now \underline{a}_B is \perp to AB, so $\underline{a}_B \cdot \underline{r}_{B/A} = 0$ which gives
 $-2\omega_x - 3\omega_y + 6\omega_z = 0$. Eliminate ω 's & get $v_B = 1.0 \frac{m}{s}$
 Then $\omega_x = -\frac{3}{49} \text{ rad/s}, \omega_y = \frac{20}{49} \text{ rad/s}, \omega_z = \frac{9}{49} \text{ rad/s}$
 $\underline{\omega} = \frac{1}{49} (-3\hat{i} + 20\hat{j} + 9\hat{k}) \text{ rad/s}$

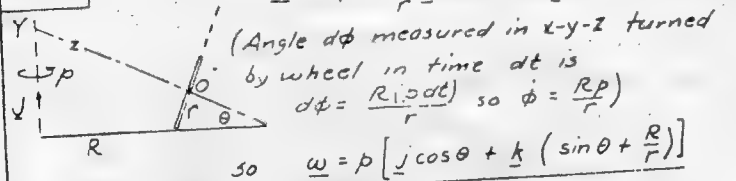
(Alternative solution for v_B)
 $x^2 + y^2 + z^2 = l^2$, $x\dot{x} + y\dot{y} + z\dot{z} = 0$, $\dot{y} = 0$, $\dot{x} = -3 \text{ m/s}$
 So $\dot{z} = -\frac{x\dot{x}}{z} = -\frac{2}{-3}(-3) = 1.0 \text{ m/s}$

7/30 Angular velocity of axes $\underline{\Omega} = \Omega \underline{k}$
 " " " panels $\underline{\omega} = -\dot{\theta} \underline{j} + \Omega \underline{k}$
 $\underline{\dot{\omega}} = -\dot{\theta} \underline{j} + \Omega \underline{k} = -\dot{\theta}(\underline{\Omega} \times \underline{j}) + \Omega(\underline{\Omega} \times \underline{k}) = \underline{\Omega} \times \underline{\omega} = \Omega \dot{\theta} \underline{i}$
 $= \frac{1}{2} \frac{1}{4} \underline{i} = \frac{1}{8} \underline{i} \text{ rad/s}^2$

$$\begin{aligned} \underline{a}_A &= \underline{a}_0 + \underline{\dot{\Omega}} \times \underline{r}_{A/O} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/O}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel} \\ \underline{a}_0 &= \underline{\dot{\Omega}} = \underline{0}, \quad \underline{\Omega} \times \underline{r}_{A/O} = \frac{1}{2} \underline{k} \times (-0.3 \underline{i} + 2.4 \underline{j} + 0.3\sqrt{3} \underline{k}) \\ &= -1.2 \underline{i} - 0.15 \underline{j} \quad \text{m/s} \\ \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/O}) &= \frac{1}{2} \underline{k} \times (-1.2 \underline{i} - 0.15 \underline{j}) = 0.075 \underline{i} - 0.6 \underline{j} \quad \text{m/s}^2 \\ 2\underline{\Omega} \times \underline{v}_{rel} &= 2 \frac{1}{2} \underline{k} \times \left(-\frac{0.6}{2} \frac{\sqrt{3}}{2} \underline{i} - \frac{0.6}{4} \frac{1}{2} \underline{k} \right) = -0.075\sqrt{3} \underline{j} \quad \text{m/s}^2 \\ \underline{a}_{rel} &= 0.6 \left(\frac{1}{4} \right)^2 \left(\frac{1}{2} \underline{i} - \frac{\sqrt{3}}{2} \underline{k} \right) = 0.01875 \underline{i} - 0.01875\sqrt{3} \underline{k} \quad \text{m/s}^2 \\ \underline{a}_A &= (0.075 + 0.01875) \underline{i} + (-0.6 - 0.075\sqrt{3}) \underline{j} - 0.01875\sqrt{3} \underline{k} \quad \text{m/s}^2 \\ &= \underline{0.0938 \underline{i} - 0.730 \underline{j} - 0.0325 \underline{k} \quad \text{m/s}^2} \end{aligned}$$

with $a_A = 0.737 \text{ m/s}^2$

7/31



Angular velocity of axes is $\underline{\Omega} = j p$ so
 $\underline{\omega} = \underline{\Omega} + \left(\frac{R p}{r} \right) k$; Now use $\left(\frac{d}{dt} \right)_{XYZ} = \left(\frac{d}{dt} \right)_{xyz} + \underline{\Omega} \times []$
 Noting $\underline{\Omega}$ is constant in XYZ & xyz .
 Thus $\underline{\alpha} = \left(\frac{d \underline{\omega}}{dt} \right)_{XYZ} = 0 + \underline{\Omega} \times \left[\underline{\Omega} + \frac{R p}{r} k \right]$
 $= \left[(p \cos \theta) j + (p \sin \theta) k \right] \times \frac{R p}{r} k$; $\underline{\alpha} = \frac{R p^2}{r} \cos \theta i$
 or merely $\underline{\alpha} = \underline{\dot{\omega}} = 0 + \frac{R p}{r} \dot{k} = \frac{R p}{r} (\underline{\Omega} \times k)$, etc.

7/32

Angular vel. of xyz is $\underline{\Omega} = i q \sin \theta - j \dot{\theta} + k q \cos \theta$
 $\underline{a}_A = \underline{a}_O + \underline{\Omega} \times \underline{r}_{A/O} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/O}) + 2 \underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$
 $\underline{a}_O = -0.15(8^2) j = -9.6 j \text{ m/s}^2$, $\underline{r}_{A/O} = 0.15 i - 0.1 k \text{ m}$
 $\underline{\dot{\theta}} = \underline{\Omega} \times \underline{\Omega} + i q \dot{\theta} \cos \theta + 0 - k q \dot{\theta} \sin \theta$ by Eq. 142
 $= q \dot{\theta} (i \cos \theta - k \sin \theta) = 40(\sqrt{3} i - k) \text{ rad/s}^2$
 $\underline{\Omega} \times \underline{r}_{A/O} = 40(0.1\sqrt{3} - 0.15) j = 0.928 j \text{ m/s}^2$
 $\underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/O}) = -25.0 i + 0.928 j + 15.76 k \text{ m/s}^2$
 $2 \underline{\Omega} \times \underline{v}_{rel} = 2(4 i - 10 j + 4\sqrt{3} k) \times (3 j) = 24(k - \sqrt{3} i) \text{ m/s}^2$
 $\underline{a}_{rel} = 0.1(30)^2 k = 90 k \text{ m/s}^2$
 Substitute in \underline{a}_A & get $\underline{a}_A = -66.5 i - 7.74 j + 129.8 k \text{ m/s}^2$
 Angular vel. of rotor is $\underline{a}_A = \sqrt{66.5^2 + 7.74^2 + 129.8^2}$
 $= 146.0 \text{ m/s}^2$
 $\underline{\omega} = \underline{\Omega} + i p$
 $\underline{\alpha} = \underline{\dot{\omega}} = \underline{\dot{\Omega}} + i \dot{p}$, $\dot{p} = 0$
 $= \underline{\dot{\Omega}} + \underline{\Omega} \times i p = 40(\sqrt{3} i - k) + (4 i - 10 j + 4\sqrt{3} k) \times 30 i$
 $= 20(2\sqrt{3} i + 6\sqrt{3} j + 13 k) \text{ rad/s}^2$
 with $\alpha = 20\sqrt{12 + 108 + 169} = 340 \text{ rad/s}^2$

7/33

$\overline{AB}^2 = 3(200)^2$, $\overline{AB} = 200\sqrt{3} \text{ mm}$; $\underline{r}_{A/B} = 200(-i + j)$
 $\underline{v}_A = \underline{v}_B + \frac{d}{dt}(\underline{r}_{A/B})$; $\frac{d}{dt}(\underline{r}_{A/B}) = \dot{\underline{r}}_{A/B} = \underline{\dot{r}}_{A/B} + \underline{\omega} \times \underline{r}_{A/B}$
 $\dot{\underline{r}}_{A/B} = \frac{d}{dt} \sqrt{x_1^2 + y_1^2 + z_1^2} = \frac{x_1 \dot{x}_1 + y_1 \dot{y}_1 + z_1 \dot{z}_1}{\underline{r}_{A/B}}$ where $\underline{r}_{A/B} = i x_1 + j y_1 + k z_1$
 $= \frac{(-200)(-150) + (200)(150)}{200\sqrt{3}} = 100\sqrt{3} \text{ mm/s}$; $\underline{v}_A = 150 j \text{ mm/s}$, $\underline{v}_B = 150 i$
 Thus $150 j = 150 i + 100\sqrt{3} \frac{-i + j + k}{\sqrt{3}} + \begin{vmatrix} i & j & k \\ \omega_x & \omega_y & \omega_z \\ -200 & 200 & 200 \end{vmatrix}$
 Equate coefficients & get
 $-1 = 4\omega_y - 4\omega_z$ Also $\underline{\omega} \cdot \underline{r}_{A/B} = 0$ for $\underline{\omega} \perp \underline{r}_{A/B}$
 $-1 = 4\omega_x + 4\omega_z$ so $-\omega_x + \omega_y + \omega_z = 0$ ---- (1)
 $-1 = 2\omega_x + 2\omega_y$ Combine (1) with any two of the i, j, k eqs. & get
 $\omega_x = -1/4 \text{ rad/s}$, $\omega_y = -1/4 \text{ rad/s}$, $\omega_z = 0$
 Hence $\underline{\omega}_n = -1/4 (i + j) \text{ rad/s}$ with $\omega_n = \sqrt{2}/4 \text{ rad/s}$

7/34

Note $\underline{\omega}_A \times \underline{r}_{A/B} = \underline{\omega} \times \underline{r}_{A/B}$ of Prob. 7/33, so the equations
 $\begin{cases} -1 = 4\omega_{Ay} - 4\omega_{Az} \\ -1 = 4\omega_{Ax} + 4\omega_{Az} \\ -1 = 2\omega_{Ax} + 2\omega_{Ay} \end{cases}$
 may be written directly from the solution for Prob. 7/33 where $\underline{\omega}_A$ replaces $\underline{\omega}$. Also from results of Prob. 7/26, the angular velocity $\underline{\omega}_A$ obeys $\underline{\omega}_A \cdot j \times (\underline{r}_{A/B} \times j) = 0$ which gives
 $\underline{\omega}_A \cdot 200(-i + k) = 200(-\omega_{Ax} + \omega_{Az}) = 0$. Combine with two of above three equations & get
 $\omega_{Ax} = -1/8 \text{ rad/s}$, $\omega_{Ay} = -3/8 \text{ rad/s}$, $\omega_{Az} = -1/8 \text{ rad/s}$
 so $\underline{\omega}_A = -1/8 (i + 3j + k) \text{ rad/s}$
 with $\omega_A = 1/8 \sqrt{1^2 + 9 + 1} = \sqrt{11}/8 \text{ rad/s}$

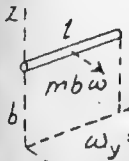
CHAPTER EIGHT

SPACE KINETICS OF RIGID BODIES

8/2

$$I_{xx} = \frac{1}{12} m l^2 + m \left(\frac{l^2}{4} + b^2 \right)$$

$$= \frac{1}{3} m l^2 + m b^2$$



$$I_{yy} = m b^2, \quad I_{zz} = \frac{1}{3} m l^2$$

$$I_{xy} = I_{xz} = 0, \quad I_{yz} = \frac{1}{2} m b l$$

$\omega_x = \omega_z = 0$ From Eq. 147,

$$H = \underline{i}(0) + \underline{j}(0 + m b^2 \omega - 0) + \underline{k}(0 - \frac{1}{2} m b l \omega + 0)$$

$$= m b^2 \omega \left(\underline{j} - \frac{1}{2} \frac{l}{b} \underline{k} \right)$$

8/3 Let $m' = m/3$

$$I_{xx} = \frac{1}{3} m' b^2 + \frac{1}{12} m' b^2 + \frac{5}{12} m' b^2 = \frac{5}{9} m' b^2 = \frac{5}{9} m b^2$$

$$I_{yy} = \frac{1}{3} m' b^2 + m' b^2 + \frac{1}{12} m' b^2 + \frac{5}{12} m' b^2 = \frac{8}{9} m' b^2 = \frac{8}{9} m b^2$$

$$I_{zz} = \frac{1}{3} m' b^2 + \frac{1}{12} m' b^2 + \frac{5}{12} m' b^2 + 2 m' b^2 = \frac{11}{9} m' b^2 = \frac{11}{9} m b^2$$

$$I_{xy} = \frac{1}{2} m' b^2 + m' b^2 = \frac{3}{2} m' b^2 = \frac{1}{2} m b^2$$

$$I_{xz} = \frac{1}{2} m' b^2 = \frac{1}{6} m b^2$$

$$I_{yz} = \frac{1}{2} m' b^2 = \frac{1}{6} m b^2$$

Dir. cosines for OM are $l=m=n=1/\sqrt{3}$

Substitute in Eq. 151

$$H_M = m b^2 \frac{1}{3} \left[\frac{5}{9} + \frac{8}{9} + \frac{11}{9} - \frac{2}{2} - \frac{1}{3} - \frac{1}{3} \right] = \frac{1}{3} m b^2$$

8/4

$$I_{xx} = \frac{1}{3} m l^2, \quad I_{yy} = \frac{1}{3} m (l \sin \theta)^2$$

$$I_{zz} = \frac{1}{3} m (l \cos \theta)^2$$

$$I_{xy} = I_{xz} = 0, \quad I_{yz} = \int yz \, dm$$

$$= \frac{1}{3} m l^2 \sin \theta \cos \theta$$

So from Eq. 147

$$H = \underline{i}(0) + \underline{j} \left(\frac{1}{3} m l^2 \sin^2 \theta \omega \right) + \underline{k} \left(-\frac{1}{3} m l^2 \sin \theta \cos \theta \omega \right)$$

$$H = \frac{1}{3} m l^2 \omega \sin \theta \left(\underline{j} \sin \theta - \underline{k} \cos \theta \right)$$

8/5

$$I_{xx} = I_{yy} = I_{zz} = 2 \left(\frac{2}{5} m r^2 + m b^2 \right) + \frac{2}{5} m r^2$$

$$= m \left(\frac{6}{5} r^2 + 2 b^2 \right)$$

$$I_{xy} = I_{yz} = I_{xz} = 0$$

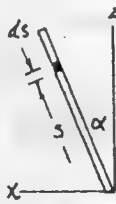
Eq. 153 for principal moment of inertia I is

$$\left[m \left(\frac{6}{5} r^2 + 2 b^2 \right) - I \right]^3 = 0, \quad I_1 = I_2 = I_3 = m \left(\frac{6}{5} r^2 + 2 b^2 \right)$$

& $q_1 = q_2 = q_3 = 1/\sqrt{m \left(\frac{6}{5} r^2 + 2 b^2 \right)}$, so ellipsoid of inertia is a sphere of radius $q = q_1 = q_2 = q_3$ & I is the same for all axes through its center O .

8/6

$$H_x = I_{xx} \omega_x - I_{xz} \omega_z, \quad \omega_x = \omega, \quad \omega_z = p, \quad I_{xy} = 0$$



$$I_{xx} = \frac{1}{3} m h^2$$

$$I_{xz} = \int_0^l (s \sin \alpha)(s \cos \alpha) \rho \, ds = \frac{\rho \sin \alpha \cos \alpha}{3} l^3$$

$$= \frac{1}{3} m r h$$

$$H_x = \frac{m h}{3} (h \omega - r p)$$

8/7

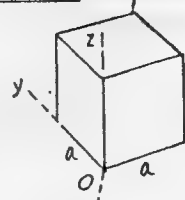
$$H_z = -I_{xz} \omega_x + I_{zz} \omega_z, \quad I_{yz} = 0, \quad \omega_x = \omega, \quad \omega_z = p$$

From Prob. 8/6, $I_{xz} = \frac{1}{3} m r h$; Also $I_{zz} = \frac{1}{3} m r^2$

$$H_z = \frac{m r}{3} (r p - h \omega)$$

8/8

$$\omega_x = \omega_y = \omega_z = \omega/\sqrt{3}$$



$$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3} m a^2$$

$$I_{xy} = I_{xz} = I_{yz} = \frac{1}{4} m a^2$$

$$H_x = H_y = H_z = \frac{2}{3} m a^2 \frac{\omega}{\sqrt{3}} - 2 \frac{1}{4} m a^2 \frac{\omega}{\sqrt{3}}$$

$$= \frac{m a^2 \omega}{6 \sqrt{3}}$$

$$H = \frac{m a^2 \omega}{6 \sqrt{3}} (\underline{i} + \underline{j} + \underline{k}), \quad H = \frac{m a^2 \omega}{6}$$

8/9

$$\omega_x = \omega_y = 0, \quad \omega_z = \omega$$

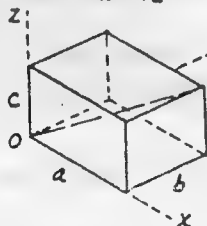
$$I_{xz} = 0, \quad I_{yz} = 0 + m \left(\frac{4r}{3\pi} \right) \left(c + \frac{b}{2} \right); \quad I_{zz} = \frac{1}{2} m r^2$$

$$\text{So } H = -I_{yz} \omega_z \underline{j} + I_{zz} \omega_z \underline{k}$$

$$H = m r \omega \left[-\frac{2(2c+b)}{3\pi} \underline{j} + \frac{r}{2} \underline{k} \right]$$

8/10 Let Ω = angular velocity of x-y-z about Z_0
 Axes; $\Omega_x = -\Omega \sin \theta$, $\Omega_y = \dot{\theta} = 0$, $\Omega_z = \Omega \cos \theta$; $\dot{\Omega} = 2\pi f$
 Body; $\omega_x = -\Omega \sin \theta$, $\omega_y = 0$, $\omega_z = \Omega \cos \theta + p$
 $H_x = I_{xx} \omega_x = mk^2(-2\pi f \sin \theta)$, $H_y = I_{yy} \omega_y = 0$,
 $H_z = I_{zz} \omega_z = mk^2(2\pi f \cos \theta + p)$
 $H = 2\pi mf(-k^2 \sin \theta \hat{i} + k^2 \cos \theta \hat{k}) + mk^2 p \hat{k}$

8/11 $I_{xx} = \frac{1}{12} m(b^2 + c^2) + m(\frac{b^2}{4} + \frac{c^2}{4}) = \frac{1}{3} m(b^2 + c^2)$



$I_{yy} = \frac{1}{3} m(a^2 + c^2)$
 $I_{zz} = \frac{1}{3} m(a^2 + b^2)$
 $I_{xy} = \frac{1}{4} mab$, $I_{xz} = \frac{1}{4} mac$
 $I_{yz} = \frac{1}{4} mbc$

Direction cosines of OA are $l = \frac{a}{r}$, $m = \frac{b}{r}$, $n = \frac{c}{r}$
 where $r^2 = a^2 + b^2 + c^2$.

From Eq. 151,

$I_M = \frac{1}{3} m \frac{1}{r^2} [(b^2 + c^2)a^2 + (a^2 + c^2)b^2 + (a^2 + b^2)c^2]$
 $- 2 \frac{1}{4} m \frac{1}{r^2} (ab[ab] + ac[ac] + bc[bc])$

$I_M = \frac{1}{6} m \frac{a^2 b^2 + a^2 c^2 + b^2 c^2}{a^2 + b^2 + c^2}$

8/12 $H_{xyz} = \bar{H} + \bar{r} \times m \bar{v}$

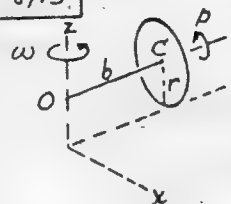
$\bar{H} = \hat{i} I_{xx} \omega_x + \hat{j} I_{yy} \omega_y + \hat{k} I_{zz} \omega_z$
 $= \hat{i} (\frac{1}{12} mb^2 + \frac{1}{4} mr^2) \omega + \hat{j} \frac{1}{2} mr^2 p + 0$

$\bar{r} = h\hat{k} - \frac{b}{2}\hat{j}$, $\bar{v} = -h\omega\hat{j} - \frac{b}{2}\omega\hat{k}$

$\bar{r} \times \bar{v} = \hat{i} (h^2 + \frac{b^2}{4}) \omega$

so $\begin{cases} H_x = (\frac{b^2}{3} + \frac{r^2}{4} + h^2) m \omega \\ H_y = \frac{1}{2} mr^2 p \\ H_z = 0 \end{cases}$

8/13



$\bar{v}_C = -b\omega \hat{i}$
 $I_{Oxx} = \frac{1}{4} mr^2 + mb^2$
 $I_{Oyy} = \frac{1}{2} mr^2$
 $I_{Ozz} = \frac{1}{4} mr^2 + mb^2$
 $I_{Oxy} = I_{Oxz} = I_{Oyz} = 0$

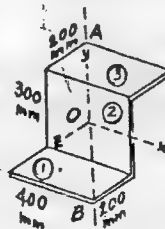
$\omega_x = 0$, $\omega_y = -\frac{b\omega}{r}$, $\omega_z = \omega$

Eq. 147 holds for point O as a fixed point on axis of disk, so

$\underline{H}_O = (0)\hat{i} + \frac{1}{2} mr^2 (-\frac{b\omega}{r})\hat{j} + (\frac{1}{4} mr^2 + mb^2)\omega\hat{k}$
 $= mr^2 \omega (-\frac{1}{2} \frac{b}{r} \hat{j} + [\frac{1}{4} + \frac{b^2}{r^2}] \hat{k})$

8/14

$m_3 = m_1 = 9.43 \text{ kg}$
 $m_2 = 14.14 \text{ kg}$



	①	②	③	Total
I_{xx}	0.338	0.1061	0.338	0.782
I_{yy}	0.251	0.1886	0.251	0.691
I_{zz}	0.338	0.295	0.338	0.971
I_{xy}	0	0	0	0
I_{xz}	0	0	0	0
I_{yz}	-0.141	0	-0.141	-0.283

$\overline{OB} = \sqrt{0.2^2 + 0.15^2 + 0.2^2} = 0.320 \text{ m}$

$l = 0.2/0.320 = 0.625$, $m = -0.15/0.320 = -0.469$, $n = l = 0.625$

From Eq. 151,

$I = 0.782(0.625)^2 + 0.691(-0.469)^2 + 0.970(0.625)^2$
 $= 0.305 + 0.1518 + 0.379 - 0.1656 - 2(-0.283)(-0.469)(0.625)$
 $I = 0.670 \text{ kg} \cdot \text{m}^2$

8/15 $I_{xx} = m(b^2 + 2b^2 + b^2) = 4b^2m$ $I_{xy} = 0$
 $I_{yy} = m(b^2 + b^2 + b^2 + 2b^2) = 5b^2m$ $I_{xz} = -mb^2$
 $I_{zz} = m(b^2 + b^2 + b^2) = 3b^2m$ $I_{yz} = mb^2$

Eq. 153 with $I' = I/mb^2$ gives

$$\begin{vmatrix} 4-I' & 0 & +1 \\ 0 & 5-I' & -1 \\ +1 & -1 & 3-I' \end{vmatrix} = 0$$
 Expansion yields $I'^3 - 12I'^2 + 45I' - 5 = 0$
 Solution gives $I_1 = 5.532 mb^2$ max
 $I_2 = 4.347 mb^2$
 $I_3 = 2.121 mb^2$ min

Dir. cosines from Eq. 154

For I_1 ;
 $(4-5.532)l_1 - 0 + n_1 = 0$
 $0 + (5-5.532)m_1 - n_1 = 0$
 $l_1 - m_1 + (3-5.532)n_1 = 0$
 $l_1^2 + m_1^2 + n_1^2 = 1$
 $l_1 = -0.293$
 $m_1 = 0.844$
 $n_1 = -0.449$

Similarly for I_2 & I_3 ;

$l_2 = 0.844$ $l_3 = -0.449$
 $m_2 = 0.449$ $m_3 = 0.293$
 $n_2 = 0.293$ $n_3 = 0.844$

$\omega_x = \omega_y = 0, \omega_z = \omega$
 $H_x = -I_{xz}\omega_z; H_y = -I_{yz}\omega_z; H_z = I_{zz}\omega_z$
 $H = mb^2\omega(\underline{i} - \underline{j} + 3\underline{k})$



8/17 From Prob. 8/16 $I_3 = 1.223 mb^2$

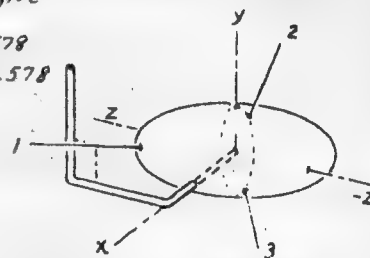
Dir. cosines for I_3 satisfy Eq. 154. Thus

$$\begin{cases} (5/9 - 1.223)l_3 - \frac{1}{6}m_3 - \frac{1}{2}n_3 = 0 \\ -\frac{1}{6}l_3 + (\frac{11}{9} - 1.223)m_3 - \frac{1}{6}n_3 = 0 \\ -\frac{1}{2}l_3 - \frac{1}{6}m_3 + (\frac{8}{9} - 1.223)n_3 = 0 \end{cases} \quad \& \quad l_3^2 + m_3^2 + n_3^2 = 1$$

sol. gives $l_3 = 1/\sqrt{3}, m_3 = n_3 = -1/\sqrt{3}$

Similarly for I_1 & I_2 which give

$l_1 = 0.790, m_1 = 0.212, n_1 = 0.578$
 $l_2 = 0.212, m_2 = 0.790, n_2 = -0.578$



8/18 Let p = spin about y relative to x - y - z

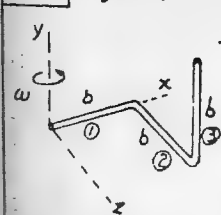
$p = \frac{R\omega}{r}$; Let ω_y = total ang. vel. about y
 $\omega_x = 0, \omega_y = \omega \sin \theta - p = (\frac{r}{R} - \frac{R}{r})\omega$
 $\omega_z = \omega \cos \theta = \omega \sqrt{1 - \frac{r^2}{R^2}}$

$H_y = I_{yy}\omega_y; H_z = I_{zz}\omega_z; I_{xy} = I_{xz} = I_{yz} = 0$
 $I_{zz} = \bar{I}_{zz} + md^2 = \frac{1}{4}mr^2 + m(R^2 - r^2) = m(R^2 - \frac{3}{4}r^2)$

So $H_0 = \frac{1}{2}mr^2(\frac{r}{R} - \frac{R}{r})\omega \underline{j} + m(R^2 - \frac{3}{4}r^2)\omega \sqrt{1 - \frac{r^2}{R^2}} \underline{k}$

or $H_0 = \frac{2\pi mr^2}{T} \left[\frac{1}{2}(\frac{r}{R} - \frac{R}{r}) \underline{j} + (\frac{R^2}{r^2} - \frac{3}{4}) \sqrt{1 - \frac{r^2}{R^2}} \underline{k} \right]$ where $\omega = \frac{2\pi}{T}$

8/16 $m_0 = m/3$



For system

$I_{xx} = \frac{5}{9}mb^2$

$I_{yy} = \frac{11}{9}mb^2$

$I_{zz} = \frac{8}{9}mb^2$

$I_{xy} = \frac{1}{6}mb^2$

$I_{xz} = \frac{1}{2}mb^2$

$I_{yz} = \frac{1}{6}mb^2$

Eq. 153 gives with $I' = I/mb^2$

$$\begin{vmatrix} \frac{5}{9} - I' & -\frac{1}{6} & -\frac{1}{2} \\ -\frac{1}{6} & \frac{11}{9} - I' & -\frac{1}{6} \\ -\frac{1}{2} & -\frac{1}{6} & \frac{8}{9} - I' \end{vmatrix} = 0$$

Simplifies to $2916I'^3 - 7776I'^2 + 5832I' - 671 = 0$

Solution gives $I_1 = 0.145 mb^2$ min
 $I_2 = 1.299 mb^2$ max
 $I_3 = 1.223 mb^2$ intermediate

8/19 $I_{xz} = \int_{-b}^b (-l \cos \theta)(l \sin \theta) 2c \rho dl = -\frac{2}{3} \rho c b^3 \sin 2\theta$

$I_{yz} = I_{xy} = 0$ by symmetry

$I_{zz} = \bar{I}_{zz} + md^2$ (per panel)

So total $I_{zz} = 2 \left\{ \frac{2c b \rho}{12} (c^2 + [2b \cos \theta]^2) + 2c b \rho (a + \frac{c}{2})^2 \right\}$
 $= 4c b \rho \left\{ \frac{c^2}{3} + \frac{b^2}{3} \cos^2 \theta + a^2 + ac \right\}$

$H_0 = -I_{xz}\omega_z \underline{i} + I_{zz}\omega_z \underline{k}; m = 4c b \rho$ (total)

$H_0 = \frac{m}{3} b^2 \omega \sin \theta \cos \theta \underline{i} + m \omega \left(\frac{c^2}{3} + \frac{b^2}{3} \cos^2 \theta + a^2 + ac \right) \underline{k}$

By symm. principal axes are 0-1, 0-2, 0-y

$I_1 = 4\rho b c \left\{ (b^2 + c^2)/3 + a^2 + ac \right\}$ (max.)

$I_2 = 4\rho b c \left\{ c^2/3 + a^2 + ac \right\}$ (intermediate)

$I_3 = I_{yy} = \frac{4}{3} \rho b^3 c$ (min.)

8/20

$x = x' \cos \phi = r \cos \theta \cos \phi, z = -r \cos \theta \sin \phi$

$I_{xz} = \int xz dA = \rho \int_0^{\pi/2} \int_0^r -\frac{r^2}{2} \sin 2\phi \cos^2 \theta r dr d\theta$

$= -\frac{\rho \pi r^4}{32} \sin 2\phi = -\frac{mr^2}{8} \sin 2\phi$

$I_{yz} = \int yz dA = \rho \int_0^{\pi/2} \int_0^r -\frac{r^2}{2} \sin \phi \sin 2\theta r dr d\theta$

$= -\frac{\rho r^4}{8} \sin \phi = -\frac{mr^2}{2\pi} \sin \phi$

$I_{zz} = \iint R_z^2 dm$ where $R_z^2 = (r \sin \theta)^2 + (r \cos \phi \cos \theta)^2$

$= \rho \int_0^{\pi/2} \int_0^r r^2 (\sin^2 \theta + \cos^2 \phi \cos^2 \theta) r dr d\theta = \frac{\rho \pi r^4}{16} (1 + \cos^2 \phi)$

$= \frac{mr^2}{4} (1 + \cos^2 \phi)$

$H = -I_{xz} \omega_z \underline{i} - I_{yz} \omega_z \underline{j} + I_{zz} \omega_z \underline{k}$

$= \frac{mr^2 \omega}{2} \left(\frac{\sin 2\phi}{4} \underline{i} + \frac{\sin \phi}{\pi} \underline{j} + \frac{1 + \cos^2 \phi}{2} \underline{k} \right)$

8/21 $H_x = 0$ since $\omega_x = \omega_y = 0$ & $I_{xz} = 0$

$H_y = -I_{yz} \omega_z, H_z = I_{zz} \omega_z$

$I_{yz} = \int (-r \sin \theta)(r \cos \theta) \rho r d\theta$

$= -\rho r^3 (-\cos \theta)_0^{\pi} = -\frac{\rho r^3}{4} (-\cos 2\theta)_0^{\pi} = -2\rho r^3 = -2\frac{mr^2}{\pi}$

For complete shell $dI_{zz} = dI_{zz_0} + x^2 dm$

$dI_{zz} = \frac{1}{2} r^2 dm + x^2 dm$

$= \left(\frac{1}{2} r^2 + x^2 \right) \rho dx, \rho = \text{mass/length} = m/2b$

$I_{zz} = \left(\frac{1}{2} r^2 x + \frac{x^3}{3} \right) \rho = 2b\rho \left(\frac{r^2}{2} + \frac{b^2}{3} \right) = m \left(\frac{r^2}{2} + \frac{b^2}{3} \right)$

So for half shell $I_{zz} = m \left(\frac{r^2}{2} + \frac{b^2}{3} \right)$ since $m = \frac{m}{2}$

Thus $H = m\omega \left(\frac{2r^2}{\pi} \underline{j} + \left[\frac{r^2}{2} + \frac{b^2}{3} \right] \underline{k} \right)$

8/22 $\omega_x = \omega_y = 0, \omega_z = \omega$

$H_x = -I_{xz} \omega_z, H_y = -I_{yz} \omega_z, H_z = I_{zz} \omega_z$

$I_{xz} = \bar{I}_{xz} + m d_x d_z = 0 + m(r)(-b/2) = -\frac{mrb}{2}$

$dI_{yz} = (r \sin \theta)(-z) \rho r d\theta dz$

$I_{yz} = \left[-\rho r^2 \frac{z^2}{2} \right]_0^b (-\cos \theta) = -\rho r^2 b^2 = -\frac{mrb}{\pi}$

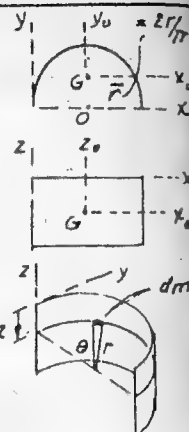
(or more simply $I_{yz} = \bar{I}_{yz} + m d_y d_z$)

$= 0 + m \left(\frac{2r}{\pi} \right) \left(-\frac{b}{2} \right)$

$I_{zz} = \bar{I}_{zz} + m d^2 = (I_0 - m\bar{r}^2) + m(r^2 + \bar{r}^2)$

$= I_0 + mr^2 = 2mr^2$

$H = mr\omega \left(\frac{b}{2} \underline{i} + \frac{b}{\pi} \underline{j} + 2r \underline{k} \right)$



8/23 $\omega_x = \phi, \omega_y = \omega \sin \phi, \omega_z = \omega \cos \phi; I_{xz} = I_{xy} = 0$

$H_x = I_{xx} \omega_x, H_y = I_{yy} \omega_y - I_{yz} \omega_z, H_z = -I_{yz} \omega_y + I_{zz} \omega_z$

$I_{xx} = \bar{I}_{xx} + m\bar{r}^2$ for complete shell of mass m

$= \frac{2}{3} m\bar{r}^2 + m\bar{r}^2 = \frac{5}{3} m\bar{r}^2$. So for half shell $m = \frac{m}{2}$

By symmetry $I_{yy} = I_{xx}$ & $I_{xx} = \frac{5}{3} m\bar{r}^2$

$I_{zz} = \frac{2}{3} m\bar{r}^2$

For differential ring,

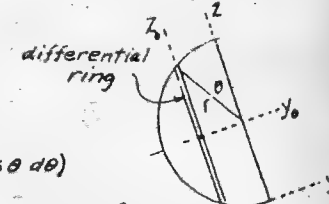
$dI_{yz} = dI_{yz_0} + dm(-r \sin \theta)(r)$

$I_{yz} = 0 - \int_0^{\pi/2} r^2 \sin \theta (2\pi \rho r^2 \cos \theta d\theta)$

$= -\frac{\pi \rho r^4}{2} (-\cos 2\theta)_0^{\pi/2} = -\pi \rho r^4 = -\frac{mr^2}{2}$

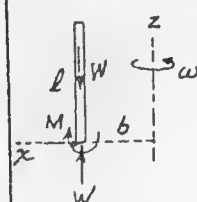
Thus

$H = mr^2 \left[\frac{5\phi}{3} \underline{i} + \omega \left(\frac{5}{3} \sin \phi + \frac{1}{2} \cos \phi \right) \underline{j} + \omega \left(\frac{1}{2} \sin \phi + \frac{3}{3} \cos \phi \right) \underline{k} \right]$

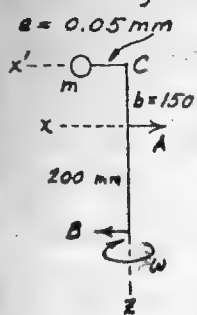


8/25 $\Sigma M_y = -I_{xz} \omega^2, -M = -m \frac{b\ell}{2} \omega^2$


$M = \frac{mb\ell}{2} \omega^2$



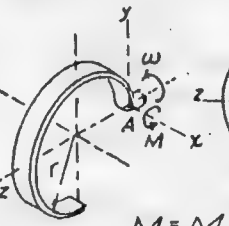
8/26 $\Sigma M_y = -I_{yz} \dot{\omega}_z - I_{xz} \omega_z^2$; $\dot{\omega}_z = 0$
 $a = 0.05 \text{ m}$ $\omega_z = \omega = 10000 \frac{2\pi}{60} = 1047 \text{ rad/s}$
 $I_{xz} = -mbe = -6(0.15)(50)(10^{-6})$
 $= -45(10^{-6}) \text{ kg} \cdot \text{m}^2$
 Thus $B(0.20) = 45(10^{-6})(1047)^2$, $B = 247 \text{ N}$
 For origin of coordinates $x'-y'-z$ at C,
 $\Sigma M_y = 0$ since $I_{x'z} = 0$, so that
 $0.35B - 0.15A = 0$; $A = \frac{0.35}{0.15} 247$
 $= 576 \text{ N}$



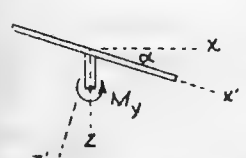
8/31 $\Sigma F_x = m\ddot{x}$; $P = 0$; $\Sigma M_y = -I_{xz} \omega_z^2$
 $I_{xz} = \int xz dm = \int_{-l/4}^{l/4} xy\sqrt{5}(\frac{1}{4}+x)p dx$
 where $p = \text{mass}/(\text{x-component of length})$
 $I_{xz} = \frac{\sqrt{5}}{48} ml^2$ $m = pl/2$
 so $-mg\frac{l}{4} = -\frac{\sqrt{5}}{48} ml^2 \omega^2$; $\omega^2 = \frac{12g}{\sqrt{5}l}$
 $\omega = \sqrt{\frac{12g}{\sqrt{5}l}} = 2\sqrt{\frac{3g}{5l}}$



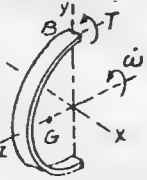
8/32 At A $M_x = I_{yz} \omega_z^2$
 $I_{yz} = \int (r \sin \theta)(r - r \cos \theta) p r d\theta$
 $= \int_0^{3\pi/2} pr^3 [-\cos \theta + \frac{1}{4} \cos 2\theta] d\theta$
 $= \frac{1}{2} pr^3$
 $M = M_x = \frac{1}{2} pr^3 \omega^2$



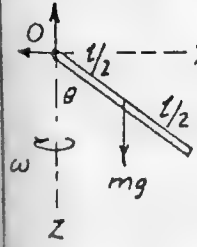
8/28 $\Sigma M_y = -I_{xz} \omega_z^2$; $I_{xz} = \int (x' \cos \alpha)(x' \sin \alpha) dm$
 $= \frac{\sin 2\alpha}{2} I_{yy}$
 $I_{yy} = \frac{1}{4} mr^2$
 so $M_y = -\frac{\sin 2\alpha}{8} mr^2 \omega^2$



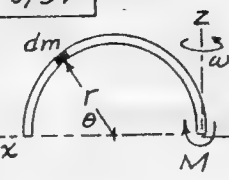
8/33 For entire ring section $\Sigma M_z = I_{zz} \dot{\omega}$
 where $I_{zz} = \frac{3}{4} \frac{1}{2} (2\pi r p) r^2 = \frac{3\pi pr^3}{4}$
 so $\dot{\omega} = \frac{4M}{3\pi pr^3}$
 For 1/2 ring, $\Sigma M_z = I_{zz} \dot{\omega}$; Force at B is zero since $\bar{a} = 0$; Thus
 $\Sigma M_z = T = \frac{\pi pr^3}{2} \frac{4M}{3\pi pr^3} = \frac{2}{3} M$



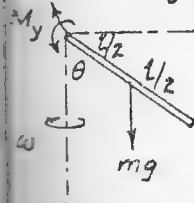
8/29 $\Sigma M_y = -I_{yz} \dot{\omega}_z - I_{xz} \omega_z^2$; $\dot{\omega}_z = \dot{\omega} = 0$
 $I_{xz} = \int xz dm = \int xz p dl$
 $= p \int_0^{l \cos \theta} z^2 \tan \theta \frac{dz}{\cos \theta} = \frac{1}{6} ml^2 \sin 2\theta$
 $-mg \frac{l}{2} \sin \theta = -\frac{1}{6} ml^2 \sin 2\theta \omega^2$
 $\theta = 0$, $\theta = \cos^{-1} \frac{3g}{2l\omega^2}$, $\omega = \sqrt{\frac{3g}{2l}}$



8/34 $\Sigma M_y = -I_{xz} \omega_z^2$
 $I_{xz} = \int xz dm = \int_0^\pi (r + r \cos \theta)(r \sin \theta) p r d\theta$
 where $p = \text{mass/unit length}$
 $I_{xz} = pr^3 [-\cos \theta - \frac{1}{4} \cos 2\theta]_0^\pi$
 $= 2pr^3 = \frac{2}{\pi} mr^2$
 $-M = -\frac{2}{\pi} mr^2 \omega^2$, $M = \frac{2mr^2 \omega^2}{\pi}$



8/30 $\Sigma M_y = -I_{yz} \dot{\omega}_z - I_{xz} \omega_z^2$; $\dot{\omega}_z = 0$, $\omega_z = \omega$
 From Prob. 8/29, $I_{xz} = \frac{1}{6} ml^2 \sin 2\theta$
 $M_y - mg \frac{l}{2} \sin \theta = -\frac{ml^2}{6} \sin 2\theta \omega^2$
 $M_y = \frac{mg l}{2} \sin \theta (1 - \frac{2l}{3g} \omega^2 \cos \theta)$



8/35

With G as origin,

$$I_{xz} = \int_{-\pi/2}^{\pi/2} (r \sin \theta)(-r + r \cos \theta) \rho r d\theta$$

$$+ \int_{\pi/2}^{\pi} (-r \sin \theta)(r - r \cos \theta) \rho r d\theta$$

$$= -\rho r^3 = -\frac{m r^3}{\pi} \quad (\text{same as for origin at O})$$

$$\left. \begin{aligned} \Sigma M_y &= -I_{xz} \dot{\omega}_z; F_A r + F_B r = \frac{m_2 r^2}{\pi} \frac{v^2}{r^2} \\ \Sigma F_x &= 0; F_A - F_B = 0 \end{aligned} \right\} F_A = F_B = \frac{m_2 v^2}{2\pi r}$$

8/36

$$\omega_x = 3600 \frac{2\pi}{60} = 377 \text{ rad/s}$$

$$\Sigma M_x = I_{yz} \omega_z^2; \quad (\text{neglect gravity unbalance})$$

$$\Sigma M_y = -I_{xz} \omega_z^2;$$

$$I_{yz} = 0.8(0.04)(0.15) + 0.6(-0.05)(\frac{1}{2})(0.2)$$

$$= -0.000396 \text{ kg} \cdot \text{m}^2$$

$$I_{xz} = 1(0.025)(0.05) + 0.6(0.05/2)(0.2)$$

$$= 0.00425 \text{ kg} \cdot \text{m}^2$$

$$\left. \begin{aligned} -0.25 B_y &= -396(10^{-6})(377)^2, \quad B_y = 225 \text{ N} \\ 0.25 B_x &= -4250(10^{-6})(377)^2, \quad B_x = -2420 \text{ N} \end{aligned} \right\} B = \sqrt{225^2 + 2420^2}$$

$$= 2430 \text{ N}$$

For origin at B with x'-y'-z,

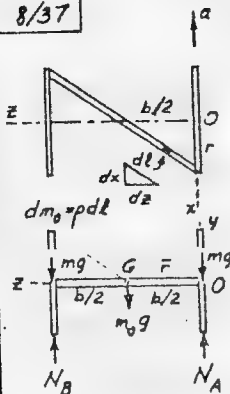
$$I_{y'z'} = 0.8(0.04)(-0.10) + 0.6(-0.05)(\frac{1}{2})(-0.05) = -1901(10^{-6}) \text{ kg} \cdot \text{m}^2$$

$$I_{x'z'} = 1(0.025)(0.2) + 0.6(0.05/2)(-0.05) = -5750(10^{-6}) \text{ kg} \cdot \text{m}^2$$

$$0.25 A_y = -1901(10^{-6})(377)^2, \quad A_y = -1081 \text{ N}$$

$$-0.25 A_x = +5750(10^{-6})(377)^2, \quad A_x = -3270 \text{ N}$$

8/37



$$\Sigma M_x = -I_{xz} \dot{\omega}_x + I_{yz} \omega_z^2; \quad \dot{\omega}_x = 0, \quad \dot{\omega}_z = \frac{a}{r}$$

$$I_{xz} = \int xz dm = \int x \frac{b}{2r} (r-x) \rho dl$$

$$= \frac{b \rho}{4r^2} \int_0^r x(r-x) dx = -\frac{1}{6} m_0 b r$$

$$\text{so } m_0 g \frac{b}{2} + (mg - N_B) b = +\frac{1}{6} m_0 b r \frac{a}{r}$$

$$N_B = mg + \frac{m_0 g}{2} \left(1 - \frac{1}{3} \frac{a}{g}\right)$$

$$\Sigma F_y = 0; N_A + N_B = 2mg + m_0 g$$

$$N_A = mg + \frac{m_0 g}{2} \left(1 + \frac{1}{3} \frac{a}{g}\right)$$

(Note: ΣM_x of Eq. 165 is valid here since $\rho_0 \times m \bar{r}$ term of Eq. 100 has no x-component.)

8/38

$$\omega_z = 0, \quad \dot{\omega}_z \neq 0 \quad (\text{Sample Prob. 8/24}), \quad I_{xz} = m_2 r^2 / \pi$$

$$\Sigma M_z = I_{zz} \dot{\omega}_z; \quad I_{zz} = \frac{2m_1}{2} r^2 + \frac{m_2}{2} r^2 = \left(m_1 + \frac{m_2}{2}\right) r^2 = \frac{m r^2}{2}$$

$$\text{where } (I_z)_{\text{complete hoop}} = \frac{1}{2} m r^2 = \frac{1}{2} 2m_2 r^2$$

$$(I_z)_{\text{two } \frac{1}{2}\text{-hoops}} = \frac{m_2}{2} r^2$$

$$\text{So } M + F_A r - F_B r = \frac{m r^2}{2} \dot{\omega}_z \quad \left\{ \begin{aligned} M &= \frac{3}{2} m r \dot{\omega}_z, \quad \dot{\omega}_z = \frac{2M}{3mr^2} \\ \Sigma F_x &= m \ddot{a}_x; \quad F_B - F_A = m r \dot{\omega}_z \end{aligned} \right.$$

$$\Sigma M_x = -I_{xz} \dot{\omega}_z; \quad mgr - N_A(2r) = -\left(\frac{m_2 r^2}{\pi}\right) \frac{2M}{3mr^2}$$

$$\Sigma F_y = 0; N_A + N_B = mg$$

$$\text{which give } \left\{ \begin{aligned} N_A &= \frac{mg}{2} - \frac{M m_2}{3\pi m r} \\ N_B &= \frac{mg}{2} + \frac{M m_2}{3\pi m r} \end{aligned} \right. \quad \begin{aligned} &(\text{Note: } \Sigma M_x \text{ of Eq. 165} \\ &\text{is valid here since} \\ &\text{the term } \rho_0 \times m \bar{r} \\ &\text{of Eq. 100 has} \\ &\text{no x-component}) \end{aligned}$$

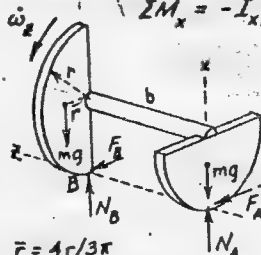


8/39

$$\Sigma M_z = I_{zz} \dot{\omega}_z; \quad -mg \frac{4r}{3\pi} = \left(3 - \frac{8}{3\pi}\right) m r^2 \dot{\omega}_z, \quad \dot{\omega}_z = \frac{-4g/r}{9\pi - 8}$$

$$\Sigma M_x = -I_{xz} \dot{\omega}_z; \quad F_B b = -m r b \frac{-4g/r}{9\pi - 8}, \quad F_B = \frac{4mg}{9\pi - 8}$$

(Note: A is valid origin for Eq. 165 since $a_A = 0$)



$$\Sigma F_y = m \ddot{a}_y;$$

$$\frac{4mg}{9\pi - 8} + F_A = m \left(r + \left[r - \frac{4r}{3\pi}\right]\right) \frac{4g/r}{9\pi - 8}$$

$$F_A = \frac{4mg}{9\pi - 8} \left(1 - \frac{4}{3\pi}\right)$$

$$\bar{r} = 4r/3\pi$$

$$I_{zzB} = \frac{3}{2} m r^2$$

$$I_{zzA} = \bar{I} + m(r - \bar{r})^2$$

$$= \frac{1}{2} m r^2 - m \bar{r}^2$$

$$+ m(r - \bar{r})^2$$

$$= \left(\frac{3}{2} - \frac{8}{3\pi}\right) m r^2$$

$$I_{zz} = \left(3 - \frac{8}{3\pi}\right) m r^2$$

$$I_{xz} = m r b$$

Similarly

$$N_A = mg, \quad N_B = \frac{mg}{3\pi} \frac{(9\pi + 4)(3\pi - 4)}{9\pi - 8}$$

$$f_A = \frac{F_A}{N_A} = \frac{4}{3\pi} \frac{3\pi - 4}{9\pi - 8} = 0.1136$$

$$f_B = \frac{F_B}{N_B} = \frac{12\pi}{(9\pi + 4)(3\pi - 4)} = 0.215 (\text{governs})$$

8/40

$\Sigma M_y = -I_{xz} \omega_z^2; N_A \sin \theta (2r) + F_A \cos \theta (2r)$
 From Prob. 8/24 $-mgr \sin \theta = \frac{m_2 r^2}{\pi} \frac{v^2}{r^2}$ or
 $I_{xz} = -\frac{m_2 r^2}{\pi}$
 $N_A \sin \theta + F_A \cos \theta = \frac{m_2 g}{2} \sin \theta$
 $= \frac{m_2 v^2}{2\pi r} \quad \text{--- (a)}$
 $\Sigma M_x = I_{yz} \omega_z^2 = 0;$
 $-N_A \cos \theta (2r) + F_A \sin \theta (2r) + mgr \cos \theta = 0$
 or $N_A \cos \theta - F_A \sin \theta = \frac{mg}{2} \cos \theta = 0 \quad \text{--- (b)}$
 Solve (a) & (b) for N_A & F_A & get
 $F_A = \frac{m_2 v^2}{2\pi r} \cos \theta, N_A = \frac{mg}{2} + \frac{m_2 v^2}{2\pi r} \sin \theta$
 From $\Sigma F_x = \Sigma F_y = 0, F_B = F_A; N_B = \frac{mg}{2} - \frac{m_2 v^2}{2\pi r} \sin \theta$

8/46

Front

 Change in normal force is ΔR
 Triad ψ, ϕ, M as shown
 ΔR is down under right rear wheel to give proper direction of M .
 Hence decrease of force under right rear wheel.

8/47

$\dot{\psi} = 20000 \frac{2\pi}{60} = 2094 \text{ rad/s}$
 $\ddot{\psi} = 2 \text{ rad/s}$
 $I = 3.5(2094)^2 + 2.4(0.071)^2$
 $= 0.0339 \text{ kg} \cdot \text{m}^2$
 $M = I \ddot{\psi}$
 $0.15C = 0.0339(2094)(2), C = D = 948 \text{ N}$

8/43

With couple shown and A deflects up

8/44

Front view of left wheel
 $\dot{\psi} = 0.5 \text{ rad/s}$
 Eq. 168, $M = I \dot{\psi} \ddot{\psi}$
 $\dot{\psi} = \frac{v}{r} = \frac{225}{3.6} / 0.4 = 156.2 \text{ rad/s}$
 $M = 31(0.25)^2 (156.2)(0.5)$
 $= 151.4 \text{ N} \cdot \text{m}$

8/45

$M = I \dot{\psi} \ddot{\psi}$
 $= 225(0.25)^2 (18000) \frac{2\pi}{60} \frac{1000/3.6}{3000}$
 $= 2450 \text{ N} \cdot \text{m}$
 Nose tends to rise

8/49

M needed on structure of ship to counteract roll to port (left).
 Reaction on gyro is opposite to M on ship. Proper direction of triad shown — requiring rotation (b) of motor.
 $M = I \dot{\psi} \ddot{\psi} = 80(1.45)^2 960 \frac{2\pi}{60} 0.320 = 5410 \text{ kN} \cdot \text{m}$

8/50

Assume right turn

 $m\bar{a} = m v^2 / R \quad \Sigma M_O = m\bar{a} h$
 $M + 0 = m v^2 h / R$
 $M = I \dot{\psi} \ddot{\psi} = m_0 k^2 p v / R$
 so $m v^2 \frac{h}{R} = m_0 k^2 p \frac{v}{R}$
 $p = \frac{m v h}{m_0 k^2}$ opposite direction to wheels
 Rear views

8/51



$$I = I_{zz} = mr^2$$

$$I_0 = I_{xx} = \frac{1}{2}mr^2 + \frac{1}{12}ml^2$$

$$\frac{I_0}{I} = \frac{1}{2} + \frac{1}{12}\left(\frac{l}{r}\right)^2$$

Direct precession if $\frac{I_0}{I} > 1$; $\frac{1}{2} + \frac{1}{12}\left(\frac{l}{r}\right)^2 > 1$, $\frac{l}{r} > \sqrt{6}$

Retrograde " " $\frac{I_0}{I} < 1$; $\frac{l}{r} < \sqrt{6}$

$$8/52 \quad I_1 = \frac{1}{12}m(a^2 + 2a^2), I_2 = \frac{1}{12}m(a^2 + 3a^2)$$

$$I_3 = \frac{1}{12}m(2a^2 + 3a^2)$$

$$I_1 = \frac{5}{12}ma^2, I_2 = \frac{5}{6}ma^2, I_3 = \frac{13}{12}ma^2$$

$$T_{0-1} = T_{0-2}; \frac{1}{2}I_2\omega_0^2 = \frac{1}{2}I_1\omega^2, \omega = \omega_0\sqrt{I_2/I_1}$$

$$\text{so } \omega = \omega_0\sqrt{\frac{5/6 ma^2}{5/12 ma^2}} = \omega_0\sqrt{2}$$

Axis 0-2 is axis of intermediate principal moment of inertia so rotation about 0-2 is unstable.

8/53

$$A; I_1 = I_2 = m\left(\frac{r^2}{2} + \frac{l^2}{12}\right) = 39\left(\frac{0.3^2}{2} + \frac{0.75^2}{12}\right)$$

$$= 3.58 \text{ kg}\cdot\text{m}^2$$

$$I_3 = mr^2 = 39(0.3)^2$$

$$= 3.51 \text{ kg}\cdot\text{m}^2$$

$$B; I_1 = I_3 = \frac{1}{12}ml^2$$

$$= \frac{12}{12}(1)^2 = 1.000 \text{ kg}\cdot\text{m}^2$$

$$I_2 \approx 0$$

$$\text{So } I_1 = 3.58 + 1.00 = 4.58 \text{ kg}\cdot\text{m}^2 \text{ (max)}$$

$$I_2 = 3.58 + 0 = 3.58 \text{ kg}\cdot\text{m}^2 \text{ (min)}$$

$$I_3 = 3.51 + 1.00 = 4.51 \text{ kg}\cdot\text{m}^2 \text{ (intermed)}$$

Rotation about axes 1 & 2 stable

" " axis 3 unstable

8/54

$$\text{From Eq. 174 } f = \frac{\dot{\psi}}{2\pi} = \frac{1}{2\pi} \frac{I\dot{\psi}}{(I - I_0)\cos\theta}$$

$$\cos\theta \approx 1, \frac{\dot{\psi}}{2\pi} = \frac{300}{60} = 5 \text{ rev/s}$$

$$\frac{I}{I_0 - I} = \frac{mr^2}{\frac{1}{2}mr^2 - mr^2} = -2$$

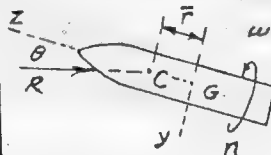
$$\text{So } f = 2(5) = 10 \text{ Hz}$$

Retrograde precession



8/55

$\Sigma M_x = M = R\bar{F}\sin\theta$ & from Eq. 170 minimum $n = \dot{\phi}$ for precession with $\theta \rightarrow 0$ is



$$n = \frac{2}{I} \sqrt{R\bar{F}\sin\theta (I_0 - I) \tan\theta}$$

$$\text{& for } \theta = 0, n = \frac{2}{I} \sqrt{R\bar{F}(I_0 - I)}$$

8/56

For zero moment, $\dot{\psi} = \frac{I\dot{\phi}}{(I_0 - I)\cos\theta}$

where $k = 0.72 \text{ m}$

$$k_0 = 0.54 \text{ m}$$

$$\dot{\phi} = 1.5 \text{ rad/s}$$

$$\theta = 2^\circ$$

$(I = k^2 m) > (I_0 = k_0^2 m)$ so retrograde precession with $\dot{\phi}$ in negative z -dir.

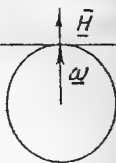
$$\text{Period } T = \left| \frac{2\pi}{\dot{\psi}} \right| = 2\pi \left| \frac{(k_0/k)^2 - 1}{\dot{\phi}} \cos\theta \right|$$

$$= 2\pi \left| \frac{(0.54/0.72)^2 - 1}{1.5} 0.9994 \right| = 1.831 \text{ s}$$



8/57

When $I_0 \rightarrow I$, the Poincaré ellipsoid approaches a sphere & $\underline{\omega}$ & \underline{H} must be collinear. Thus body spins about one axis with no precession. Space & body cones degenerate into the common line which is the invariable line.



8/58

$$\omega_z = \frac{v}{r}, \omega_x = 0, \omega_y = \omega_0$$

$$\bar{v} = b\omega_0 \text{ (rel. to airplane)}$$

$$\bar{H} = I_{xx}\omega_x\bar{i} + I_{yy}\omega_y\bar{j} + I_{zz}\omega_z\bar{k}$$

$$\text{since } I_{xy} = I_{xz} = I_{yz} = 0$$

$$\bar{H} = m(k_0^2\omega_0\bar{j} + k^2\frac{v}{r}\bar{k})$$

$$\underline{\omega} = \omega_0\bar{j} + \frac{v}{r}\bar{k}$$

$$T = \frac{1}{2}\underline{\omega} \cdot \bar{H} + \frac{1}{2}\bar{v} \cdot \underline{G} = \frac{1}{2} \left[mk_0^2\omega_0^2 + mk^2\frac{v^2}{r^2} \right] + \frac{1}{2}b\omega_0 m b\omega_0$$

$$\text{So } T = \frac{1}{2}m \left[(k_0^2 + b^2)\omega_0^2 + k^2\frac{v^2}{r^2} \right]$$

8/59 $I_{zz} = \frac{1}{2}mr^2 = \frac{1}{2}(30)(0.075)^2 = 0.0844 \text{ kg}\cdot\text{m}^2$

$I_{xx} = I_{yy} = \frac{1}{4}mr^2 + \frac{1}{12}mh^2$

$= \frac{30}{4}[(0.075)^2 + \frac{1}{3}(0.225)^2] = 0.1688 \text{ kg}\cdot\text{m}^2$

$\dot{\psi} = \frac{150(2\pi)}{60} = 15.71 \text{ rad/s} = 5\pi \text{ rad/s}$

$\dot{\phi} = 300 \frac{2\pi}{60} = 10\pi \text{ rad/s}$

$\omega_x = \dot{\phi} = 0, \omega_y = \dot{\psi} \sin \theta = \frac{5\sqrt{3}}{2} \pi \text{ rad/s}$

$\omega_z = \dot{\phi} + \dot{\psi} \cos \theta = \frac{25\pi}{2} \text{ rad/s}$

$\bar{H}_x = I_{xx}\omega_x = 0; \bar{H}_y = I_{yy}\omega_y = 0.1688 \frac{5\sqrt{3}}{2} \pi \text{ kg}\cdot\text{m}^2/\text{s}$

$\bar{H}_z = I_{zz}\omega_z = 0.0844 \frac{25\pi}{2} \text{ kg}\cdot\text{m}^2/\text{s}$

$T = \frac{1}{2}\omega \cdot \bar{H} = \frac{1}{2}[\omega_x \bar{H}_x + \omega_y \bar{H}_y + \omega_z \bar{H}_z]$

$= \frac{1}{2}[0 + 0.1688(\frac{5\sqrt{3}}{2}\pi)^2 + 0.0844(\frac{25\pi}{2})^2]$

$= \frac{0.0844}{8}(25\pi^2)(31) = 80.7 \text{ J}$

8/60 $I = mr^2, I_0 = \frac{1}{2}mr^2, \text{Eq. 173, } \tan \theta = \frac{1}{2} \tan \beta$

so $\tan \beta = 2 \tan 10^\circ, \beta = 19.43^\circ$

Eq. 174, $\dot{\psi} = \frac{300(2\pi/60)}{(\frac{1}{2}-1) \cos 10^\circ} = -63.8 \text{ rad/s}$

$T_{\text{rotation}} = \frac{1}{2}\omega \cdot \bar{H} = \frac{1}{2}(I_{xx}\omega_x^2 + I_{yy}\omega_y^2 + I_{zz}\omega_z^2)$

$\omega_x = \Omega_x = \dot{\phi} = 0; \omega_y = \Omega_y = \dot{\psi} \sin \theta$

$= -63.8 \sin 10^\circ = -11.08 \frac{\pi}{s}$

$\omega_z = \dot{\psi} \cos \theta = 31.4 - 63.8 \cos 10^\circ$

$= -31.4 \text{ rad/s}$

$I_{xx} = I_{yy} = I_0 = \frac{1}{2}mr^2 = \frac{1}{2}(8)(0.3)^2$

$= 0.360 \text{ kg}\cdot\text{m}^2$

$I_{zz} = I = mr^2 = 0.720 \text{ kg}\cdot\text{m}^2$

$T = \frac{1}{2}(0.360)(0^2 + 11.08^2 + 2[31.4]^2)$

$= 377 \text{ J}$

8/61 From the second of Eqs. 171 with θ replaced by $\pi - \theta$

$\dot{\psi} = \frac{M}{I\dot{\phi} \sin \theta}; M = mg\bar{r} \sin \theta = \frac{9}{4}mgr \sin \theta$

$I = \frac{3}{10}mr^2, \dot{\phi} = 2700 \frac{2\pi}{60} = 283 \text{ rad/s}$

$\dot{\psi} = \frac{\frac{9}{4}mgr \sin \theta}{\frac{3}{10}mr^2(283) \sin \theta} = \frac{15 \cdot 9.81}{2(0.1)(283)}$

$= 2.60 \text{ rad/s}$

Result independent of θ

Period $T = \frac{2\pi}{\dot{\psi}} = \frac{2\pi}{2.60} = 2.41 \text{ s}$

8/62 For cone $I_0 = I_{xx} = \frac{3}{5}mH^2 + \frac{3}{20}mr^2$

with $H=3r, I_0 = \frac{111}{20}mr^2; I = I_{zz} = \frac{3}{10}mr^2$

$I_0 - I = \frac{21}{4}mr^2, I/(I_0 - I) = 2/35$

1st of Eqs. 171 } $\dot{\psi}_{\text{fast}} = \frac{I\dot{\phi}}{(I_0 - I) \cos(\pi - \theta)} = \frac{2}{35} 2700$

with $\theta \rightarrow 0$ } $= 154 \text{ rev/min (neg. Z-dir.)}$

8/63 $H_x = I_{xx}\omega_x = I_{xx}\dot{\theta}$, small so H_x is essentially in y-z plane

Friction force acts into paper at C so exerts moment M_f as shown.

Thus change in H_0 due to M_f is in direction of M_f , hence toward Z-axis. Thus axis rises.

From Eq. 170 min. value of $n = \dot{\phi}$ for solution is $n = \dot{\phi} = \frac{2}{I} \sqrt{M_0(I_0 - I) \cot \theta}$

But $M = W\bar{r} \sin \theta$, so for $\theta \rightarrow 0$

$n = \frac{2}{I} \sqrt{mg\bar{r}(I_0 - I)}$, (assumes $I_0 > I$)

8/64 $\dot{\theta} = 0, \dot{\phi} = p = \text{const.}, M_z = \text{torque of electric field to maintain } \dot{\phi} \text{ constant (}\omega_z \text{ is not constant)}$

Apply Eqs. 166 with $I = mk_x^2, I_0 = mk_z^2, \gamma = \pi/2 - \theta$

$x; 0 = mk_x^2(-\ddot{\gamma} - 0) + 0$, so $\ddot{\gamma} = 0$

$y; F_c = F_z b = \frac{mk_x^2}{\cos \gamma} \frac{d}{dt}(\dot{\psi} \cos^2 \gamma) + mk_z^2 \dot{\gamma} \omega_z$

$= mk_x^2 \dot{\psi} \cos \gamma$

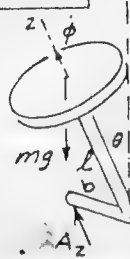
$z; -M_z = -F_y b = mk_z^2(\ddot{\psi} \sin \gamma + 0)$

But $M = F_z b \cos \gamma - F_y b \sin \gamma$

$= mk_x^2 \ddot{\psi} \cos^2 \gamma + mk_z^2 \sin^2 \gamma$

so $\ddot{\psi} = \frac{M/m}{k_x^2 \cos^2 \gamma + k_z^2 \sin^2 \gamma}$

48/65


 $\dot{\psi} = \dot{\phi} = 0$; From moment equations Eqs 166

$$x: mgl \sin \theta = m \left(\frac{r^2}{4} + l^2 \right) \ddot{\theta} \quad \text{--- (a)}$$

$$\text{where } I_0 = I_{xx} = \frac{1}{4}mr^2 + ml^2$$

$$y: -b(-A_z + B_z) = -\frac{1}{2}mr^2 \dot{\phi} \ddot{\theta} \quad \text{--- (b)}$$

$$\text{where } I = \frac{1}{2}mr^2$$

$$z: 0 = I \ddot{\phi} \text{ where } \dot{\omega}_z = 0 + \dot{\phi} \quad \text{--- (c)}$$

From (c), $\dot{\phi} = \text{constant}$ $\pi/2$

$$\text{From (a) with } \dot{\theta} d\theta = \ddot{\theta} d\theta, \int gl \sin \theta d\theta = \left(\frac{r^2}{4} + l^2 \right) \int \ddot{\theta} d\theta$$

$$\text{which gives } \dot{\theta}^2 = 8gl / (r^2 + 4l^2)$$

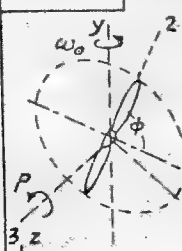
$$\text{From (b) } -A_z + B_z = \frac{1}{2}m \frac{r^2}{b} \dot{\phi} \dot{\theta} \text{ for } \theta = \pi/2$$

$$\text{Also for } \theta = \pi/2, \Sigma F_z = m\ddot{a}_z \text{ gives } -A_z - B_z = m\ddot{\theta}^2$$

$$\text{Solve \& get } \left. \begin{aligned} A_z &= -\frac{m\dot{\theta}}{2} \left(\frac{r^2}{2b} \dot{\phi} + l\dot{\theta} \right) \\ B_z &= \frac{m\dot{\theta}}{2} \left(\frac{r^2}{2b} \dot{\phi} - l\dot{\theta} \right) \end{aligned} \right\} \text{ where } \dot{\theta} = 2\sqrt{\frac{2gl}{r^2 + 4l^2}}$$

48/66

From Eqs. 158 with 1,2,3 sub. for x,y,z,



$$M_1 = I_1 \dot{\omega}_1 - (I_2 - I_3) \omega_2 \omega_3$$

$$M_2 = I_2 \dot{\omega}_2 - (I_3 - I_1) \omega_3 \omega_1$$

$$M_3 = I_3 \dot{\omega}_3 - (I_1 - I_2) \omega_1 \omega_2$$

$$I_1 = I_3 = I; I_2 = 0$$

$$\omega_1 = -\omega_0 \cos \phi, \omega_2 = \omega_0 \sin \phi, \omega_3 = p = \dot{\phi}$$

$$\dot{\omega}_1 = \omega_0 p \sin \phi, \dot{\omega}_2 = \omega_0 p \cos \phi, \dot{\omega}_3 = 0$$

Thus

$$M_0 = M_1 = I\omega_0 p \sin \phi + Ip\omega_0 \sin \phi = 2I\omega_0 p \sin \phi$$

(bending)

$$M_2 = 0 - 0 = 0$$

$$M_z = M_3 = 0 - I(-\omega_0 \cos \phi)(\omega_0 \sin \phi) = \frac{1}{2}I\omega_0^2 \sin 2\phi$$

(torsion)

48/67

x-y-z fixed to aircraft

Angular velocity of axes $\Omega_x = 0, \dot{\Omega}_x = 0$

$$\Omega_y = \omega_0, \dot{\Omega}_y = 0$$

$$\Omega_z = 0, \dot{\Omega}_z = 0$$



Angular velocity of propeller $\omega_x = 0, \dot{\omega}_x = 0$

$$\omega_y = \omega_0, \dot{\omega}_y = 0$$

$$\omega_z = p = \dot{\phi}, \dot{\omega}_z = 0$$

$$I_{xy} = \int (y \cos \phi)(x \sin \phi) dm$$

$$= \frac{1}{2} \sin 2\phi \int x^2 dm = \frac{1}{2} \sin 2\phi I$$

$$I_{yy} = \int x^2 \cos^2 \phi dm = I \cos^2 \phi$$

$$H_x = I_{xx} \omega_x - I_{xy} \omega_y - I_{xz} \omega_z = 0 - \frac{I}{2} \sin 2\phi \omega_0 + 0, \dot{H}_x = -I\omega_0 p \cos 2\phi$$

$$H_y = -I_{xy} \omega_x + I_{yy} \omega_y - I_{yz} \omega_z = 0 + I\omega_0 \cos^2 \phi + 0, \dot{H}_y = -I\omega_0 p \sin 2\phi$$

$$H_z = -I_{xz} \omega_x - I_{yz} \omega_y + I_{zz} \omega_z = 0 + 0 + Ip; \dot{H}_z = 0$$

$$\Sigma M_x = \dot{H}_x - H_y \Omega_z + H_z \Omega_y = -I\omega_0 p \cos 2\phi - 0 + I\omega_0 p = 2I\omega_0 p \sin 2\phi$$

$$\Sigma M_y = \dot{H}_y - H_z \Omega_x + H_x \Omega_z = -I\omega_0 p \sin 2\phi + 0$$

$$\Sigma M_z = \dot{H}_z - H_x \Omega_y + H_y \Omega_x = 0 + \frac{1}{2}I\omega_0^2 \sin 2\phi + 0$$

$$M_1 = M_x \sin \phi - M_y \cos \phi = 2I\omega_0 p \sin \phi (\sin^2 \phi + \cos^2 \phi) = 2I\omega_0 p \sin \phi$$

$$M_2 = 0, M_3 = \frac{1}{2}I\omega_0^2 \sin 2\phi$$

48/68

The second of Eqs. 159 with $M_y = 0$

& with $I_{xx} = I_{yy} = I_0, I_{zz} = I, \phi = p,$

$$\Omega_x = -\omega \cos \gamma \sin \beta, \Omega_y = \omega \sin \gamma + \beta, \Omega_z = \omega \cos \gamma \cos \beta$$

$$\text{gives } 0 = I_0(0 + \beta) + (I - I_0)\omega^2 \cos^2 \gamma \sin \beta \cos \beta$$

$$+ Ip\omega \cos \gamma \sin \beta = 0$$

Neglect term in ω^2 compared with $p\omega$ & get

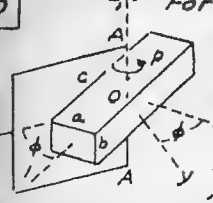
$$\beta + K^2 \sin \beta = 0 \text{ where } K^2 = \frac{I}{I_0} \omega p \cos \gamma$$


which is the same form as that for the simple pendulum. With β small, this equation describes the linear harmonic oscillator

$$\text{and has a period } \tau = \frac{2\pi}{K} = 2\pi \sqrt{\frac{I_0}{I\omega p \cos \gamma}}$$

Thus gyro axis will oscillate about the north direction. With some damping it will point north.

$\tau = 2\pi/\dot{\psi} = \text{const.}$ $\dot{\psi} = -2r\sqrt{10}\dot{\psi}/2r = -\frac{2\pi\sqrt{10}}{\tau}$
 $\theta = \tan^{-1} 3$ (Seen by allowing ground to rotate under axle at rate $\dot{\psi}$)
 $= 71.6^\circ$
 $I = I_{zz} = \frac{1}{2} 4m(2r)^2 + \frac{1}{2} mr^2 = \frac{17}{2} mr^2$
 $I_{xx} = I_o = \frac{1}{4} 4m(2r)^2 + 4m(6r)^2 + \frac{1}{2} mr^2 + m(3r)^2 = \frac{629}{4} mr^2$
 Eq. 166 $\Sigma M_x = I_o \ddot{\theta} + (I - I_o) \dot{\psi}^2 \sin \theta \cos \theta + I \dot{\psi} \dot{\psi} \sin \theta$
 substitute & get
 $mg(3r \sin \theta + 4(6r) \sin \theta) - r\sqrt{10}(2N_1 + N_2)$
 $= 0 + \left(\frac{17}{2} - \frac{629}{4}\right) mr^2 \left(\frac{2\pi}{\tau}\right)^2 \frac{3}{\sqrt{10}} \frac{1}{\sqrt{10}} - \frac{17}{2} mr^2 \frac{2\pi\sqrt{10}}{\tau} \frac{2\pi}{\tau} \frac{3}{\sqrt{10}}$
 $= -\frac{280.5 mr^2 \pi^2}{\tau^2}$ or
 $27 mg \sin \theta - \sqrt{10}(2N_1 + N_2) = -\frac{280.5 mr^2 \pi^2}{\tau^2}$
 so $\Sigma F_z = 0$; $N_1 + N_2 = 5 mg$
 solve simultaneously } & get $N_1 = 3.94 mg$
 with $\tau = 4.3$ $N_2 = 1.06 mg$
 $r = 0.15 \text{ m}$
 $g = 9.81 \text{ m/s}^2$

8/70  For constant ω_0 , M applied to shaft about x-axis equals moment applied to block by shaft A-A.
 Use Euler's Eqs 158 where 1, 2, 3 replace x, y, z
 Angular velocity of block is ω
 $\omega_1 = \omega_0 - (I_2 - I_3) \omega_2 \omega_3$ & $M_2 = I_2 \dot{\omega}_2 - (I_3 - I_1) \omega_3 \dot{\omega}_1$
 $\omega_1 \cos \phi$, $\omega_2 = -\omega_0 \sin \phi$, $\omega_3 = \dot{\phi} = p$
 $I_1 = \frac{m}{12}(a^2 + b^2)$, $I_2 = \frac{m}{12}(b^2 + c^2)$, $I_3 = \frac{m}{12}(a^2 + c^2)$
 $M_1 = \frac{m}{12}(a^2 + b^2)(-\omega_0 \dot{\phi} \sin \phi) - \frac{m}{12}(b^2 - a^2)(-\dot{\phi} \omega_0 \sin \phi)$
 $= -\frac{1}{6} ma^2 p \omega_0 \sin \phi$
 $M_2 = \frac{m}{12}(b^2 + c^2)(-\omega_0 \dot{\phi} \cos \phi) - \frac{m}{12}(c^2 - b^2)(\dot{\phi} \omega_0 \cos \phi)$
 $= -\frac{1}{6} mc^2 p \omega_0 \cos \phi$
 $M = M_1 \cos \phi - M_2 \sin \phi$
 $M_1 = \frac{1}{12} m(c^2 - a^2) p \omega_0 \sin 2\phi$

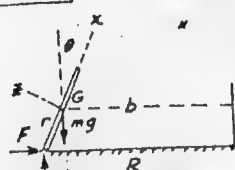
 $I_o = I_{xx} = I_{yy} = \frac{1}{2} mr^2$, $I = I_{zz} = mr^2$
 Sample Prob. 8/71,
 $n_{\min} = \sqrt{\frac{\frac{1}{2} mr^2 mgr}{mr^2(mr^2 + mr^2)}} = \frac{1}{2} \sqrt{\frac{g}{r}}$
 so $v = rn = \frac{1}{2} \sqrt{gr} = \frac{1}{2} \sqrt{9.81(0.3)}$
 $= 0.858 \text{ m/s}$

8/73 From Sample Prob. 8/71, cones will be stable if
 $n > n_{\min} = \sqrt{\frac{I_{xx} mgr}{I_{zz}(I_{zz} + mr^2)}}$
 where $I_{xx} = 2 \frac{m}{20} (3r^2 + 2h^2)$ } mass of each cone
 $I_{zz} = 2 \frac{3}{10} mr^2$
 $h = 600 \text{ mm}$
 $r = 300 \text{ mm}$
 Substitute & get
 $v = r \sqrt{\frac{\frac{1}{10} (3r^2 + 2h^2) 2gr}{\frac{3}{10} r^2 (\frac{3}{10} + 2) r^2}} = \sqrt{\frac{5gr}{39} (3 + \frac{2h^2}{r^2})}$
 $= \sqrt{\frac{5(9.81)(0.3)}{39} (3 + \frac{2(0.6)^2}{(0.3)^2})} = 2.04 \text{ m/s}$

8/74 Use the coordinates and notation of Sample Prob. 8/71
 $I = I_{zz} = 0$, $I_o = I_{xx} = I_{yy} = \frac{1}{12} mL^2$
 The differential equation (j) becomes
 $m(\frac{L^2}{12} + r^2) \ddot{\theta} - mgr \theta = 12 \frac{r^2}{L^2} Cn$
 Coeff. of θ is negative, hence motion grows with time & is unstable.

8/75 For disk, $\omega_x = -\frac{v}{b} \cos \theta$, $\omega_y = 0$, $\omega_z = \frac{v}{r} - \frac{v}{b} \sin \theta$
 so angular velocity is
 $\omega = -\frac{v}{b} \cos \theta \mathbf{i} + \left(\frac{v}{r} - \frac{v}{b} \sin \theta\right) \mathbf{k}$
 Angular momentum about center is
 $\vec{H} = \frac{1}{4} mr^2 \left(-\frac{v}{b} \cos \theta\right) \mathbf{i} + \frac{1}{2} mr^2 \left(\frac{v}{r} - \frac{v}{b} \sin \theta\right) \mathbf{k}$
 $T = \frac{1}{2} \vec{v} \cdot \vec{G} + \frac{1}{2} \omega \cdot \vec{H}$
 $= \frac{1}{2} v(mv) + \frac{1}{2} \left(\frac{1}{4} mr^2\right) \left(\frac{v}{b} \cos \theta\right)^2 + \frac{1}{2} \left(\frac{1}{2} mr^2\right) \left(\frac{v}{r} - \frac{v}{b} \sin \theta\right)^2$
 $= \frac{1}{4} mv^2 \left[3 - \frac{2r}{b} \sin \theta + \frac{1}{2} \frac{r^2}{b^2} (1 + \sin^2 \theta)\right]$

8/76



velocity of G
 $x = v_j$ (into paper)

$$I = I_{zz} = \frac{1}{2} m r^2$$

$$I_{xx} = I_{yy} = \frac{1}{4} m r^2$$

Also the force eqs. are $F = m \frac{v^2}{b}$; $N = mg$; $0 = 0$

Substitution gives

$$m g r \sin \theta - m \frac{v^2}{b} r \cos \theta = \frac{1}{4} m \frac{v^2}{b} r \cos \theta (2 + \frac{r}{b} \sin \theta)$$

which simplifies to

$$\tan \theta = \frac{v^2}{2 g b} (3 + \frac{r}{2 b} \sin \theta)$$

Ang. vel. axes $\omega_x = -\frac{v}{b} \cos \theta$, $\omega_y = 0$, $\omega_z = -\frac{v}{b} \sin \theta$

disk $\omega_x = -\frac{v}{b} \cos \theta$, $\omega_y = 0$, $\omega_z = \frac{v}{r}$

$$\bar{H}_x = \frac{1}{4} m r^2 (-\frac{v}{b} \cos \theta), \bar{H}_y = 0, \bar{H}_z = \frac{1}{2} m r^2 \frac{v}{r}$$

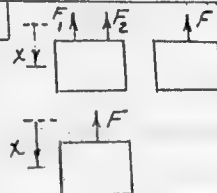
Substitute in Eq. 156 & get

$$\begin{cases} 0 = 0 - 0 + 0 \\ N r \sin \theta - F r \cos \theta = 0 - \frac{1}{2} m r v (-\frac{v}{b} \cos \theta) \\ \quad + \frac{1}{4} m r^2 \frac{v}{b} \cos \theta \frac{v}{b} \sin \theta \\ 0 = 0 - 0 + 0 \end{cases}$$

CHAPTER NINE

VIBRATION AND TIME RESPONSE

9/2



$$(a) F = F_1 + F_2$$

$$kx = k_1 x + k_2 x, \quad k = k_1 + k_2$$

$$(b) F = F_1 = F_2$$

$$x_1 = \frac{F}{k_1}, x_2 = \frac{F}{k_2}, \quad x = \frac{F}{k}$$

$$x = x_1 + x_2; \quad \frac{F}{k} = \frac{F}{k_1} + \frac{F}{k_2}; \quad \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

9/3

$$M = I \ddot{\theta}, \quad -\frac{JG}{L} \theta = I \ddot{\theta}, \quad \ddot{\theta} + \frac{JG}{IL} \theta = 0,$$

$$\tau = 2\pi/p = 2\pi \sqrt{\frac{IL}{JG}}; \quad \text{for } I = \frac{1}{12} m l^2, \quad \tau = \pi l \sqrt{\frac{mL}{3JG}}$$

8/77

Use the notation & equations (a) & (b) in Sample Prob. 8/71 with $\dot{\theta}$, $\ddot{\theta}$, θ small. Neglect products θ^2 , $\dot{\theta}^2$, $\dot{\theta}\ddot{\theta}$, $\ddot{\theta}\theta$. Eqs. (a) & (b) of Prob. 8/71 become for θ small

$$0 = I_o \ddot{\psi}, \quad \ddot{\psi} = n \text{ constant} \quad (c)$$

$$N r \theta - F_1 r = I_o \ddot{\theta} - I \omega_z n + I_o n^2 \theta \quad (d)$$

$$F_1 r = I \omega_z \quad (e)$$

$$N - mg + F_2 \theta = -mr \ddot{\theta} \quad (f)$$

$$-F_1 = mr (\dot{\omega}_z + n \dot{\theta}) \quad (g)$$

$$(N - mg) \theta - F_2 = mr (-\ddot{\theta} + n \omega_z) \quad (h)$$

(e) & (g); $(I + mr^2) \dot{\omega}_z + mr^2 n \dot{\theta} = 0$ which is integrated to give $(I + mr^2) \omega_z + mr^2 n \theta = 0$ since $\omega_z = 0$ when $\theta = 0$

Combine with combination of (d) & (h) to get

$$(I_o + mr^2) \ddot{\theta} + ([I_o + mr^2] n^2 - mgr) \theta = 0$$

Stable if coeff. of θ is positive. Thus $n > \sqrt{\frac{mgr}{I_o + mr^2}}$

$$\text{But for } I_o = \frac{1}{4} m r^2, \quad n > 2 \sqrt{\frac{g}{5r}}$$

9/4

$$\text{From Prob. 9/3 } \tau = 2\pi \sqrt{\frac{IL}{JG}}$$

$$\text{so } \frac{\tau_1^2}{\tau_2^2} = \frac{IL/JG}{(I + 2mr^2)L/JG} = \frac{I}{I + 2mr^2}$$

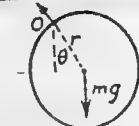
$$\text{Solve for } I \text{ & get } I = \frac{2mr^2}{(\tau_2/\tau_1)^2 - 1}$$

9/5

$$\Sigma M_o = I_o \ddot{\theta}; \quad -mgr \sin \theta = 2mr^2 \ddot{\theta}$$

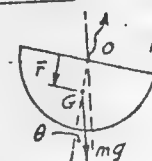
$$\text{For } \theta \text{ small, } \ddot{\theta} + \frac{g}{2r} \theta = 0$$

$$\tau = \frac{2\pi}{p} = 2\pi \sqrt{\frac{2r}{g}}$$



9/6

$$\Sigma M_o = I_o \ddot{\theta}; \quad -mg \frac{4r}{3\pi} \sin \theta = \frac{1}{2} m r^2 \ddot{\theta}$$



$$F = \frac{4r}{3\pi}$$

$$\text{For small } \theta, \sin \theta \approx \theta \text{ & } I_o = \frac{1}{2} m r^2$$

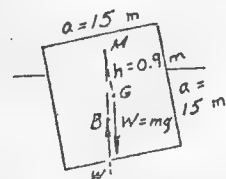
$$\ddot{\theta} + \frac{8g}{3\pi r} \theta = 0, \quad p^2 = \frac{8g}{3\pi r}$$

$$f = \frac{p}{2\pi} = \frac{1}{\pi} \sqrt{\frac{2g}{3\pi r}}$$

9/7

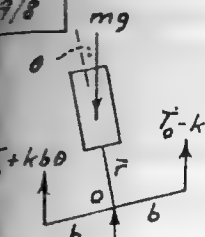
$$\bar{I} = \frac{1}{12} m (a^2 + a^2) = \frac{1}{6} m a^2$$

$$\Sigma \bar{M} = \bar{I} \ddot{\theta}; \quad -mgh \theta = \frac{1}{6} m a^2 \ddot{\theta} \quad \text{where } \theta =$$

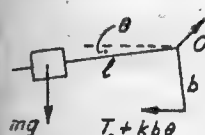


$$\text{so } \ddot{\theta} + \frac{6gh}{a^2} \theta = 0, \quad p = \frac{1}{a} \sqrt{6gh}$$

$$\tau = \frac{2\pi}{p} = \frac{2\pi a}{\sqrt{6gh}} = \frac{2\pi(15)}{\sqrt{6(4.21)(0.9)}} = 12$$

9/8  $\Sigma M_O = I_O \alpha$; for θ small
 $mg \bar{r} \theta + (T_0 - kb \theta)b - (T_0 + kb \theta)b = I_O \ddot{\theta}$
 $T_0 - kb \theta$ or $\ddot{\theta} + \frac{2kb^2 - mg\bar{r}}{I_O} \theta = 0$
 $T = \frac{2\pi}{p} = 2\pi \sqrt{\frac{I_O}{2kb^2 - mg\bar{r}}}$

Note: instability if $k < \frac{W\bar{r}}{2b^2}$

9/9 For equil. with $\theta = 0$, $T_0 b = mg\bar{l}$, $T_0 = \frac{mg\bar{l}}{b}$
 $\Sigma M_O = I_O \ddot{\theta}$
 $mg\bar{l} \cos \theta - (mg\frac{l}{2} + kb\theta)b \cos \theta = m\bar{l}^2 \ddot{\theta}$
 $\ddot{\theta} + \frac{kb^2}{m\bar{l}^2} \theta = 0$ for $\cos \theta = 1$
 $p^2 = \frac{kb^2}{m\bar{l}^2}$ so $f = \frac{p}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

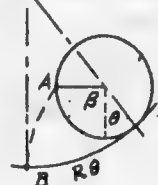
9/10 From Prob. 9/3, the period for each rotor is proportional to $\sqrt{I_L}$, so with $T_1 = T_2$, there results
 $I_1 l_1 = I_2 l_2$. Combination with $l = l_1 + l_2$ gives
 $l_1 = \frac{I_2}{I_1 + I_2} l$, $l_2 = \frac{I_1}{I_1 + I_2} l$. Thus for I_1 or I_2 the period from Prob. 9/3 is

$$T = 2\pi \sqrt{\frac{I_1 I_2}{I_1 + I_2} \frac{l}{g}}$$

9/11 $F_1 = 422/2 - 2k(0.2\theta) = 211 - 0.4k\theta$; $F_2 = 211 + 0.4k\theta$
 $\bar{k} = 100 \text{ mm}$, $W = mg = 43(9.81) = 422 \text{ N}$ $\Sigma M_O = I_O \ddot{\theta}$
 $(211 - 0.4k\theta)0.2 - (211 + 0.4k\theta)0.2 = 43(0.1)^2 \ddot{\theta}$
 $\ddot{\theta} + \frac{0.16}{0.43} k \theta = 0$
 $f = \frac{360}{60} = 6 \text{ Hz}$; $f = \frac{p}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{0.16}{0.43} k}$
 $k = \frac{0.43}{0.16} (12\pi)^2 = 3820 \text{ N/m}$

9/12 $V = 0$ for $\theta = 0$; $V_{\theta} = V_{\max} = mg\frac{l}{2}(1 - \cos \theta_0)$
 $= mg\frac{l}{2}(1 - [1 - \frac{\theta_0^2}{2} + \dots])$
 $T_{\max, \theta=0} = \frac{1}{2} I_C \omega^2 = \frac{1}{2} \frac{1}{3} m l^2 (\theta_0 p)^2$, $\omega = \theta_0 p$
 $V_{\max} = T_{\max}$; $\frac{mg l \theta_0^2}{4} = \frac{m l^2 \theta_0^2 p^2}{6}$
 $p^2 = \frac{3g}{2l}$, $T = \frac{2\pi}{p} = 2\pi \sqrt{\frac{2l}{3g}}$

9/13 $\Sigma M_C = I_C \ddot{\alpha}$; $-mgr \sin \theta = I_C \ddot{\beta}$
 But $R\theta = r(\beta + \theta)$ or $\beta = \frac{R-r}{r} \theta$
 Also $\sin \theta \approx \theta$, $I_C = \frac{3}{2} mr^2$
 so $-mgr \theta = \frac{3}{2} mr^2 \frac{R-r}{r} \ddot{\theta}$
 $\ddot{\theta} + \frac{2g}{3(R-r)} \theta = 0$



Thus $T = \frac{2\pi}{p} = 2\pi \sqrt{\frac{3(R-r)}{2g}}$

9/14 $T + V = E$, const. ; $\beta = \frac{R-r}{r} \theta$
 $T = \frac{1}{2} I_C \dot{\beta}^2 = \frac{1}{2} I_C \left(\frac{R-r}{r}\right)^2 \dot{\theta}^2 = \frac{3}{4} m (R-r)^2 \dot{\theta}^2$
 $V = mgr(R-r)(1 - \cos \theta)$; so $\frac{3}{4} (R-r)^2 \dot{\theta}^2 + g(1 - \cos \theta) = \frac{E}{m(R-r)}$
 Differentiate with time t get
 $\frac{3}{2} (R-r)^2 \dot{\theta} \ddot{\theta} + g \dot{\theta} \sin \theta = 0$; $\dot{\theta} \neq 0$ for θ small,
 $\ddot{\theta} + \frac{2g}{3(R-r)} \theta = 0$ $f = p/2\pi = \frac{1}{2\pi} \sqrt{\frac{2g}{3(R-r)}}$

9/15 With $m = 0$, Eq. 178 becomes $\dot{x} = -\frac{k}{c} x$
 or $\int_{x_0}^x \frac{dx}{x} = \int_0^t -\frac{k}{c} dt$; $\ln \frac{x}{x_0} = -\frac{k}{c} t$, $x = x_0 e^{-\frac{k}{c} t}$

9/16 $x_1 = 4.65 \text{ mm}$, $x_2 = 4.30 \text{ mm}$, $m = 1.10 \text{ kg}$
 $\ln \frac{x_1}{x_2} = b\tau = \frac{c}{2m} \frac{1}{f}$, $c = 2mf \ln \frac{x_1}{x_2}$
 Thus $c = 2(1.10)(10) \ln \frac{4.65}{4.30} = 1.722 \text{ N.s/m}$

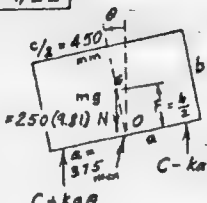
9/17 For critical damping $c = c_{cr} = 2\sqrt{km}$
 $\neq b = c/2m = \sqrt{k/m}$
 so $x = e^{-bt}(A_1 + A_2 t)$; $x = x_0$ & $\dot{x} = 0$ when $t = 0$
 which gives $A_1 = x_0$, $A_2 = b x_0 = \sqrt{k/m} x_0$
 Thus $x = x_0 e^{-\frac{\sqrt{k}}{m} t} (1 + \sqrt{\frac{k}{m}} t)$ for $x = x_1$, $t = t_1$

9/18 $E = (T+V)_1 - (T+V)_2$
 But at x_1 & x_2 , $T_1 = T_2 = 0$ since $\dot{x} = 0$
 Thus $E = V_1 - V_2 = \frac{1}{2} k (x_1^2 - x_2^2)$
 But $x_1/x_2 = e^{b\tau}$
 so $E = \frac{1}{2} k x_1^2 (1 - e^{-2b\tau}) = \frac{1}{2} k x_1^2 (1 - e^{-\frac{2\pi c}{mg}})$
 where $g = \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}$


9/19 $\frac{x_1}{x_2} = \frac{e^{-bt_1}}{e^{-b(t_1+12t)}}$ where $12 = 19-7$
 $\ln \frac{x_1}{x_2} = 12b\tau = 12 \frac{2\pi\eta}{\sqrt{1-\eta^2}}$ where $\eta = c/c_{cr}$
 Given $\frac{x_1}{x_2} = 30$, so $\ln 30 = 3.4012$
 $3.4012 = \frac{24\pi\eta}{\sqrt{1-\eta^2}}$, $\left(\frac{24\pi}{3.4012}\right)^2 = \frac{1}{1-\eta^2} - 1$
 $\eta = \frac{c}{c_{cr}} = 0.0451$

9/20 From Eq. 180 with $b = \frac{c}{2m}$, $c_{cr} = 2\sqrt{km}$
 $\frac{c}{2m} \tau = 2\pi \frac{c/c_{cr}}{\sqrt{1-(c/c_{cr})^2}}$, $\sqrt{1-(c/c_{cr})^2} = \frac{4\pi m}{c_{cr} \tau}$
 Solve for c & get $c = 2\sqrt{km} \sqrt{1 - \frac{4\pi^2 m}{k \tau^2}}$
 $c = 2\sqrt{850(1)} \sqrt{1 - \frac{4\pi^2(1)}{850(0.32)^2}} = 43.1 \text{ N}\cdot\text{s/m}$
 $c_{cr} = 2\sqrt{850(1)} = 58.3 \text{ N}\cdot\text{s/m}$

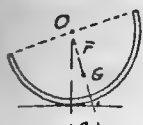
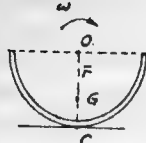
9/21 For the half cycle from $x = 0.30 \text{ mm}$ to $x = -0.15 \text{ mm}$,
 $\ln \frac{|x_1|}{|x_2|} = \frac{b\tau}{2} = \frac{\pi\delta}{\sqrt{1-\delta^2}}$ From Eq. 180; $b = \frac{c}{2m}$, $\eta = \frac{c}{2\sqrt{km}}$
 $\frac{0.300}{0.150} = \frac{0.150}{h}$, $h = 0.075 \text{ mm}$
 $\ln \frac{0.300}{0.150} = 0.693 = \frac{c}{4(4)} (3.8 - 3.2)2$; $c = 9.24 \text{ N}\cdot\text{s/m}$
 $0.693 = \frac{\pi\eta}{\sqrt{1-\eta^2}}$, $\eta = 0.215 = \frac{9.24}{2\sqrt{k(4)}}$
 Solve for k & get $k = 115.0 \text{ N/m}$

9/22 $\Sigma M_O = I_O \ddot{\theta}$; $mg \frac{b}{2} \sin \theta + (C - ka\theta)a - (C + ka\theta)a = I_O \ddot{\theta}$

 For small θ , $\sin \theta \approx \theta$ &
 $\ddot{\theta} + \frac{2ka^2 - mgb/2}{I_O} \theta = 0$
 Coeff. of θ must be (+) for harmonic oscillation.
 Thus $k_{min} = \frac{mgb}{4a^2} = \frac{250(9.81)(0.6)}{4(0.375)^2} = 2616 \text{ N/m}$

9/23 $L \sin \theta = 2r \sin \frac{\beta}{2}$ & for small β , $2\theta = r\beta$
 $h = L(1 - \cos \theta) \approx L\theta^2/2$; $T_{max} = \frac{1}{2} \left(\frac{1}{2} mr^2\right) \dot{\beta}_{max}^2$
 But $\dot{\beta}_{max} = p\dot{\beta}_0$, so $T_{max} = \frac{mr^2 p^2 \dot{\beta}_0^2}{4}$
 Also $V_{max} = mgh = \frac{mgL\theta_0^2}{2} = \frac{mgr^2 \dot{\beta}_0^2}{2L}$
 $V_{max} = T_{max}$, $\frac{mgr^2 \dot{\beta}_0^2}{2L} = \frac{mr^2 p^2 \dot{\beta}_0^2}{4}$
 $p^2 = \frac{2g}{L}$, $T = \frac{2\pi}{p} = 2\pi \sqrt{\frac{L}{2g}} = \pi \sqrt{2L/g}$
 independent of r

9/24 Let $V = 0$ when $x = 0$; $x_0 = \text{max displacement}$

 $V_{max} = V_g + V_e = -40(9.81)x_0 + 20(9.81)(2x_0) + \frac{1}{2}(450)(2x_0)^2 = 1080 x_0^2$
 $T_{max} = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} I \dot{\omega}^2$, $\dot{\omega} = \frac{\dot{x}}{0.6} = \frac{p x_0}{0.6}$
 so $T_{max} = \frac{1}{2} 40 p^2 x_0^2 + \frac{1}{2} 15(0.45)^2 \left(\frac{p x_0}{0.6}\right)^2 = 24.2 p^2 x_0^2$
 $T_{max} = V_{max}$; $24.2 p^2 x_0^2 = 1080 x_0^2$, $p^2 = \frac{1080}{24.2} = 44.6 \left(\frac{\text{rad}}{\text{s}}\right)^2$
 $f = \frac{p}{2\pi} = \frac{6.68}{2\pi} = 1.063 \text{ Hz}$

9/25

 $T=0$ $V=V_{max}$  $V=0$ $T=T_{max}$

$$V_{max} = mgr(1 - \cos \theta_0)$$

$$= mgr \frac{2r}{\pi} (1 - \cos \theta_0)$$

For small angles,

$$\cos \theta_0 \approx 1 - \frac{\theta_0^2}{2}$$

$$\therefore V_{max} = mgr \frac{\theta_0^2}{\pi}$$

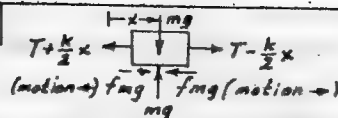
$$T_{max} = \frac{1}{2} I_c \omega^2 = \frac{1}{2} (I_G + m[r - \bar{r}]^2) \omega^2 = \frac{1}{2} (I_G - m\bar{r}^2 + m(r - \bar{r})^2) \omega^2$$

$$= mr^2 (1 - \frac{2}{\pi}) \theta_0^2 p^2 \text{ where } \omega = \theta_0 p$$

$$T_{max} = V_{max}; mr^2 (1 - \frac{2}{\pi}) \theta_0^2 p^2 = mgr \frac{\theta_0^2}{\pi}$$

$$p^2 = \frac{g}{\pi r (1 - \frac{2}{\pi})} \quad \& \quad T = \frac{2\pi}{p} = 2\pi \sqrt{\frac{\pi - 2}{g} r}$$

9/28



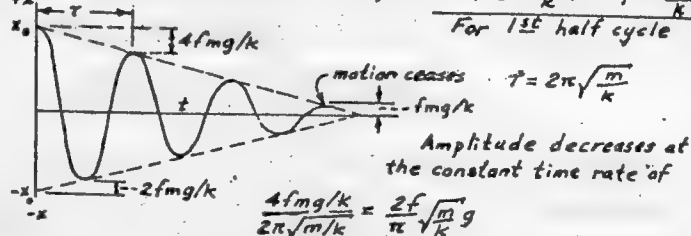
$$\Sigma F_x = m\ddot{x}$$

$$\ddot{x}(t); (T - \frac{k}{2}x) - (T + \frac{k}{2}x) + fmg = m\ddot{x}; \ddot{x} + \frac{k}{m}x = \frac{fmg}{m}$$

$$\text{Sol. is } x = A \cos pt + B \sin pt + \frac{fmg}{k}; p = \sqrt{\frac{k}{m}}$$

$$\text{Boundary conditions } \left. \begin{array}{l} \dot{x} = 0 \text{ when } t = 0 \\ x = x_0 \end{array} \right\} \text{ (+) sign for } fmg/k$$

$$\text{Gives } B = 0, A = x_0 - \frac{fmg}{k}; \text{ so } x = (x_0 - \frac{fmg}{k}) \cos pt + \frac{fmg}{k}$$



9/26

Let y_0 = max. vertical deflection (up) above neutral position, m

δ = spring deflection in neutral position, m

$$(T_{max})_{y=0} = \Delta V; \Delta V = \Delta V_1 + \Delta V_2$$

$$= 24(9.81)y_0 + 2 \frac{9000}{2} (\delta - \frac{150}{250}y_0)^2 - 2 \frac{1}{2} 9000 \delta^2$$

$$= 235y_0 + 3240y_0^2 - 10800y_0\delta$$

But from equil. of a lower member,

$$\Sigma M_0 = 0; 9000\delta(150) - 12(9.81)(250) = 0; \delta = 21.8$$

$$\text{So } \Delta V = 235y_0 + 3240y_0^2 - 235y_0 = 3240y_0^2$$

$$\& T_{max} = \frac{1}{2} 24(y_{max})^2 = 12y_{max}^2$$

But $y_{max} = y_0 p$ (p = circular frequency)

$$\text{so } 12y_0^2 p^2 = 3240y_0^2; p = \sqrt{270} = 16.43 \text{ rad/s}$$

$$f = \frac{p}{2\pi} = 2.62 \text{ Hz}$$

9/27

Equil. $0.8(9.81) = 1.03(9.81) \frac{\pi(0.6)^2}{4} h$, h = equil. submerged depth = 2.75 m

Mean submerged area of vertical surface is

$$A = \pi(0.6)(2.75) = 5.18 \text{ m}^2$$

$$\Sigma F_x = m\ddot{x}; -1.03(9.81) \frac{\pi(0.6)^2}{4} x - 5.18 f \ddot{x} (10^{-3})$$

$$= 0.8 \ddot{x}$$

Damping coefficient is $c = 5.18(10^{-3}) f$

Critical damping coefficient is

$$c_{cr} = 2\sqrt{km} = 2\sqrt{1.03(9.81)\pi(0.6)^2 \cdot 0.8}$$

$$= 3.02 \text{ kN}\cdot\text{s/m}$$

$$\ln \frac{x_1}{x_2} = 2\pi \frac{c/c_{cr}}{\sqrt{1 - (c/c_{cr})^2}}; \ln \frac{0.3}{0.3 - 0.1} = 0.4055$$

$$\text{So } (c/c_{cr})^2 \left[\left(\frac{2\pi}{0.4055} \right)^2 + 1 \right] = 1; (c/c_{cr})^2 = \frac{1}{241}; c = \frac{3.02}{\sqrt{241}} = 0.1947 \text{ kN}\cdot\text{s/m}$$

$$\text{So } f = \frac{c}{5.18(10^{-3})} = \frac{0.1947}{0.00518} = 37.6 \text{ N}\cdot\text{s/m}^3$$

9/29

r_0 = radial distance at rest; δ = steady-state deflection at speed ω ; x = added displacement due to vibration.

$$\text{For steady state, } \Sigma F_r = m a_r$$

$$C_1 - C_2 = m a_r;$$

$$C = \text{Compression for rest condition } (C - \frac{k}{2}\delta) - (C + \frac{k}{2}\delta) = -m(r_0 + \delta)\omega^2$$

$$k\delta = m(r_0 + \delta)\omega^2 \quad \text{--- (a)}$$

During vibration $\Sigma F_r = m a_r; C_1 - C_2 = m(\ddot{r} - r\omega^2)$

$$[C - \frac{k}{2}(\delta + x)] - [C + \frac{k}{2}(\delta + x)] = m \left[\frac{d^2}{dt^2} (r_0 + \delta + x) - (r_0 + \delta + x)\omega^2 \right]$$

$$-k\delta - kx = m\ddot{x} - m(r_0 + \delta)\omega^2 - m\omega^2 \text{ since } r_0 \& \delta \text{ are constant}$$

Substitute (a) & get $\ddot{x} + (\frac{k}{m} - \omega^2)x = 0$

$$\text{So } f = \frac{p}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m} - \omega^2} \quad \text{For } \omega^2 = \frac{k}{m}, \ddot{x} = 0 \& \dot{x} = \text{const}$$

For $\omega^2 > \frac{k}{m}, \ddot{x} - [\]x = 0$ & solution is hyperbolic function

9/31

From Fig. 90 for $\eta = 0.2$ & $x_0/\delta_0 = 2$,

$\mu < 0.78$ or $\mu > 1.11$ where $\mu = f/f_0$, $f_0 = 4 \text{ Hz}$

Thus $f < [0.78(4) = 3.12] \text{ Hz}$

$f > [1.11(4) = 4.44] \text{ Hz}$

9/32

$$\omega = 2\pi f = 2\pi(5) = 31.42 \text{ rad/s}$$

$$p^2 = \frac{k/m}{0.15} = 14400 \text{ (rad/s)}^2$$

$p = 120.0 \text{ rad/s}$, $\omega/p = 0.262$; hence it is small & instrument acts like an accelerometer

$$\& a_0 \approx x_0 p^2 = 0.0052(14400) = 74.9 \text{ m/s}^2 = 7.6 g$$

9/33 $\ddot{x} + \frac{k}{m}x = 0$, $\dot{x} = -\frac{k}{m}x$
 But $x = x_0 \sin pt$ where $p = \sqrt{k/m}$
 so $\dot{x} = -\frac{k}{m}x_0 \sqrt{\frac{k}{m}} \cos pt$, $\dot{x}_0 = -\left(\frac{k}{m}\right)^{3/2} x_0$
 $|x_0| = \left(\frac{m}{k}\right)^{3/2} \dot{x}_0$

9/34 Equiv. spring constant $= k = P/\delta = 10/12 = 0.833 \text{ kN/m}$
 $x_0 = \frac{\delta_0}{1 - (\omega/p)^2}$ where $p^2 = \frac{k}{m} = \frac{833}{1.5} = 556 \text{ (rad/s)}^2$
 $p = \sqrt{556} = 23.6 \text{ rad/s}$
 Also $\omega = 2(2\pi) = 12.57 \text{ rad/s}$
 $(\omega/p)^2 = \frac{16\pi^2}{556} = 0.284$
 Thus $x_0 = \frac{6}{1 - 0.284} = 8.38 \text{ mm}$

9/35 For seismic instrument, Sample Prob. 9/30
 gives $\frac{x_0}{\delta_0} = \frac{\mu^2}{\sqrt{(1-\mu^2)^2 + (2\eta\mu)^2}}$, $x_0 = 0.75 \text{ mm}$
 $\eta = 0.5$
 $\mu = \frac{\omega}{p} = \frac{180}{60} \cdot \frac{3}{3} = 3$
 $\delta_0 \cdot \frac{0.75}{\delta_0} = \frac{3^2}{\sqrt{(1-3^2)^2 + (2[0.5]3)^2}} = \frac{9}{\sqrt{73}}$, $\delta_0 = 0.712 \text{ mm}$

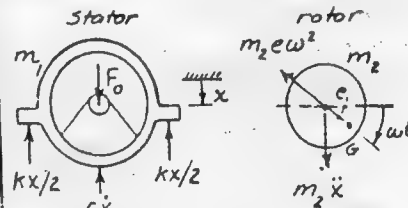
9/36 $\frac{x_0}{\delta_0} = \frac{1}{1 - (\omega/p)^2}$ where $\omega = 2\pi f$
 $p^2 = k/m = \frac{4(7.2 \cdot 10^3)}{43} = 670 \text{ (rad/s)}^2$
 $|x_0/\delta_0| = \frac{0.15}{0.10} = 1.5$
 $\left(\frac{\omega}{p}\right)^2 = \frac{4\pi^2 f^2}{670} = 0.0589 f^2$
 For $\omega < p$, $1.5 = \frac{1}{1 - 0.0589 f^2}$, $f = 2.38 \text{ Hz}$
 For $\omega > p$, $-1.5 = \frac{1}{1 - 0.0589 f^2}$, $f = 5.32 \text{ Hz}$
 Prohibited range $2.38 < f < 5.32 \text{ Hz}$

9/37 With negligible damping,
 $\frac{x_0}{\delta_0} = \frac{1}{1 - (\omega/p)^2}$ & $p^2 = k/m = \frac{k}{24}$, k in N/m
 where $x_0 = 0.30 \text{ mm}$, $\delta_0 = 0.60 \text{ mm}$
 $\omega = 2\pi f = 2\pi(4) = 8\pi \text{ rad/s}$
 For $x_0/\delta_0 = 1/2 < 1$, $1 - (\omega/p)^2$ is negative
 Thus $1 - \frac{(8\pi)^2}{k/24} = -2$; $k = 5050 \text{ N/m}$

9/38 $F_n = m\ddot{a}_n$; $k\delta = m(\delta + e)\omega^2$; $\delta(k - m\omega^2) = m e \omega^2$
 $\delta = \frac{m e \omega^2}{k - m\omega^2} = e \frac{\mu^2}{1 - \mu^2}$ where $\mu = \omega/p$
 $\omega_c = p = \sqrt{k/m}$



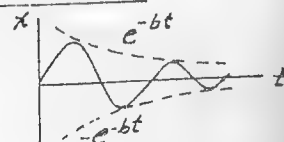
9/39 For simplicity assume rotation about vertical axis
 Stator m_1 rotor m_2
 $F_0 = \text{reaction to vertical component of forces on rotor} = m_2(e\omega^2 \sin \omega t - \ddot{x})$



For stator $\Sigma F_x = m\ddot{x}$; $-c\dot{x} - kx + m_2(e\omega^2 \sin \omega t - \ddot{x}) = m_1\ddot{x}$
 so $\ddot{x} + \frac{c}{m_1+m_2}\dot{x} + \frac{k}{m_1+m_2}x = \frac{m_2 e \omega^2}{m_1+m_2} \sin \omega t$
 which is identical with Eq. 181a when m_1+m_2 is used for m .

9/40 From the steady-state term of Eq. 182:
 $\delta_0 = F_0/k = m'e\omega^2/k$ & $p^2 = k/m$, $\mu = \omega/p$
 $\frac{x_0}{\delta_0} = \frac{x_0 m p^2}{m' e \omega^2} = \frac{m}{m'} \frac{x_0}{e} \frac{1}{\mu^2} = \frac{1}{\sqrt{(1-\mu^2)^2 + (2\eta\mu)^2}}$
 Thus $\frac{mx_0}{m'e} = \frac{\mu^2}{\sqrt{(1-\mu^2)^2 + (2\eta\mu)^2}}$ same as Sample Prob. 9/30

9/41 $\int F dt = I = m \Delta v$; $I = m\dot{x}$ at $t=0$
 After impulse Eq. 179 holds &
 $x = x_0 e^{-bt} \sin(qt + \phi)$, $\dot{x} = x_0 e^{-bt} (-b \sin[qt + \phi] + q \cos[qt + \phi])$
 $t=0$, $x=0$; $0 = x_0 \sin \phi$, $\phi=0$
 " $\dot{x} = I/m$, $I/m = x_0(-b \sin(0) + q \cos(0)) = qx_0$
 so $x_0 = \frac{I}{mq}$ & $x = \frac{I}{mq} e^{-bt} \sin qt$
 where $b = c/2m$
 $q = \sqrt{k/m - (c/2m)^2}$



9/42 $F_0 = 2 m \omega^2 = 2(1)(0.012)(1800 \frac{2\pi}{60})^2 = 853 \text{ N}$

Force transmitted is $kx = 1500 \text{ N}$

3-1 $kx = \frac{F_0 k/m}{p^2 - \omega^2}$, $p^2 = \frac{k}{m}$ so $\frac{kx}{F_0} = \frac{1}{1 - (\frac{\omega}{p})^2}$

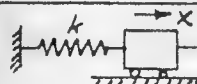
Thus $\frac{1500}{853} = \left| \frac{1}{1 - (\frac{\omega}{p})^2} \right|$ or $\left| 1 - (\frac{\omega}{p})^2 \right| = 0.568$

$(\frac{\omega}{p})^2 = 1.568$ or 0.432 , $p^2 = \frac{\omega^2}{1.568}$ or $p^2 = \frac{\omega^2}{0.432}$

$\omega^2 = [1800 \frac{2\pi}{60}]^2 = 35500 \text{ (rad/s)}^2$

$p^2 = \frac{35500}{1.568} = 22650$, $p^2 = \frac{35500}{0.432} = 82340$

$k = p^2 m = p^2(1) = 226.5 \text{ kN/m}$ or 823.4 kN/m

9/43  $F = F_0 e^{-bt}$

Diff. eq. is $\ddot{x} + \frac{k}{m}x = \frac{F_0}{m}e^{-bt}$; $x = x_c + x_p$

$x_c = C_1 \sin pt + C_2 \cos pt$ where $p^2 = k/m$

Try $x_p = Be^{-bt}$; substitution gives

$Bb^2 e^{-bt} + \frac{k}{m}Be^{-bt} = \frac{F_0}{m}e^{-bt}$, so $B = \frac{F_0}{mb^2 + k}$

Hence $x = C_1 \sin pt + C_2 \cos pt + \frac{F_0}{mb^2 + k}e^{-bt}$

$\dot{x} = C_1 p \cos pt - C_2 p \sin pt - \frac{F_0 b}{mb^2 + k}e^{-bt}$

$x = 0$ & $\dot{x} = 0$ when $t = 0$; give $C_2 = -\frac{F_0}{mb^2 + k}$

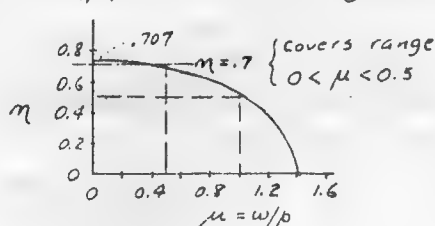
$\dot{x} = \frac{F_0}{mb^2 + k} \left[\frac{b}{p} \sin pt - \cos pt + e^{-bt} \right]$ $C_1 = \frac{F_0 b/p}{mb^2 + k}$

9/44 In Sample Prob. 9/30 $x_0 = \frac{1}{p^2} |\ddot{s}|_{\max}$

when denominator in expression for x_0/δ_0 is unity

Thus $\sqrt{(1 - \mu^2)^2 + (2\eta\mu)^2} = 1$ which gives

$\eta = \frac{1}{2} \sqrt{2 - \mu^2}$



9/45 Take road contour to be $x = x_0 \sin \omega t$

Wavelength $\lambda = vT = v \frac{2\pi}{\omega}$ so $\omega = 2\pi v/\lambda$

& $x = x_0 \sin \frac{2\pi vt}{\lambda}$. Equivalent system is that of

Fig. 84 b & Eq. 175 b with $c = 0$.

From Eq. 182 with $p = \sqrt{k/m}$, $\mu = \omega/p$, $F_0 = k\delta_0$, steady-state portion yields $x_0 = \frac{\delta_0}{1 - \mu^2}$

$\omega = 2\pi \frac{25}{3.6}/1.2 = 36.4 \text{ rad/s}$

$p^2 = k/m = \frac{75(9.81)}{0.003}/500 = 490 \text{ (rad/s)}^2$

$\mu^2 = (\omega/p)^2 = (36.4)^2/490 = 2.70$; $\delta_0 = \frac{0.050}{2} = 0.025 \text{ m}$

$x_0 = \left| \frac{0.025}{1 - 2.70} \right| = 0.0147 \text{ m} = 14.75 \text{ mm}$

$v_c = \frac{\lambda \omega_c}{2\pi} = \frac{\lambda p}{2\pi} = \frac{1.2 \sqrt{490}}{2\pi} = 4.23 \text{ m/s} = 15.23 \text{ km/h}$

9/46 $\ddot{x} + \frac{k}{m}x = \frac{F}{m} = \frac{bt}{m}$, $F = bt$, $x = x_c + x_p$

$x_c = C_1 \sin pt + C_2 \cos pt$; try $x_p = At$ & get $A = b/k$

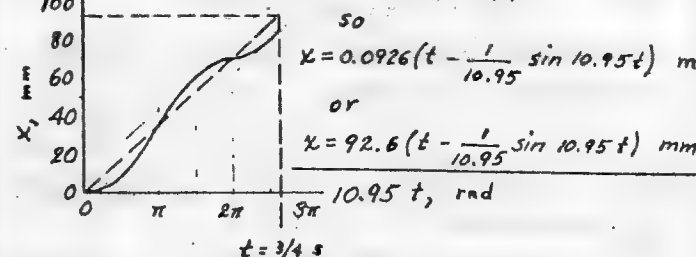
so $x = C_1 \sin pt + C_2 \cos pt + \frac{bt}{k}$

$\dot{x} = 0$, $x = 0$ when $t = 0$ gives $C_1 = -\frac{b}{kp}$, $C_2 = 0$

$x = \frac{b}{k} (t - \frac{1}{p} \sin pt)$ for $0 < t < \frac{3}{4} \text{ s}$; $p = \sqrt{k/m} = \sqrt{90/0.75}$

$x = \frac{6.25}{3/4} \frac{1}{90} (t - \frac{1}{10.95} \sin 10.95 t) \text{ m} = 10.95 \text{ rad/s}$

where $b = \frac{6.25}{3/4} = 8.33 \text{ N/s}$



9/47 Energy loss during motion dx is

$dE = (c\dot{x}) dx = c\dot{x}^2 dt$

& over one complete cycle is $E = c \int_0^{2\pi/\omega} \dot{x}^2 dt$

where $x = \frac{F_0/k}{\sqrt{(1 - \mu^2)^2 + (2\eta\mu)^2}} \sin(\omega t - \phi)$

Let $x = A \sin(\omega t - \phi)$ so $\dot{x} = A\omega \cos(\omega t - \phi)$

$E = CA^2 \omega^2 \int_0^{2\pi/\omega} \cos^2(\omega t - \phi) dt = CA^2 \omega^2 \left[\frac{\omega t - \phi}{2} - \frac{1}{4} \sin 2(\omega t - \phi) \right]_0^{2\pi/\omega}$

$= CA^2 \omega^2 [\pi]$

Power $P = (\text{Energy loss per cycle})(\text{cycles per sec.})$

$= E \frac{\omega}{2\pi} = CA^2 \omega^2/2$

or $P = \frac{F_0^2 c \omega^2 / (2k^2)}{(1 - \mu^2)^2 + (2\eta\mu)^2}$

CHAPTER TEN
DYNAMICS OF NONRIGID SYSTEMS

10/4 Thrust $T = m'(u-v)$
 $= 30(40 - 10(1.852)/3.6) = 1046 \text{ N}$
 For constant hull speed $R = T = 1046 \text{ N}$

10/5 Vertical thrust $= m' \Delta v_y = 2(60+1)(600)$
 $= 73\,200 \text{ N}$
 $\Sigma F_y = ma_y; 73.2 - 7(9.81) = 7a, a = 0.647 \text{ m/s}^2$

10/6 $\Sigma M = m'(v_2 d_2 - v_1 d_1), M = mrv(r) = m\dot{r}\omega$

10/7 $\Sigma F_x = m' \Delta v_x$
 $-T \cos 15^\circ = (43 + 0.8)(0 - 720)$
 $T = 32\,600 \text{ N} = 32.6 \text{ kN}$

10/8 Thrust $T = m'_x u - m'_y v = D$, drag
 $T = D = (110 + 0.97) 780 - 110 \frac{1500}{3.6}$
 $= 86\,600 - 45\,800 = 40\,700 \text{ N} = 40.7 \text{ kN}$
 $P = TV, P = 40.7 \left(\frac{1500}{3.6} \right) = 16\,970 \text{ kW} = 16.97 \text{ MW}$

10/9 $\Sigma F = m' \Delta v; 2T = (0.025)(1.2)(0.4)(7.83) \left(\frac{25}{19} a - 0 \right)$
 $\rho_{\text{steel}} = 7.83 \text{ t/m}^3$
 $T = 0.00593 \text{ kN} = 5.93 \text{ N}$

10/10 $mg = 4(9.81) \text{ kN}$
 $T = m'_x(u-v) + m'_y u$
 $= 90(600 - 800/3.6) + 0.9(600)$
 $= 34\,500 \text{ N}$
 $\Sigma F_x = 0; 34.5 - 24 - 4(9.81) \sin \alpha = 0$
 $\sin \alpha = 0.2686, \alpha = 15.58^\circ$

10/11 $kx = m' \Delta v, 15(150) = 1000 \frac{\pi(0.030)^2}{4} v$
 $v = 56.4 \text{ m/s}$

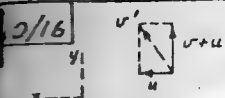
$\Sigma M_A = m' v d; M = 15(150)(15 \sin 75^\circ - 4.8 \cos 75^\circ)$
 $= 2250(13.25) = 29\,800 \text{ N}\cdot\text{m}$

10/12 $m' = 30/60 = 0.5 \text{ t/s}$
 $\Sigma F_x = m' \Delta v_x$
 $F = 0.5(15/3.6 - 0) = 2.08 \text{ kN}$
 $\Sigma F_y = m' \Delta v_y; R = 0.5(12/\sqrt{2} - 0) = 4.24 \text{ kN}$

10/13 $\Sigma F_y = m' \Delta v_y; m' = \rho A_1 v$
 $\Delta v_y = 0 - (-v) = v$
 $\text{so } \rho A_2 - mg = \rho A_1 v^2$
 $\rho(\pi 3^2) - 2.1(9.81)$
 $= 1.217(10^{-3})(\pi 1^2)(45^2)$
 $\rho = 1.002 \text{ kPa}$

10/14 $\Delta v_x = 2v \sin \frac{\theta}{2}; m' = \rho A v$
 $\Sigma F_x = m' \Delta v_x$ (entire system)
 $2T \sin \frac{\theta}{2} - 2\rho A \sin \frac{\theta}{2} = \rho A v(2v \sin \frac{\theta}{2})$
 $\text{so } T - \rho A = \rho A v^2, T = A(\rho + \rho v^2)$

10/15 $\Delta v_x = -(v-u)(1 - \cos \theta)$ See Sample Prob. 10/1
 $m' = \rho A v$
 $\Sigma F_x = m' \Delta v_x; -F_{\text{on}} = \rho A v [-(v-u)(1 - \cos \theta)]$
 Power $P = F_{\text{on}} u = \rho A v u (v-u)(1 - \cos \theta)$
 $dP/du = \rho A(1 - \cos \theta)(v^2 - 2uv) = 0$ for max. P
 Thus $u = v/2$ & $P = \frac{1}{4} \rho A v^3(1 - \cos \theta)$



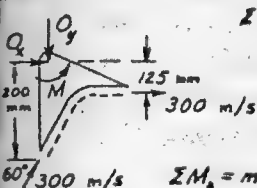
Final velocity of diverted water is $\underline{u'} = \underline{u} + \underline{u_{rel}}$
where $\underline{u_{rel}} = \underline{u} + \underline{u}$

$$m' = \rho A u_{rel} = 1000 \frac{\pi}{4} (0.025)^2 (20 + 10) = 14.73 \text{ kg/s}$$

$$\Sigma F_x = m' \Delta u_x; F_x = 14.73 (10 - [-20]) = 442 \text{ N}$$

$$\Sigma F_y = m' \Delta u_y; F_y = 14.73 (30 - 0) = 442 \text{ N}$$

10/17 $m' = 3 \text{ kg/s}$

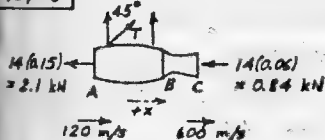


$$\Sigma F_x = m' \Delta u_x; Q_x = 3(300 - 300 \cos 60^\circ) = 450 \text{ N}$$

$$\Sigma F_y = m' \Delta u_y; Q_y = 3(0 - [300 \sin 60^\circ]) = 779 \text{ N}$$

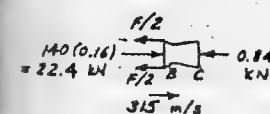
$$\Sigma M_o = m' (u_2 d_2 - u_1 d_1); M = 3(300[0.125] - 300 \frac{0.200}{2}) = 22.5 \text{ N}\cdot\text{m}$$

10/18



$$\Sigma F_x = m' \Delta u_x = 2.1 - 0.84 + T \cos 45^\circ = [30.6(600) - 30(120)](10^{-3})$$

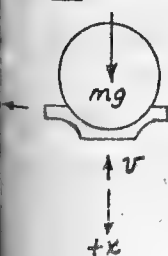
$$T = 25.0 \text{ kN}$$



$$\Sigma F_x = m' \Delta u_x = 22.4 - 0.84 - F = 30.6(600 - 315)(10^{-3})$$

$$F = 12.84 \text{ kN}$$

10/19



$$\Sigma F_x = m' \Delta u_x$$

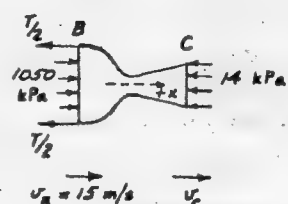
$mg = \rho \frac{\pi d^2}{4} u (0 - [-v]);$ But for water with energy conserved, $u^2 - v^2 = 2gh$

Solve for h & get

$$h = \frac{1}{2g} (u^2 - v^2) = \frac{u^2}{2g} - \frac{2m}{\pi \rho u d^2}$$

10/20

$$A_c = \frac{\pi}{4} (0.1)^2 = 0.00785 \text{ m}^2; A_b = 4A_c = 0.0314 \text{ m}^2$$



$$m' = \rho A u$$

$$m'_b = 13.5 (0.0314) (15) = 6.36 \text{ kg/s}$$

$$\phi \text{ at } C, 6.36 = 1.217 (0.00785) u_c$$

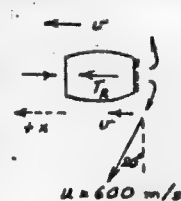
$$u_c = 666 \text{ m/s}$$

$$\Sigma F_x = m' \Delta u_x; -T + 1050 (0.0314) - 14 (0.00785) = 6.36 (666 - 15)(10^{-3})$$

$$T = 28.7 \text{ kN}$$

10/21

$$u = 250/3.6 = 69.4 \text{ m/s}$$

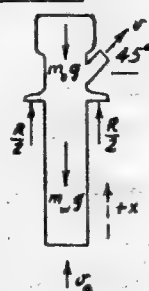


With reversers, $T_R = m'_g u \sin 20^\circ + m'_a v$
so $T_R = (40 + 0.6)(600)(0.3420) + 40(69.4) = 8330 + 2780 = 11100 \text{ N}$

Without reversers, $T = m'_g u - m'_a v$
 $T = (40 + 0.6)(600) - 40(69.4) = 24400 - 2780 = 21600 \text{ N}$

$$n = \frac{T_R}{T} = \frac{11100}{21600} = 0.515$$

10/22



$$\Sigma F_x = m' \Delta u_x; R - m_g g - m_w g = \rho Q (u \cos 45^\circ - u_c)$$

$$m_g = 310 \text{ kg}$$

$$\text{mass of water } m_w = \rho V = 1000 \frac{\pi}{4} (0.2)^2 (6) = 188.5 \text{ kg}$$

$$Q = 0.125 \text{ m}^3/\text{s}$$

$$A = \frac{\pi}{4} (0.1)^2 = 0.00785 \text{ m}^2; A_c = \frac{\pi}{4} (0.25)^2 = 0.0491 \text{ m}^2$$

$$u = Q/A = 0.125/0.00785 = 15.92 \text{ m/s}$$

$$u_c = Q/A_c = 0.125/0.0491 = 2.55 \text{ m/s}$$

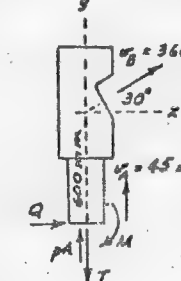
$$\text{Thus } R - (310 + 188.5)9.81 = 1000(0.125)(15.92 \cos 45^\circ - 2.55)$$

$$= 1088 \text{ N}$$

$$R = 5980 \text{ N}$$

10/23

$$m' = 6 \text{ kg/s}; pA = 1400(10^3)(7500)(10^{-4}) = 10500 \text{ N}$$



$$\Sigma F_x = m' \Delta u_x; Q = 6(360 \frac{\sqrt{3}}{2} - 0) = 1871 \text{ N}$$

$$\Sigma F_y = m' \Delta u_y; 10500 - T = 6(360 \frac{1}{2} - 45)$$

$$T = 9690 \text{ N}$$

$$\Sigma M_A = m' (u_2 d_2 - u_1 d_1); M = 6(360 \frac{\sqrt{3}}{2} 0.6 - 0)$$

$$M = 1122 \text{ N}\cdot\text{m}$$

10/24

$$Q = 0.05 \text{ m}^3/\text{s}; v_1 = \frac{Q}{A_1} = \frac{0.05}{0.0125} = 4 \text{ m/s}$$

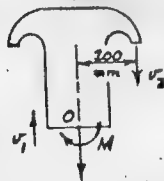
$$v_2 = \frac{Q}{A_2} = \frac{0.05}{2(0.002)} = 12.5 \text{ m/s}$$

$$m' = 0.05(1030) = 51.5 \text{ kg/s}$$



$$\Sigma F_x = m' \Delta v_x; 4500 - 875 - 10p = 1030(0.05)(12.5 - [-4])$$

$$p = 278 \text{ kPa}$$

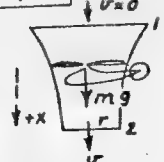


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 & with $Q = 0.05/2 = 0.025 \text{ m}^3/\text{s}$,
 $v_2 = Q/A_2 = 0.025/0.002 = 12.5 \text{ m/s}$,
 $m' = 51.5/2 = 25.8 \text{ kg/s}$

$$\Sigma M_o = m'(v_2 d_2 - v_1 d_1)$$

$$M = 25.8(12.5)(0.2) = 64.4 \text{ N}\cdot\text{m}$$

10/25



mg = weight of helicopter
 = force on air stream

For system between sections 1 & 2,

$$\Sigma F_x = m' \Delta v_x; mg = \rho \pi r^2 v (v - 0)$$

$$v = \frac{1}{r} \sqrt{\frac{mg}{\pi \rho}}$$

Power = rate of increase of kinetic energy

$$P = \frac{1}{2} m' v^2 = m' v \frac{v}{2} = mg \frac{v}{2} = \frac{mg}{2r} \sqrt{\frac{mg}{\pi \rho}}$$

10/26

$$v = 18 \text{ m/s}$$

$$\text{Power } P = M\omega;$$

$$M = \frac{40000}{900(2\pi/60)} = 424 \text{ N}\cdot\text{m}$$

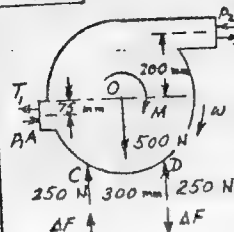
$$\Sigma M_o = m'(v_2 d_2 - v_1 d_1)$$

$$424 + 0.3\Delta F = (20/60)(1000)[18(0.2) - (-18)(0.075)]$$

$$\Delta F = \frac{1650 - 424}{0.3} = 4090 \text{ N}$$

$$\text{Thus } C = 250 + 4090 = 4340 \text{ N up}$$

$$D = 4090 - 250 = 3840 \text{ N down}$$



10/27

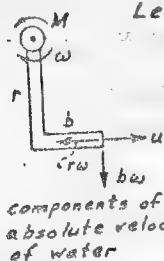
For entire system $\Sigma M = m'(v_2 d_2 - v_1 d_1)$
 Let u = velocity of water relative to nozzle
 to nozzle $= \frac{Q}{4A}$

$$m' = \rho Q$$

$$-M = \rho Q(r^2 \omega + b^2 \omega - \frac{Q}{4A} r - 0)$$

$$M = \rho Q(\frac{Qr}{4A} - (r^2 + b^2)\omega)$$

$$\text{For } M = 0, \omega = \frac{Qr}{4A(r^2 + b^2)}$$



components of
 absolute velocity
 of water

10/28

$$\Sigma F_x = m' \Delta v_x; Q = A_o v_o; v_o = \frac{1.2/60}{\pi/4(0.1)^2} = 2.55 \text{ m/s}$$

$$\text{Area of discharge } 0.006(0.5)\pi(0.1) = 942(10^{-6}) \text{ m}^2$$

$$v = \frac{1.2/60}{942(10^{-6})} = 21.2 \text{ m/s}$$

$$pA = 280(10^3) \frac{\pi}{4}(0.1)^2 = 2200 \text{ N}$$

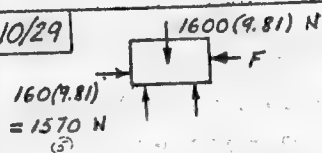
$$m' = 1000(1.2/60) = 20 \text{ kg/s}$$

$$\text{Thus } 2200 - C = 20(21.2 \sin 60^\circ - 2.55)$$

$$C = 2200 - 317 = 1882 \text{ N}$$

$$\& \text{ from levers, } \frac{25(9.81)(75+b)}{75} = \frac{1882}{3}, b = 116.9 \text{ mm}$$

4/10/29



$$160(9.81) = 1570 \text{ N}$$

From Prob. 10/1

$$F = pA(v-u)^2(1+\cos 30^\circ)$$

$$= 1000 \frac{\pi}{4}(0.15)^2(1.866)(180-u)$$

$$= 33.0(180-u)^2 \text{ N}$$

$$\Sigma F = m\ddot{u}; 33.0(180-u)^2 - 1570 = 1600\ddot{u}$$

$$\text{or } \int \frac{du}{0.0206(180-u)^2 - 0.981} = \int dt; \text{ integrate \& get}$$

$$\frac{1}{2\sqrt{0.0206(0.981)}} \ln \frac{0.1422(180-u)+0.981}{0.1422(180-u)-0.981} \Big|_0^u = \int_0^t dt = 4$$

$$3.52 \ln \frac{1-0.00535u}{1-0.00578u} = 4, \frac{1-0.00535u}{1-0.00578u} = 3.119$$

$$\text{Solve for } u \& \text{ get } u = 167.3 \text{ m/s}$$

4/10/30

$$\text{Entrance: } \frac{v_\theta}{315} = \frac{\sin 110^\circ}{\sin 43^\circ}, v_\theta = 434 \text{ m/s}$$

$$\frac{v_R}{315} = \frac{\sin 27^\circ}{\sin 43^\circ}, v_R = 210 \text{ m/s}$$

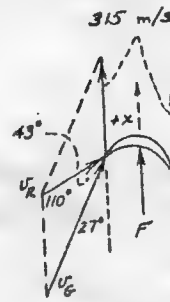
$$\text{Exit: } v_R' = v_R \text{ (negligible friction)}$$

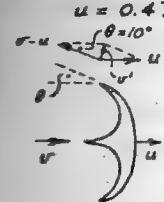
$$v_{\theta x}' = 315 - 210 \frac{\sqrt{3}}{2} = 133.4 \text{ m/s}$$


$$|\Delta v_x| = 434 \cos 27^\circ - 133.4 = 253 \text{ m/s}$$


$$\Sigma F_x = m' \Delta v_x; \Sigma F_{\text{all blades}} = 15(253) = 3800 \text{ N}$$

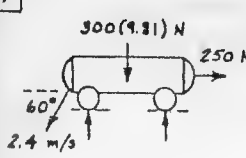
$$P = 2Fv; P = 3800(315) = 1.197(10^6) \text{ W} = 1.197 \text{ MW}$$




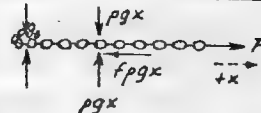
10/31 $u = \sqrt{2gh} = \sqrt{2(9.81)(300)} = 76.7 \text{ m/s}$
 $u = 0.47 \text{ } u = 36.1 \text{ m/s}$

 $\Delta u_x = [u - (u-u) \cos \theta] - u = -(u-u)(1 + \cos \theta)$
 $= -(76.7 - 36.1)(1 + 0.985) = -80.7 \text{ m/s}$
 Average tangential thrust per jet is
 $F_x = m' \Delta u_x = 1000(76.7A) - 80.7 = 6.19(10^6)A \text{ N}$
 where $A = \text{jet area, m}^2$
 Theor. power $P = 6F_x u = 6(6.19)(10^6)A(36.1) = 1.340A(10^9) \text{ W}$
 Actual power = $\frac{22(10^6)}{1.340A(10^9)} = 0.90(0.85)$, $A = 0.0215 \text{ m}^2$
 Theor. power
 Thus $\pi d^2/4 = 0.0215$, $d = 0.1653 \text{ m} = 165.3 \text{ mm}$

10/35 $m = 3160 \text{ t}$

 $T = m'u$; $5(7500) = \frac{2280}{160} u$
 $u = 2630 \text{ m/s}$
 $\Sigma F = ma$; $5(7500) - 3160(9.81) = 3160 a$
 $a = 2.06 \text{ m/s}^2$

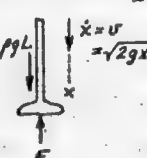
10/36 mg

 Relative velocity of attachment of mass is $u = v$. Thus with
 $\Sigma F = m\dot{v} + m\dot{u}$
 $mg - R = m\dot{v} + m\dot{u} = \frac{d}{dt}(mv)$
 so $\Sigma F = \frac{d}{dt}(mv)$


10/37 
 In x -dir, $\Sigma F_x = m\dot{v} + m\dot{u}$
 $250 = 300 a - 20(2.4 \cos 60^\circ)$
 $a = \frac{250 + 24}{300}$
 $a = 0.913 \text{ m/s}^2$

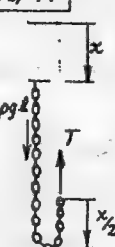
10/38 $\Sigma F = ma$; $-\dot{m}u - mg = ma$
 $-\frac{dm}{dt}u = m(a+g)$
 $\int_{m_0}^m -\frac{dm}{m} = \int_0^t \frac{a+g}{u} dt$
 $\ln \frac{m}{m_0} = -\frac{a+g}{u} t$, $m = m_0 e^{-\frac{a+g}{u} t}$



10/39 Sol. I: entire chain
 $\Sigma F_x = \dot{G}_x$; $P - fpgx = \frac{d}{dt}(0 + \rho x \dot{x})$
 $= \rho(\dot{x}^2 + x\ddot{x})$
 $a = \ddot{x} = \frac{P}{\rho x} - fg - \frac{\dot{x}^2}{x}$


Sol. II: Eq. 191 for moving portion; $\Sigma F = m\dot{v} + m\dot{u}$
 $P - fpgx = \rho x a + \rho x \dot{x}$ where $u = \dot{x}$
 $a = \frac{P}{\rho x} - fg - \frac{\dot{x}^2}{x}$

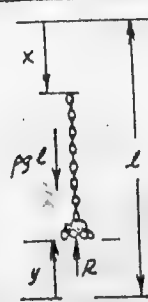
10/40 $G_x = \rho(L-x)\sqrt{2gx} = \rho\sqrt{2g}(Lx^{1/2} - x^{3/2})$

 $\Sigma F_x = \dot{G}_x$ for entire system
 $pgL - F = \rho\sqrt{2g}(\frac{1}{2}Lx^{-1/2} - \frac{3}{2}x^{1/2})\dot{x}$
 $= \rho\sqrt{2g}(\frac{1}{2}L\sqrt{2g} - \frac{3}{2}\sqrt{2g}x)$
 $= pgL - 3pgx$, $F = 3pgx$

Alternatively,

 $R = m'\dot{u} = \rho\dot{x}(\dot{x}) = \rho(2gx) = 2pgx$
 $\Sigma F_x = 0$; $F = pgx + 2pgx = 3pgx$

10/41 Take entire chain as system of constant mass
 $\Sigma F_x = \dot{G}_x$; $pgl - T = \frac{d}{dt}(\rho(l - \frac{x}{2})\dot{x} + 0)$
 $= \rho(l - \frac{x}{2})\ddot{x} - \rho\frac{\dot{x}^2}{2}$
 But $\ddot{x} = g$ & $\dot{x}^2 = 2gx$ so that
 $pgl - T = pgl - pg\frac{x}{2} - pgx$
 & $T = \frac{3pgx}{2}$

 $\Delta E = |\Delta V_g| = pgl(\frac{l}{2} + \frac{l}{2}) = pgl^2$ energy loss

10/42 $\Sigma F_x = \dot{G}_x$; $pA - pgV = \frac{d}{dt}(\rho V \frac{\dot{x}}{2}) = \frac{\rho V}{2} \ddot{x}$
 $V = \pi r^2 x$, $\dot{V} = \pi(2r\dot{r}x + r^2\dot{x}) = 0$
 so $\dot{r} = -\frac{r}{2}\frac{\dot{x}}{x}$ where $\dot{r} = -c$, constant
 $\& \ddot{r}r + \dot{r}^2 = 2c\dot{x}$ or $\ddot{x} = \frac{3c\dot{x}}{r} = \frac{6c^2x}{r^4} = \frac{6\pi c^2x^3}{V}$

 Thus $p\frac{V}{x} - pgV = \frac{\rho V}{2} \frac{6\pi c^2x^3}{V}$
 $\& p = pgx + \frac{3\pi\rho c^2x^4}{V} = pgx(1 + \frac{3\pi c^2x^4}{9V})$

10/43 Take entire chain as system (constant mass)



$$\ddot{x} = g, \ddot{y} = a$$

$$\dot{x} = gt, \dot{y} = at$$

$$x = \frac{1}{2}gt^2, y = \frac{1}{2}at^2$$

$$G_x = \rho(l-x-y)\ddot{x}$$

$$- \rho(x+y)\ddot{y}$$

$$\Sigma F_x = \dot{G}_x; pgl - R = \rho[(-\ddot{x}-\ddot{y})x + (l-x-y)\ddot{x}$$

$$-(\ddot{x}+\ddot{y})y + (x+y)\ddot{y}]$$

$$pgl - R = \rho[-(\ddot{x}+\ddot{y})^2 - (x+y)(\ddot{x}+\ddot{y}) + l\ddot{x}]$$

$$= \rho[-(a+g)^2 t^2 - (a+g)\frac{1}{2}t^2(a+g) + l(a+g)]$$

$$= -\rho(a+g)^2 t^2 \frac{1}{2} + pgl$$

S. $R = \frac{1}{2}\rho(a+g)^2 t^2$

10/44 For airplane plus moving portions of chains

$$\Sigma F = 0 = m\dot{v} + \dot{m}u = (m + 2\rho \frac{x}{2})\dot{v} + [2\frac{d}{dt}(\rho \frac{x}{2})]v$$

$$-(m + \rho x)dv = \rho v dx \text{ so } -\int \frac{dv}{v} = \int \frac{\rho dx}{m + \rho x}$$

$$\text{or } \ln \frac{v}{v_0} = \ln \frac{m}{m + \rho x}, v = v_0 \frac{m}{m + \rho x} = \frac{v_0}{1 + \rho x/m}$$

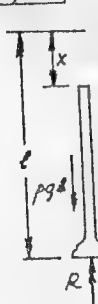
& for $x = 2L$, $v = \frac{v_0}{1 + \frac{2\rho L}{m}}$

Also, $v = \dot{x} = \frac{v_0}{1 + \frac{\rho x}{m}}$ so $\int_0^x (1 + \frac{\rho x}{m}) dx = \int_0^t v_0 dt$

$$v_0 t = x(1 + \frac{\rho x}{2m}) \text{ or}$$

$$x = \frac{m}{\rho} [\sqrt{1 + \frac{2v_0 t \rho}{m}} - 1] \text{ for } +x$$

10/45 For entire chain or rope of constant mass



$$\Sigma F_x = \dot{G}_x; pgl - R = \frac{d}{dt}(\rho[l-x]\dot{x} + 0)$$

$$\Delta T + \Delta V = 0; \frac{1}{2}\rho(l-x)\dot{x}^2 = \rho g x(l - \frac{x}{2})$$

$$\dot{x}^2 = \frac{x(2l-x)g}{l-x}$$

$$\text{or } \dot{x}(l-x) = \frac{x}{2}(2l-x)g$$

Substitute into ΣF equation & get.

$$pgl - R = \rho g \frac{d}{dt}(\frac{x[2l-x]}{\dot{x}}) = \rho g \frac{\dot{x}[2l-2x] - x[2l-x]\ddot{x}}{\dot{x}^2}$$

$$= 2\rho g(l-x) - \rho g x(2l-x) \frac{\ddot{x}}{\dot{x}^2}$$

Differentiate \dot{x}^2 & get $\ddot{x} = [1 + \frac{x(l-x/2)}{(l-x)^2}]g$. Substitute

& get $pgl - R = 2\rho g(l-x) - \rho g x(2l-x) \frac{[1 + \frac{x(l-x/2)}{(l-x)^2}]g}{\frac{x(2l-x)}{l-x}g}$

Simplify, solve for R, & get

$$R = \rho g x \frac{4l-3x}{2(l-x)}; (\text{Less than } R_{\text{aro}} \text{ Prob. 10/43 for } x < \frac{2l}{3})$$

10/46 For falling part $\Sigma F = m\dot{v} + \dot{m}u$

where $\Sigma F = p g x$, $m = \rho x$, $\dot{m} = \rho v$, $u = v = \dot{x}$

Thus $p g x = \rho x \dot{v} + \rho v \dot{x}$, $g x dt = x dv + v dx$

or $g x dt = d(xv)$; $g x^2 v dt = x v d(xv)$

or $g x^2 dx = \frac{1}{2} d((xv)^2)$, so $g \int_0^x x^2 dx = \frac{1}{2} \int_0^{(xv)^2} d((xv)^2)$

$$\frac{g x^3}{3} = \frac{1}{2} (xv)^2, v = \sqrt{\frac{2gx}{3}}$$

$$a = \dot{v} = \sqrt{\frac{2g}{3}} \frac{1}{2} x^{-1/2} \dot{x} = \sqrt{\frac{2g}{3}} \frac{1}{2\sqrt{x}} \sqrt{\frac{2gx}{3}} = \frac{g}{3} \text{ constant}$$

$$\Delta E = -\Delta V_g - \Delta T = + \frac{\rho g L^2}{2} - \frac{\rho L}{2} v_{x=L}^2 = \frac{\rho g L^2}{2} - \frac{\rho g L^2}{3} = \frac{\rho g L^2}{6}$$

10/47

Entire system is conservative so $\Delta V_g + \Delta T = 0$

$$-p g x \frac{x}{2} + \frac{1}{2} \rho x v^2 = 0, v = \sqrt{g x}, a = \dot{v} = g/2$$

For entire system

$$\Sigma F_x = \dot{G}_x; p g L - R = \frac{d}{dt}(\rho x v) = \frac{d}{dt}(\rho \sqrt{g} x^{3/2}) = \frac{3}{2} \rho \sqrt{g} x^{1/2} \dot{x}$$

Thus $R = p g L - \frac{3}{2} \rho g x$, $R = p g (L - \frac{3}{2} x)$

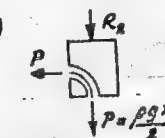
Explanation of $R=0$ for $x = 2L/3$:

For vertical section $\Sigma F_x = m a_x$; $p g x - P = \rho x \ddot{x}$

$P = p g x/2$. For idealized flow considered, a frictionless guide must be introduced

$$\Sigma F_x = m' \Delta v_x; p g x/2 + R_2 = \rho v^2$$

$$R_2 = \rho v^2 - p g x/2 = \rho(g x) - p g x/2 = p g x/2$$



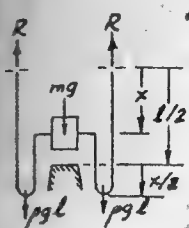
R = total force exerted on rope by platform & guide & is

$$R = R_1 - R_2 = p g (L-x) - p g x/2$$

so $R=0$ when $p g (L-x) = p g x/2$ or $x = 2L/3$

10/48 System is conservative during fall

$$\Delta V_g + \Delta T = 0; -mgx - 2pgx\left(\frac{l}{2} + \frac{x}{4}\right) + \frac{1}{2}m\dot{x}^2 + 2\frac{1}{2}\rho\left(\frac{l}{2} + \frac{x}{2}\right)\dot{x}^2 = 0$$



$$\text{or } [m + \rho(l-x)]\dot{x}^2 = 2g\left[mx + \rho x\left(l - \frac{x}{2}\right)\right]$$

$$\text{Now for system } \Sigma F_x = \dot{G}_x$$

$$2pgl + mg - 2R = \frac{d}{dt}\left[m\dot{x} + 2\rho\left(\frac{l}{2} - x + \frac{\delta}{2}\right)\dot{x}\right]$$

$$= \frac{d}{dt}\left[(m + \rho(l-x))\dot{x}\right]$$

$$\text{Substitute 1st Eq. into 2nd \& get}$$

$$2pgl + mg - 2R = 2\frac{d}{dt}\left[\frac{mgx + \rho g x(l - x/2)}{\dot{x}}\right]$$

$$\text{Upon simplification, } = 2[mg + \rho g(l-x)] - [m + \rho(l-x)]\ddot{x}$$

$$\text{or } mg + 2R - 2pgx = [m + \rho(l-x)]\ddot{x}$$

$$\text{Take } \frac{d}{dt}(\dot{x}^2) \text{ from 1st Eq. \& get } \ddot{x} = g\left[1 + \rho x \frac{m + \rho(l - x/2)}{[m + \rho(l-x)]^2}\right]$$

which shows that $\ddot{x} > g$.

Finally, substitute \ddot{x}

\& get

$$R = \frac{\rho g}{2}\left\{(l-x) + x \frac{m + \rho(l - x/2)}{m + \rho(l-x)}\right\}$$

10/49 Eq. 195; $\Sigma F = m\ddot{u} - \dot{m}u - \frac{d^2}{dt^2}(mp_e)$ for

x-direction becomes

$$\Sigma F_x = m\ddot{x} + \dot{m}\dot{x} - \frac{d^2}{dt^2}(m\frac{x}{2})$$

where $m = \rho x = \rho kt^2$; \dot{u} becomes $-\dot{x}$

$$\text{Thus } \Sigma F_x = T + \rho gx - T_0$$

$$m\ddot{x} = \rho x(2k); \dot{m}\dot{x} = 2\rho kt(2kt) = 4\rho k^2 t^2$$

$$\frac{d^2}{dt^2}(m\frac{x}{2}) = \frac{d^2}{dt^2}(\frac{1}{2}\rho k^2 t^4) = 6\rho k^2 t^2$$

$$\text{Substitute \& get } T + \rho gx - T_0 = 2\rho k^2 t^2 + 4\rho k^2 t^2 - 6\rho k^2 t^2$$

$$\text{so } T + \rho gx - T_0 = 0$$

10/50

$$H_0 = \Sigma m_i r_i^2 \omega = I_0 \omega = \frac{1}{2} \pi \rho r^4 \omega; \rho = \text{mass per unit disk area}$$

$$\dot{H}_0 = \frac{1}{2} \pi \rho (4r^3 \dot{r} \omega + r^4 \dot{\omega})$$

\dot{r}_0 = velocity of mass center of leaving particles just at separation. Magnitude is $r\omega$

$$|\dot{r}_0| = r\omega$$

$$|\dot{r}_0 \times m \dot{r}_0| = m r^2 \omega \text{ C.W.}; \dot{m} = \frac{d}{dt}(\rho \pi r^2) = 2\rho \pi r \dot{r}$$

$$\text{So from Eq. 197, } \Sigma \dot{M}_0 = \dot{H}_0 - \dot{r}_0 \times m \dot{r}_0$$

$$M = \frac{1}{2} \pi \rho (4r^3 \dot{r} \omega + r^4 \dot{\omega}) - 2\rho \pi r \dot{r} (r^2 \omega)$$

$$= \frac{1}{2} \pi \rho r^4 \dot{\omega} \text{ so } \dot{\omega} = \frac{2M}{\rho \pi r^4}$$

{This is identical with application of $\Sigma M = I\dot{\omega}$ at the instant under consideration}

Note: Eq. $M_0 = \dot{H}_0$ gives $M = \frac{1}{2} \pi \rho (4r^3 \dot{r} \omega + r^4 \dot{\omega})$

\& $\dot{\omega} = \frac{2M}{\rho \pi r^4} + \frac{4r\dot{r}\omega}{r}$ which is wrong since

$\Sigma M_0 = \dot{H}_0$ is invalid for $m = f(t)$

10/51

Let m_0 = mass of rocket structure \& machinery

m_f = mass of grain = $\rho \pi r_0^2 (l-y)$

h_0 = distance from nozzle to mass center of m_0

$$\text{Correction term } C = \frac{d^2}{dt^2}(mh) = \frac{d^2}{dt^2}(m_0 h_0 + m_f \frac{l+y}{2})$$

$$\text{so } C = 0 + \frac{d^2}{dt^2}(\rho \pi r_0^2 \frac{[l-y][l+y]}{2}) = -\rho \pi r_0^2 [\dot{y}^2 + y\ddot{y}]$$

$$\text{But } T = \dot{m}U = U \frac{d}{dt}(\rho \pi r_0^2 [l-y]) = -U \rho \pi r_0^2 \dot{y}$$

$$\text{so } n = \left|\frac{C}{T}\right| = \frac{\rho \pi r_0^2 (\dot{y}^2 + y\ddot{y})}{U \rho \pi r_0^2 \dot{y}} = \frac{y}{U} \left(\frac{\dot{y}}{\dot{y}} + \frac{\ddot{y}}{\dot{y}}\right)$$

$$\text{And } n = \frac{1.2}{1200} \left(\frac{0.050}{1.2} + \frac{0.150}{0.050}\right) = 0.00304$$

10/52

Let m_0 = mass of rocket structure \& machinery

m_f = mass of grain = $\rho \pi (r_0^2 - r^2) L$

h_0 = distance from nozzle to mass center of m_0

h_f = distance from nozzle to mass center of m_f

$$\text{Correction term } C = \frac{d^2}{dt^2}(mh) = \frac{d^2}{dt^2}(m_0 h_0 + m_f h_f)$$

$$\text{so } C = 0 + \frac{d^2}{dt^2}(\rho \pi [r_0^2 - r^2] L h_f) = -2\rho \pi L h_f (\dot{r}^2 + r\ddot{r})$$

$$\text{But } T = \dot{m}U = U \frac{d}{dt}(\rho \pi [r_0^2 - r^2] L) = -2\rho \pi L r \dot{r} U$$

$$\text{so } n = \left|\frac{C}{T}\right| = \frac{2\rho \pi L h_f (\dot{r}^2 + r\ddot{r})}{2\rho \pi L r \dot{r} U} = \frac{h_f}{U} \left(\frac{\dot{r}}{r} + \frac{\ddot{r}}{\dot{r}}\right)$$

$$\text{And } n = \frac{2.4}{1200} \left(\frac{0.050}{0.025} + \frac{0.150}{0.050}\right) = 0.01$$

10/53

$$u = \frac{Q}{4A}, \dot{m} = -\rho Q$$

$$\text{Eq. 197, } \Sigma \dot{M}_0 = \dot{H}_0 - \dot{r}_0 \times m \dot{r}_0$$

Let m = mass of water in tanks

Angular momentum about O due to

m is $\dot{H}_0 = m r^2 \omega (-\hat{k})$

Pipes are always filled so their angular momentum is constant.

$$\text{Thus } \dot{H}_0 = m r^2 \omega (-\hat{k}) = \rho Q r^2 \omega \hat{k}$$

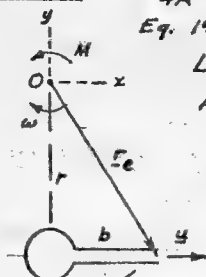
$$\dot{r}_0 \times m \dot{r}_0 = \dot{r}_0 \times \dot{m} (\dot{r}_0 + u) = \dot{m} (r_0^2 \omega - ru) (-\hat{k}) \text{ C.W.}$$

Thus for the clockwise sense $(-\hat{k})$

$$-M = -\rho Q r^2 \omega - \dot{m} (r_0^2 \omega - ru)$$

$$M = \rho Q (ru - b^2 \omega) = \rho Q \left(\frac{Qr}{4A} - b^2 \omega\right)$$

$$\text{With } M = 0, \omega = \omega_0 = \frac{Qr}{4Ab^2}$$



$$|\dot{r}_0| = r\omega$$

10/54 For 4 tubes $dH_o' = 4(\rho A dx)(x\omega)(x) = 4\rho A x^2 \omega dx$

$H_o' = 4\rho A \omega \int_r^R x^2 dx = \frac{4\rho A \omega}{3} (R^3 - r^3)$

so total $H_o = H_o' + I_o \omega$

$4I_o \times \ddot{\omega} = 4R(R\omega)(\ddot{\omega})(\frac{k}{k})$

$= 4R^2 \omega (-\ddot{r} \rho A) k$

$\Sigma M_o = H_o - I_o \times \ddot{\omega}$

$0 = \frac{4}{3} \rho A [\dot{\omega}(R^3 - r^3) - 3\omega r^2 \ddot{r}] + I_o \ddot{\omega}$

$+ 4\rho A r R^2 \omega$

A = cross-sectional area of tube
 ρ = density of liquid
 I_o = moment of inertia of disk & empty tubes

Simplify & get

$$[(R^3 - r^3) + \frac{3}{4\rho A} I_o] \ddot{\omega} = 3\omega r^2 (r^2 - R^2)$$

$$\text{or } \int_{\omega_o}^{\omega} \frac{d\omega}{\omega} = 3 \int_0^R \frac{r^2 - R^2}{(R^3 - r^3) + \frac{3I_o}{4\rho A}} dr$$

Now neglect mass of disk & hollow tubes

so $I_o \rightarrow 0$

$$\int_{\omega_o}^{\omega} \frac{d\omega}{\omega} = 3 \int_0^R \frac{-(R+r)}{R^2 + Rr + r^2} dr = -3 \int_0^1 \frac{1+x}{1+x+x^2} dx \quad \text{where } x = \frac{r}{R}$$

$$= -3 \left[\frac{1}{\sqrt{3}} \tan^{-1} \frac{2x+1}{\sqrt{3}} + \frac{1}{2} \ln(1+x+x^2) \right]_0^1$$

$$= -3 \left[\frac{1}{\sqrt{3}} \tan^{-1} \frac{1}{\sqrt{3}} + \frac{1}{2} \ln 3 \right] = -3 \left[\frac{\pi}{6\sqrt{3}} + \frac{1.099}{2} \right] = -2.555$$

Thus $\ln \frac{\omega}{\omega_o} = -2.555$, $\omega = \omega_o e^{-2.555}$, $\omega = \frac{\omega_o}{12.87}$

$$\omega = 0.0777 \omega_o$$

10/56 $2l = ct$ where $c = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{210(10^9)}{7.83}} = 5180 \text{ m/s}$

Thus $t = \frac{2(2)}{5180} = 0.772(10^{-3}) \text{ s}$ or $t = 0.772 \text{ ms}$

10/57 $l = ct$, where $c = \sqrt{\frac{T}{\rho'}} = \sqrt{\frac{T}{\rho A}}$, ρ' = mass/length
 ρ = density
 A = cross-sectional area

$$\sqrt{\frac{T}{7830 \frac{\pi}{4} (0.006)^2}} = \frac{l}{t} = \frac{30}{0.3}, T = 2210 \text{ N}$$

10/58 If rotational acceleration is neglected,
 $Q dx = \frac{\partial M}{\partial x} dx$, $Q = \frac{\partial M}{\partial x}$

$\Sigma F_y = m \ddot{y}$; $-\frac{\partial Q}{\partial x} dx = \rho dx \frac{\partial^2 y}{\partial t^2}$

or $-\frac{\partial}{\partial x} \left(\frac{\partial M}{\partial x} \right) = \rho \frac{\partial^2 y}{\partial t^2}$

But for small deflections

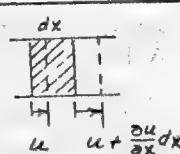
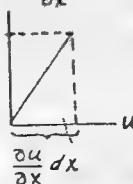
$$EI \frac{\partial^3 y}{\partial x^3} = M$$

Thus $-\frac{\partial^4}{\partial x^4} (EI \frac{\partial y}{\partial x^3}) = \rho \frac{\partial^2 y}{\partial t^2}$

or $\frac{\partial^4 y}{\partial x^4} + \frac{\rho}{EI} \frac{\partial^2 y}{\partial t^2} = 0$

10/59

$$\sigma A = \frac{\partial u}{\partial x} EA$$



dV_e = elastic energy for element of volume $A dx$

$$= \frac{1}{2} \sigma A \frac{\partial u}{\partial x} dx$$

$$V_e = \frac{dV_e}{A dx} = \frac{\sigma}{2} \frac{\partial u}{\partial x} = \frac{E}{2} \left(\frac{\partial u}{\partial x} \right)^2$$

dT = kinetic energy for element of volume $A dx$

$$= \frac{1}{2} \rho A dx \left(\frac{\partial u}{\partial t} \right)^2, T = \frac{dT}{A dx} = \frac{\rho}{2} \left(\frac{\partial u}{\partial t} \right)^2$$

Now let $w = x - ct$ so sol. is $u = f(w)$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial w} \frac{\partial w}{\partial x} = \frac{\partial u}{\partial w}, \frac{\partial u}{\partial t} = \frac{\partial u}{\partial w} \frac{\partial w}{\partial t} = -c \frac{\partial u}{\partial w}$$

Thus $V_e = \frac{E}{2} \left(\frac{\partial u}{\partial w} \right)^2$ & $T = \frac{\rho}{2} c^2 \left(\frac{\partial u}{\partial w} \right)^2$

But $c^2 = E/\rho$, so $V_e = T$

10/60 V_e = elastic potential energy per unit volume
 T = kinetic energy per unit volume

dU = total energy for length dx

$$= (V_e + T) A dx$$

From Prob. 10/59, $V_e = T = \frac{E}{2} \left(\frac{\partial u}{\partial x} \right)^2$; Also $u = u_o \sin p(t - \frac{x}{c})$

so $\frac{\partial u}{\partial x} = -\frac{p u_o}{c} \cos p(t - \frac{x}{c})$ & $\sigma_o = \sigma_{\max} = \left| \left(\frac{\partial u}{\partial x} \right) \right|_{\max} (E) = \frac{p u_o E}{c}$

$\therefore \frac{\partial u}{\partial x} = -\frac{\sigma_o}{E} \cos p(t - \frac{x}{c})$ so $u_o = \frac{c \sigma_o}{p E}$

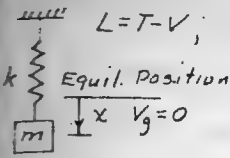
Thus $dU = E \left(\frac{\partial u}{\partial x} \right)^2 A dx = \left[EA \frac{\sigma_o^2}{E^2} \cos^2 p(t - \frac{x}{c}) \right] dx$

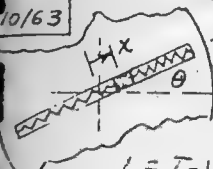
At a given x , the time average of dU is

$$\bar{dU} = \frac{1}{T} \int_0^T dU dt = \left\{ \frac{\rho}{2\pi} \int_0^{2\pi/p} A \sigma_o^2 \cos^2 p(t - \frac{x}{c}) dt \right\} dx$$

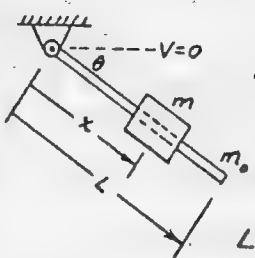
$$= \frac{\rho A \sigma_o^2}{2\pi E} \frac{1}{p} \left[\frac{p(t - \frac{x}{c})}{2} + \frac{1}{4} \sin 2p(t - \frac{x}{c}) \right]_0^{2\pi/p}$$

$$= \frac{A \sigma_o^2}{2\pi E} [\pi] dx \quad \text{so } \bar{U} = \frac{A \bar{U}}{\pi} = \frac{\sigma_o^2}{2E} = \frac{1}{2} \frac{\sigma_o^2}{E}$$

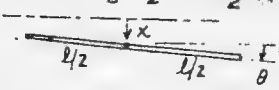
10/62 $T = \frac{1}{2} m \dot{x}^2$; $V = V_e + V_g = \frac{1}{2} k \left(\frac{mg}{k} + x \right)^2 - mgx$
 $L = T - V$; $\frac{\partial L}{\partial \dot{x}} = m \dot{x}$; $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = m \ddot{x}$

 Equil. Position $\frac{\partial L}{\partial x} = -k \left(\frac{mg}{k} + x \right) + mg = -kx$
 $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = 0$; $m \ddot{x} + kx = 0$

10/63 
 For system of spring & slider
 $T = \frac{1}{2} m (\dot{x}^2 + x^2 \dot{\theta}^2)$
 $\dot{\theta} = \omega$ $V = \frac{1}{2} k x^2$
 $L = T - V = \frac{1}{2} m (\dot{x}^2 + x^2 \dot{\theta}^2) - \frac{1}{2} k x^2$
 $\frac{\partial L}{\partial \dot{x}} = m \dot{x}$; $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = m \ddot{x}$; $\frac{\partial L}{\partial x} = m x \dot{\theta}^2 - kx$
 $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = 0$; $m \ddot{x} - m x \dot{\theta}^2 + kx = 0$
 or $\ddot{x} + \left(\frac{k}{m} - \omega^2 \right) x = 0$ where $\omega = \dot{\theta}$, constant

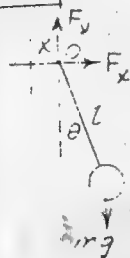
$T = 2\pi/p = \frac{2\pi}{\sqrt{\frac{k}{m} - \omega^2}}$; $\omega_{max} = \sqrt{k/m}$

10/64 Collar: $T = \frac{1}{2} m (\dot{x}^2 + x^2 \dot{\theta}^2)$
 $V = -mgx \sin \theta$
 Rod: $T = \frac{1}{2} I \dot{\theta}^2 = \frac{1}{2} m_0 k^2 \dot{\theta}^2$
 $V = -m_0 g \frac{L}{2} \sin \theta$

 For system, $L = T - V$
 $L = \frac{1}{2} m (\dot{x}^2 + x^2 \dot{\theta}^2) + \frac{1}{2} m_0 k^2 \dot{\theta}^2 + (mx + m_0 \frac{L}{2}) g \sin \theta$

$\frac{\partial L}{\partial \dot{x}} = m \dot{x}$; $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = m \ddot{x}$
 $\frac{\partial L}{\partial x} = m x \dot{\theta}^2 + mg \sin \theta$
 $\frac{\partial L}{\partial \dot{\theta}} = m x^2 \dot{\theta} + m_0 k^2 \dot{\theta}$; $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) = m x^2 \ddot{\theta} + 2 m x \dot{x} \dot{\theta} + m_0 k \ddot{\theta}$
 $\frac{\partial L}{\partial \theta} = (mx + m_0 \frac{L}{2}) g \cos \theta$
 $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = 0$; $j = x, \theta$
 $j = x$: $m \ddot{x} - m x \dot{\theta}^2 - mg \sin \theta = 0$
 $\ddot{x} - x \dot{\theta}^2 = g \sin \theta$
 $j = \theta$: $m x^2 \ddot{\theta} + 2 m x \dot{x} \dot{\theta} + m_0 k \ddot{\theta} - (mx + m_0 \frac{L}{2}) g \cos \theta = 0$
 $(x^2 + \frac{m_0 k^2}{m}) \ddot{\theta} + 2 x \dot{x} \dot{\theta} = (x + \frac{m_0 L}{m}) g \cos \theta$

10/65 $T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} \frac{1}{12} m l^2 \dot{\theta}^2$
 $V = V_e = \frac{1}{2} k x^2 + \frac{1}{2} k \left(x - \frac{l\theta}{2} \right)^2$

 $L = T - V$
 $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = 0$
 For x ; $\frac{\partial L}{\partial x} = -kx - k \left(x - \frac{l\theta}{2} \right)$; $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = \frac{d}{dt} (m \dot{x}) = m \ddot{x}$
 So $m \ddot{x} + 2kx - k \frac{l\theta}{2} = 0$
 For θ ; $\frac{\partial L}{\partial \theta} = \frac{k l}{2} \left(x - \frac{l\theta}{2} \right)$; $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) = \frac{d}{dt} \left(\frac{1}{12} m l^2 \dot{\theta} \right) = \frac{m l^2}{12} \ddot{\theta}$
 $\frac{m l^2}{12} \ddot{\theta} - \frac{k l}{2} \left(x - \frac{l\theta}{2} \right) = 0$
 So $m \ddot{\theta} - 6k \left(\frac{x}{l} - \frac{\theta}{2} \right) = 0$

10/66 $x = x_0 \sin \omega t$; $\dot{x} = x_0 \omega \cos \omega t$; $\ddot{x} = -\omega^2 x$



Velocity of m is v

$$v^2 = (\dot{x} + l\dot{\theta} \cos \theta)^2 + (l\dot{\theta} \sin \theta)^2$$

$$= \dot{x}^2 + 2l\dot{x}\dot{\theta} \cos \theta + l^2\dot{\theta}^2$$

$$T = \frac{1}{2} m [\dot{x}^2 + 2l\dot{x}\dot{\theta} \cos \theta + l^2\dot{\theta}^2]$$

$$Q_x \delta x = -\dot{x} \delta \dot{x}, \quad Q_{\dot{x}} = \dot{x}$$

$$Q_{\theta} \delta \theta = -mg \sin \theta l \delta \theta, \quad Q_{\dot{\theta}} = -mg l \sin \theta$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{x}} \right) - \frac{\partial T}{\partial x} = Q_x \quad \text{where } j = x, \theta$$

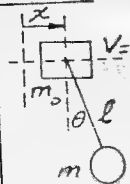
$$x; \quad \frac{d}{dt} m(\dot{x} + l\dot{\theta} \cos \theta) - 0 = \dot{x}$$

$$\ddot{x} + l\ddot{\theta} \cos \theta - l\dot{\theta}^2 \sin \theta = \dot{x}/m$$

$$\theta; \quad \frac{d}{dt} m(l\dot{x} \cos \theta + l^2\dot{\theta}) + ml\dot{x}\dot{\theta} \sin \theta = -mg l \sin \theta$$

$$\ddot{x} \cos \theta + l\ddot{\theta} = -g \sin \theta, \quad \ddot{\theta} + \frac{g}{l} \sin \theta = \frac{x_0 \omega^2 \sin \omega t \cos \theta}{l}$$

10/67 Velocity of m is $v^2 = (\dot{x} + l\dot{\theta} \cos \theta)^2 + (l\dot{\theta} \sin \theta)^2$



$$T = \frac{1}{2} m (\dot{x}^2 + 2l\dot{x}\dot{\theta} \cos \theta + l^2\dot{\theta}^2) + \frac{1}{2} m_0 \dot{x}^2$$

$$V = -mg l \cos \theta; \quad L = T - V$$

$$\theta; \quad \frac{\partial L}{\partial \theta} = m(l\dot{x} \cos \theta + l^2\dot{\theta}); \quad \frac{\partial L}{\partial \theta} = m(-l\dot{x} \sin \theta - gl \sin \theta)$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) = m(l\dot{x} \cos \theta - l\dot{x} \sin \theta + l\ddot{\theta})$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = 0; \quad m(l\dot{x} \cos \theta - l\dot{x} \sin \theta + l\ddot{\theta}) + m l \sin \theta (\dot{x} + g) = 0$$

$$\ddot{x} \cos \theta + l\ddot{\theta} + g \sin \theta = 0$$

$$x; \quad \frac{\partial L}{\partial x} = m(\dot{x} + l\dot{\theta} \cos \theta) + m_0 \dot{x}; \quad \frac{\partial L}{\partial x} = 0$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = m(\ddot{x} + l\ddot{\theta} \cos \theta - l\dot{\theta}^2 \sin \theta) + m_0 \ddot{x}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = 0; \quad (m+m_0)\ddot{x} + ml(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) = 0$$

$$\text{For } \sin \theta = 0, \cos \theta = 1, \dot{\theta}^2 = 0; \quad \ddot{x} + l\ddot{\theta} + g\theta = 0$$

$$\phi (m+m_0)\ddot{x} + ml\ddot{\theta} = 0. \quad \text{Eliminate } \ddot{x} \text{ \& yet}$$

$$\ddot{\theta} + \frac{m+m_0}{m} \frac{g}{l} \theta = 0, \quad T = 2\pi \sqrt{\frac{m_0}{m} \frac{l}{g}}$$

10/68 For bar $T = \frac{1}{2} (I_1 \omega_1^2 + I_2 \omega_2^2 + I_3 \omega_3^2)$

where $\omega_1 = \omega_0 \cos \phi$, $\omega_2 = -\omega_0 \sin \phi$, $\omega_3 = \rho = \dot{\phi}$

Also let $\omega_0 = \dot{\theta}$

$$\text{Now } T = \frac{1}{2} (I_1 \omega_0^2 \cos^2 \phi + I_2 \omega_0^2 \sin^2 \phi + I_3 \rho^2), \quad V = 0$$

$$\frac{\partial T}{\partial \theta} = 0, \quad \frac{\partial T}{\partial \phi} = \frac{\partial T}{\partial \omega_0} = (I_1 \cos^2 \phi + I_2 \sin^2 \phi) \omega_0$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \omega_0} \right) = 2(-I_1 \cos \phi \sin \phi + I_2 \sin \phi \cos \phi) \dot{\phi} \omega_0$$

$$= (I_2 - I_1) \rho \omega_0 \sin 2\phi$$

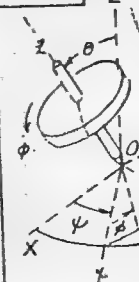
$$Q_{\theta} \delta \theta = M \delta \theta \quad \text{so } Q_{\theta} = M$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\theta}} \right) - \frac{\partial T}{\partial \theta} = Q_{\theta} \quad \text{so } (I_2 - I_1) \rho \omega_0 \sin 2\phi - 0 = M$$

$$\text{But } I_2 = \frac{1}{12} m(b^2 + c^2) \text{ \& } I_1 = \frac{1}{12} m(a^2 + b^2), \quad \text{so}$$

$$M = \frac{1}{12} m(c^2 - a^2) \rho \omega_0 \sin 2\phi$$

10/69 $I_{zz} = I, \quad I_{xx} = I_{yy} = I_0, \quad \Sigma M_0 = \Sigma \Sigma M_x + \Sigma \Sigma M_y + \Sigma \Sigma M_z$



$$\text{About } O, \quad T = \frac{1}{2} (I_0 \dot{\omega}_x^2 + I_0 \dot{\omega}_y^2 + I \dot{\omega}_z^2)$$

$$\omega_x = \dot{\theta}, \quad \omega_y = \dot{\psi} \sin \theta, \quad \omega_z = \dot{\psi} \cos \theta + \dot{\phi}$$

$$\text{so } T = \frac{1}{2} (I_0 \dot{\theta}^2 + I_0 \dot{\psi}^2 \sin^2 \theta + I (\dot{\psi} \cos \theta + \dot{\phi})^2)$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\theta}} \right) - \frac{\partial T}{\partial \theta} = Q_{\theta}, \quad \theta_j = \theta, \phi, \psi$$

$$\text{For } \theta, \quad Q_{\theta} \delta \theta = \Sigma M_x \delta \theta, \quad Q_{\theta} = \Sigma M_x$$

$$\frac{\partial T}{\partial \theta} = I_0 \dot{\psi}^2 \sin \theta \cos \theta - I (\dot{\psi} \cos \theta + \dot{\phi}) \dot{\psi} \sin \theta$$

$$\frac{\partial T}{\partial \theta} = I_0 \dot{\theta}$$

$$\text{Thus } I_0 \ddot{\theta} - I_0 \dot{\psi}^2 \sin \theta \cos \theta + I (\dot{\psi} \cos \theta + \dot{\phi}) \dot{\psi} \sin \theta = \Sigma M_x$$

$$\text{or } \Sigma M_x = I_0 (\ddot{\theta} - \dot{\psi}^2 \sin \theta \cos \theta) + I \dot{\psi} \omega_z \sin \theta$$

(which is the first of Eqs. 166)

10/70 Let axes 1, 2, 3 be principal axes of half-disk

Angular velocity components of half-disk are

$$\omega_1 = \dot{\theta}, \omega_2 = \dot{\psi} \sin \theta, \omega_3 = \dot{\psi} \cos \theta$$

$$I_1 = \frac{1}{2} m r^2, I_2 = \frac{1}{4} m r^2, I_3 = \frac{1}{4} m r^2$$

For system

$$T = \frac{1}{2} I_0 \dot{\psi}^2 + \frac{1}{2} m r^2 \left(\frac{1}{2} \dot{\theta}^2 + \frac{1}{4} \dot{\psi}^2 \sin^2 \theta + \frac{1}{4} \dot{\psi}^2 \cos^2 \theta \right)$$

$$= \frac{1}{2} I_0 \dot{\psi}^2 + \frac{1}{4} m r^2 (\dot{\theta}^2 + \dot{\psi}^2)$$

$$V = m g r (1 - \cos \theta); L = T - V; \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = 0, q_j = \theta, \psi$$

$$\text{For } \theta; \frac{\partial L}{\partial \theta} = -m g r \sin \theta, \frac{\partial L}{\partial \dot{\theta}} = \frac{1}{2} m r^2 \dot{\theta}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) = \frac{1}{2} m r^2 \ddot{\theta}$$

$$\text{So } \frac{1}{2} m r^2 \ddot{\theta} + m g r \sin \theta = 0$$

or $\ddot{\theta} + \frac{g}{3r} \sin \theta = 0$ which is independent of ψ and is the same equation as for a nonrotating support for m .

10/71 Take $V=0$ at $x=0$; $V = -m g \left(\frac{l}{2} \cos \theta + x \right)$

Equil.

$$\text{position } V_c = (m g + \frac{k x}{2}) x, V = -m g \frac{l}{2} \cos \theta + \frac{1}{2} k x^2$$

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} I \dot{\omega}^2 = \frac{1}{2} m \left(\dot{x} - \frac{l}{2} \dot{\theta} \sin \theta \right)^2 + \left(\frac{l}{2} \dot{\theta} \cos \theta \right)^2$$

$$L = T - V$$

$$x; \frac{\partial L}{\partial x} = m \left(\dot{x} - \frac{l}{2} \dot{\theta} \sin \theta \right); \frac{\partial L}{\partial x} = -k x$$

$$m \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = m \left(\ddot{x} - \frac{l}{2} \ddot{\theta} \sin \theta - \frac{l}{2} \dot{\theta}^2 \cos \theta \right)$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = 0; m \left(\ddot{x} - \frac{l}{2} \ddot{\theta} \sin \theta - \frac{l}{2} \dot{\theta}^2 \cos \theta \right) + k x = 0$$

$$\theta; \frac{\partial L}{\partial \theta} = \frac{1}{2} m \left(-\dot{x} l \sin \theta + \frac{1}{2} l^2 \dot{\theta} \right) + \frac{1}{2} m l^2 \dot{\theta}$$

$$\frac{\partial L}{\partial \theta} = \frac{1}{2} m \left(-\dot{x} l \sin \theta \right) - m g \frac{l}{2} \sin \theta$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) = \frac{1}{2} m \left(-\ddot{x} l \sin \theta - \dot{x} l \cos \theta + \frac{1}{2} l^2 \ddot{\theta} \right) + \frac{1}{2} m l^2 \ddot{\theta}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = 0; -\ddot{x} l \sin \theta + \frac{2}{3} l \ddot{\theta} + g \sin \theta = 0$$

10/72 Take $V=0$ at $x=0, \theta=0$; $V = \Delta V$

$$\Delta V_c = (m_1 + m_2) g x + \frac{1}{2} k x^2$$

WHEEL

$$\text{EQUIL. } \Delta V_g = -m_2 g x - m_1 g (x - l [1 - \cos \theta])$$

$$\text{POSITION SO } V = \frac{1}{2} k x^2 + m_1 g l (1 - \cos \theta)$$

$$T = \frac{1}{2} m_2 \dot{x}^2 + \frac{1}{2} m_1 \left[(l \dot{\theta} \cos \theta)^2 + (\dot{x} - l \dot{\theta} \sin \theta)^2 \right]$$

$$= \frac{1}{2} (m_1 + m_2) \dot{x}^2 + \frac{1}{2} m_1 l^2 \dot{\theta}^2 - m_1 l \dot{x} \dot{\theta} \sin \theta$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0 \text{ where } L = T - V, q = x, \theta$$

$$\text{For } x; \frac{\partial L}{\partial x} = -k x, \frac{\partial L}{\partial \dot{x}} = (m_1 + m_2) \dot{x} - m_1 l \dot{\theta} \sin \theta$$

$$\text{So } (m_1 + m_2) \ddot{x} - m_1 l \ddot{\theta} \sin \theta - m_1 l \dot{\theta}^2 \cos \theta + k x = 0 \quad (1)$$

$$\text{For } \theta; \frac{\partial L}{\partial \theta} = -m_1 l \dot{x} \dot{\theta} \cos \theta - m_1 g l \sin \theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = m_1 l^2 \dot{\theta} - m_1 l \dot{x} \sin \theta$$

$$\text{So } m_1 l \ddot{\theta} - m_1 l \ddot{x} \sin \theta - m_1 l \dot{x} \dot{\theta} \cos \theta + m_1 l \dot{x} \dot{\theta} \cos \theta + m_1 g l \sin \theta = 0$$

$$\text{or } l \ddot{\theta} - \ddot{x} \sin \theta + g \sin \theta = 0 \quad (2)$$

For x & θ small; neglect $\dot{x}^2, \dot{\theta}^2, \dot{x} \dot{\theta}$; let $\sin \theta = \theta, \cos \theta = 1$

Eqs. 1 & 2 become

$$(m_1 + m_2) \ddot{x} - m_1 l \ddot{\theta} + k x = 0 \quad (1a)$$

$$l \ddot{\theta} - \ddot{x} + g \theta = 0 \quad (2a)$$

10/73 From previous analysis (Prob. 7/31) the

components of angular velocity of wheel are

$$\omega_x = 0, \omega_y = p \cos \theta, \omega_z = p \left(\sin \theta + \frac{R}{r} \right)$$

$$\text{For } I_x = I_z = \frac{1}{2} m r^2$$

$$\frac{1}{2} I_y = I_y = \frac{1}{4} m r^2$$

For system For axis $I_x = I_y$

$$T = \frac{1}{2} I_x p^2 + \frac{1}{2} m (R + r \sin \theta)^2 p^2 + \frac{1}{2} \left[\frac{I_y}{2} p^2 \cos^2 \theta + I_z \left(\sin \theta + \frac{R}{r} \right)^2 p^2 \right]$$

$$= \frac{1}{2} \left\{ I_x + m \left[\frac{3}{2} R^2 + 3 R r \sin \theta + \frac{3}{2} r^2 \sin^2 \theta + \frac{1}{4} r^2 \cos^2 \theta \right] \right\} p^2$$

$$= \frac{1}{2} K p^2 \text{ where } K = \left\{ \dots \right\} = \left\{ I_x + m \left[\frac{3}{2} (R + r \sin \theta)^2 + \frac{1}{4} r^2 \cos^2 \theta \right] \right\}$$

$$\frac{\partial T}{\partial \psi} = 0, \frac{\partial T}{\partial \dot{\psi}} = \frac{\partial T}{\partial p} = K p, \frac{d}{dt} \left(\frac{\partial T}{\partial p} \right) = K \dot{p}; Q_\psi \delta \psi = \Delta \delta \psi$$

$$\text{Thus } \frac{d}{dt} \left(\frac{\partial T}{\partial p} \right) - \frac{\partial T}{\partial \psi} = Q_\psi \text{ gives}$$

$$K \dot{p} = M, \text{ so}$$

$$M = \left\{ I_x + m \left[\frac{3}{2} (R + r \sin \theta)^2 + \frac{1}{4} r^2 \cos^2 \theta \right] \right\} \dot{p}$$

10/74

Given: $\beta = k$, constant, $\omega = \text{constant}$

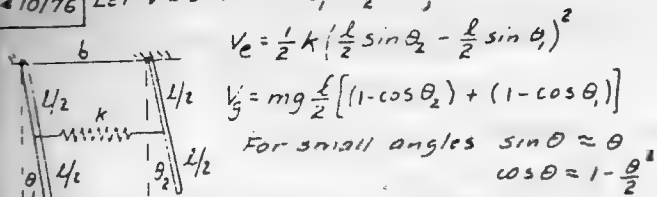
Replace spherical coordinates

 R, θ, ϕ by $r, \theta, -\beta$ where $\dot{\theta} = \omega$, $\dot{\phi} = -\dot{\beta} = -k$.Take $V=0$ at $r=0$ $V = mgr \cos \beta$; $T = \frac{1}{2} m (\dot{r}^2 + r^2 \omega^2 \sin^2 \beta + (-rk)^2)$ $\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} = Q_j$; where $q_j = r, \theta, \beta$ For r ; $Q_r \delta r = -mg \cos \beta \delta r$; $Q_r = -mg \cos \beta$ $\frac{\partial T}{\partial r} = m r (\omega^2 \sin^2 \beta + k^2)$ $\frac{\partial T}{\partial r} = m \ddot{r}$; $\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{r}} \right) = m \ddot{r}$ Thus $m \ddot{r} - m r (\omega^2 \sin^2 \beta + k^2) = -mg \cos \beta$ or $\ddot{r} - r \omega^2 \sin^2 \beta - rk^2 + g \cos \beta = 0$ ----- (1)For θ ; $Q_\theta \delta \theta = M \delta \theta$, $Q_\theta = M$ $\frac{\partial T}{\partial \theta} = 0$, $\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial \omega} = m r^2 \omega \sin^2 \beta$ $\frac{d}{dt} \left(\frac{\partial T}{\partial \omega} \right) = m \omega (2 r \dot{r} \sin^2 \beta + 2 r^2 \dot{\beta} \sin \beta \cos \beta)$, $\omega \text{ const.}$
 $= 2 m r \omega \sin \beta (\dot{r} \sin \beta + r \dot{\beta} \cos \beta)$ Thus $2 m r \omega \sin \beta (\dot{r} \sin \beta + r \dot{\beta} \cos \beta) - 0 = M$ or $M = (2 m r \omega \sin \beta) \frac{d}{dt} (r \sin \beta)$ ----- (2)For β ; $Q_\beta \delta \beta = mg \sin \beta r \delta \beta$, $Q_\beta = mgr \sin \beta$ $\frac{\partial T}{\partial \beta} = m r^2 \omega^2 \sin \beta \cos \beta$, $\frac{\partial T}{\partial \beta} = \frac{\partial T}{\partial k} = m r^2 k$ Thus $2 m r k - m r^2 \omega^2 \sin \beta \cos \beta = mgr \sin \beta$ or $2 k \dot{r} = (r \omega^2 \cos \beta + g) \sin \beta$ ----- (3)

10/75

 $T = \frac{1}{2} I_0 \dot{\psi}^2 + \frac{1}{2} \frac{1}{3} m l^2 \dot{\theta}^2 + \frac{1}{2} \frac{1}{3} m (l \sin \theta)^2 \dot{\psi}^2$ $V = mg \frac{l}{2} (1 - \cos \theta)$ $L = T - V$; $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = 0$, $q_j = \theta, \psi$ For θ ; $\frac{\partial L}{\partial \theta} = \frac{1}{3} m l^2 \dot{\psi}^2 \sin \theta \cos \theta - mg \frac{l}{2} \sin \theta$ $\frac{\partial L}{\partial \theta} = \frac{1}{3} m l^2 \ddot{\theta}$ Thus $\frac{1}{3} m l^2 \ddot{\theta} - \frac{1}{3} m l^2 \dot{\psi}^2 \sin \theta \cos \theta + mg \frac{l}{2} \sin \theta = 0$ or $\ddot{\theta} - \dot{\psi}^2 \sin \theta \cos \theta + \frac{3g}{2l} \sin \theta = 0$ ----- (1)For ψ ; $\frac{\partial L}{\partial \psi} = 0$; $\frac{\partial L}{\partial \psi} = I_0 \dot{\psi} + \frac{1}{3} m l^2 \sin^2 \theta \dot{\psi}$ Thus $\frac{d}{dt} \left(\left(I_0 + \frac{1}{3} m l^2 \sin^2 \theta \right) \dot{\psi} \right) = 0$;so $\left(I_0 + \frac{1}{3} m l^2 \sin^2 \theta \right) \dot{\psi} = C$, a constantNow $\theta = 0$ when $\dot{\psi} = \dot{\psi}_0$; Thus $\dot{\psi} = \frac{I_0 \dot{\psi}_0}{I_0 + \frac{1}{3} m l^2 \sin^2 \theta}$ So $C = I_0 \dot{\psi}_0$ For θ small & I_0 large, $\dot{\psi} \approx \dot{\psi}_0$, & Eq. 1becomes $\ddot{\theta} - \dot{\psi}_0^2 \theta + \frac{3g}{2l} \theta = 0$ or $\ddot{\theta} + \left(\frac{3g}{2l} - \dot{\psi}_0^2 \right) \theta = 0$ where $\sin \theta \approx \theta$, $\cos \theta \approx 1$.Solution is $\theta = \theta_0 \sin \omega t$ where $\omega^2 = \frac{3g}{2l} - \dot{\psi}_0^2$ $\dot{\theta} = \Omega_0$ for $t=0$, so $\theta_0 = \Omega_0 / \omega$.Thus $\theta = \frac{\Omega_0}{\omega} \sin \omega t$.Period $T = \frac{2\pi}{\omega} = 2\pi / \sqrt{\frac{3g}{2l} - \dot{\psi}_0^2}$ When $\dot{\psi}_0 > \sqrt{\frac{3g}{2l}}$, motion is no longer periodic & rod tends to swing out

10/76 Let $V=0$ when $\theta_1 = \theta_2 = 0$;



$$V_e = \frac{1}{2} k \left(\frac{l}{2} \sin \theta_2 - \frac{l}{2} \sin \theta_1 \right)^2$$

$$V_g = mg \frac{l}{2} [(1 - \cos \theta_2) + (1 - \cos \theta_1)]$$

For small angles $\sin \theta \approx \theta$
 $\cos \theta \approx 1 - \frac{\theta^2}{2}$

$$\text{So } V_e = \frac{1}{8} k l^2 (\theta_2 - \theta_1)^2, V_g = mg \frac{l}{4} (\theta_2^2 + \theta_1^2)$$

$$V = \frac{1}{8} k l^2 (\theta_2 - \theta_1)^2 + \frac{1}{4} m g l (\theta_2^2 + \theta_1^2)$$

$$T = \frac{1}{2} \left(\frac{1}{3} m l^2 \right) \dot{\theta}_1^2 + \frac{1}{2} \left(\frac{1}{3} m l^2 \right) \dot{\theta}_2^2 + \frac{1}{6} m l^2 (\dot{\theta}_1^2 + \dot{\theta}_2^2)$$

With $L = T - V$ & $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) - \frac{\partial L}{\partial \theta_1} = 0$, $\theta = \theta_1, \theta_2$,

$$\text{For } \theta_1; \frac{\partial L}{\partial \theta_1} = \frac{1}{4} k l^2 (\theta_2 - \theta_1) - \frac{1}{2} m g l \theta_1$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = \frac{1}{3} m l^2 \dot{\theta}_1, \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) = \frac{1}{3} m l^2 \ddot{\theta}_1$$

$$\text{Thus } \frac{1}{3} m l^2 \ddot{\theta}_1 - \frac{1}{4} k l^2 (\theta_2 - \theta_1) + \frac{1}{2} m g l \theta_1 = 0 \quad (1)$$

$$\text{For } \theta_2; \frac{\partial L}{\partial \theta_2} = \frac{1}{4} k l^2 (\theta_2 - \theta_1) - \frac{1}{2} m g l \theta_2$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = \frac{1}{3} m l^2 \dot{\theta}_2, \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) = \frac{1}{3} m l^2 \ddot{\theta}_2$$

$$\text{Thus } \frac{1}{3} m l^2 \ddot{\theta}_2 + \frac{1}{4} k l^2 (\theta_2 - \theta_1) + \frac{1}{2} m g l \theta_2 = 0 \quad (2)$$

Add (1) & (2) & get with $\beta = \theta_1 + \theta_2$

$$\frac{1}{3} m l^2 \ddot{\beta} + \frac{1}{2} m g l \beta = 0 \text{ or } \ddot{\beta} + \frac{3}{2} \frac{g}{l} \beta = 0$$

So $\beta = \theta_1 + \theta_2 = A \sin pt + B \cos pt$ where $p^2 = \frac{3g}{2l}$

When $t=0$, $\theta_1 = 0$, $\theta_2 = \theta_0$ & $\dot{\theta}_1 = \dot{\theta}_2 = 0$. These conditions

give $B = \theta_0$ & $A = 0$ so

$$\theta_1 + \theta_2 = \theta_0 \cos pt \text{ & } \dot{\theta}_1 + \dot{\theta}_2 = -\theta_0 p \sin pt$$

Eliminate θ_2 from Eq. (2) & get

$$\ddot{\theta}_1 + \mu^2 \theta_1 = C^2 \cos pt \text{ where } \mu^2 = \frac{3}{2} \left(\frac{k}{m} + \frac{g}{l} \right)$$

Sol. is

$$\theta_1 = D \sin \mu t + E \cos \mu t + F \cos pt$$

$$\text{where } F = C^2 / (\mu^2 - p^2) = \theta_0 / 2$$

Boundary conditions $\theta_1 = 0$ & $\dot{\theta}_1 = 0$ for $t=0$ give

$$D = 0 \text{ and } E = -F = -\frac{\theta_0}{2} \text{ Thus}$$

$$\theta_1 = \frac{\theta_0}{2} (\cos pt - \cos \mu t)$$

$$\text{where } p = \sqrt{\frac{3g}{2l}}, \mu = \sqrt{\frac{3}{2} \left(\frac{k}{m} + \frac{g}{l} \right)}$$

APPENDIX A

REVIEW PROBLEMS

A-1

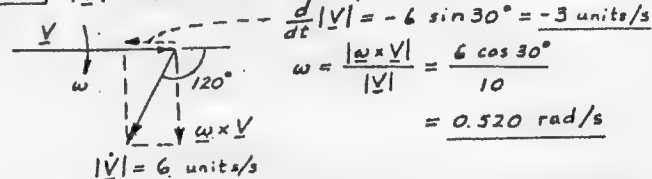
For constant acceleration a ,

$$\int v dv = a \int ds, v^2 = 2as$$

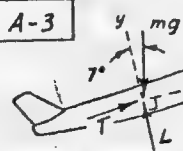
$$F = ma, s = \frac{v^2}{2F/m} = \frac{v^2 m}{2F} = \frac{(270/3.6)^2 115}{2(4) 90} = 898 \text{ m}$$

A-2

$|V| = 10$ units



A-3



$m = 90 \text{ t}, T = 240 \text{ kN}$

$$\Sigma F_x = ma_x, 240 \cos 3^\circ - 90(9.81) \sin 7^\circ = 90 a$$

$$a = 1.467 \text{ m/s}^2$$

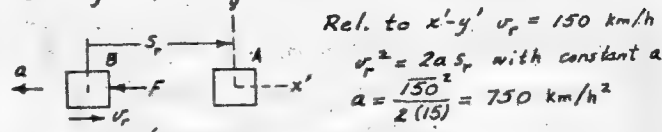
A-4

$$\Delta V_g + \Delta T = 0; -p g L \left(\frac{h}{2} + \frac{l}{2} \right) + \frac{1}{2} p L v^2 = 0$$

$$v^2 = g(L+h), v = \sqrt{g(L+h)}$$

A-5

A Newtonian coordinate system may be attached to A since it moves with constant velocity.



Rel. to $x'-y'$ $v_r = 150 \text{ km/h}$

$$v_r^2 = 2a s_r \text{ with constant } a$$

$$a = \frac{150^2}{2(15)} = 750 \text{ km/h}^2$$

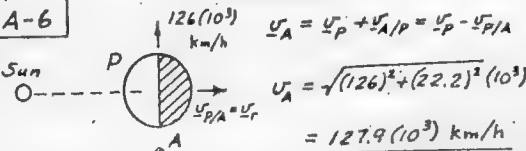
$$\text{or } a = \frac{750/3.6}{3600} = 0.0579 \text{ m/s}^2$$

$$F = ma; F = 800(0.0579) = 46.3 \text{ N}$$

$$v_r = at; t = \frac{150}{750} = 0.2 \text{ h or } 720 \text{ s}$$

$$\text{so } I = \int F dt = Ft = 46.3(720) = 33300 \text{ N}\cdot\text{s} = 33.3 \text{ kN}\cdot\text{s}$$

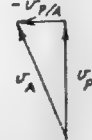
A-6



$$v_A = v_P + v_{A/P} = v_P - v_{P/A}$$

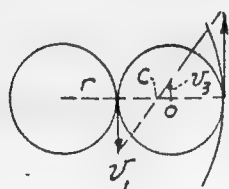
$$v_A = \sqrt{(126)^2 + (22.2)^2} (10^3)$$

$$= 127.9 (10^3) \text{ km/h}$$



A-7 Potential energy loss of rope equals kinetic energy gain of successively smaller portion of rope. At end of motion, A impacts cylinder with infinite velocity, assuming conservative system, and energy is dissipated in impact losses.

A-8



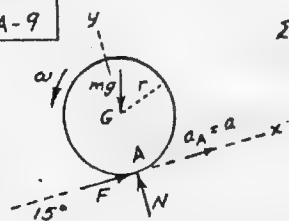
$$\frac{v_2 - v_3}{r} = \frac{v_1 + v_2}{2r}, v_3 = \frac{1}{2}(v_2 - v_1)$$

$$v_1 = rN_1, v_2 = 3rN_2, v_3 = 2rN_3$$

$$2N_3 = \frac{1}{2}(3N_2 - N_1)$$

$$N_3 = \frac{1}{4}(3N_2 - N_1)$$

A-9



$$\Sigma F_x = ma_x = 0; F = mg \sin 15^\circ = 0.259 mg$$

$$\Sigma \bar{M} = \bar{I} \alpha; Fr = \frac{1}{2} m r^2 \frac{a}{r}$$

$$a = \frac{2F}{m} = 2(9.81)(0.259) = 5.08 \text{ m/s}^2$$

A-10

Use polar coordinates where

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$ is replaced by $-a_x$ &

$a_r = \ddot{r} - r\dot{\theta}^2$ is replaced by $+a_y$

$$r = 2.4 + y = 2.4 + 0.15 \sin \pi t \text{ m}, \theta = \frac{\pi}{4} \sin \frac{\pi}{2} t$$

$$\dot{r} = \dot{y} = 0.15 \pi \cos \pi t \quad \dot{\theta} = \frac{\pi^2}{8} \cos \frac{\pi}{2} t$$

$$\ddot{r} = \ddot{y} = -0.15 \pi^2 \sin \pi t \quad \ddot{\theta} = -\frac{\pi^3}{16} \sin \frac{\pi}{2} t$$

For $t = 2 \text{ s}$,

$$r = 2.4 \text{ m}, \dot{r} = 0.15 \pi \text{ m/s}, \theta = 0, \dot{\theta} = -\frac{\pi^2}{8} \text{ rad/s}$$

$$\ddot{r} = 0 \quad \ddot{\theta} = 0$$

$$a_y = a_r = 0 - 2.4 \left(\frac{\pi^4}{64} \right) = -\frac{3\pi^4}{80} \text{ m/s}^2$$

$$a_x = -a_\theta = -(2.4)(0) - 2(0.15\pi)(-\pi^2/8) = \frac{3\pi^3}{80} \text{ m/s}^2$$

$$\underline{a = \frac{3\pi^3}{80} (\underline{i} - \pi \underline{j}) \text{ m/s}^2}$$

A-11

$$\Delta V_c + \Delta T = 0; \frac{1}{2} C (\theta^2 - \theta_0^2) + \frac{1}{2} m K^2 (\dot{\theta}^2 - 0) = 0$$

$$\dot{\theta}^2 = \frac{C}{m K^2} (\theta_0^2 - \theta^2)$$

$$\text{Power } P = M\dot{\theta} = C\theta \frac{1}{K\sqrt{m}} \sqrt{\theta_0^2 - \theta^2}$$

$$\frac{dP}{d\theta} = \frac{C}{K\sqrt{m}} \left(\frac{\theta_0^2 - 2\theta^2}{\sqrt{\theta_0^2 - \theta^2}} \right) = 0 \text{ for max. } P \text{ so } \underline{\theta_m = \theta_0/\sqrt{2}}$$

$$P = \frac{C\theta_0}{\sqrt{2}} \frac{1}{K\sqrt{m}} \theta_0 \sqrt{1 - \frac{1}{2}} = \underline{\frac{C}{2K} \sqrt{\frac{C}{m}} \theta_0^2}$$

A-12

$$\Sigma F_n = ma_n; T - mg \cos \theta = m l \dot{\theta}^2$$

$$\Delta T + \Delta V = 0; \frac{1}{2} m l^2 \dot{\theta}^2 + mg l [(1 - \cos \theta) - (1 - \cos \theta_0)] = 0$$

$$\frac{l \dot{\theta}^2}{2g} + (\cos \theta_0 - \cos \theta) = 0$$

$$\text{so } T = mg \left(\frac{l \dot{\theta}^2}{g} + \cos \theta \right) = \underline{mg (3 \cos \theta - 2 \cos \theta_0)}$$

A-13

$$e = \frac{v_2 - v_1}{v}; \Sigma G_x = \text{constant}$$

$$2mv = m(v + v_1) + mv_2 \quad 2mv = m(v + v_1) + mv_2$$

$$v = v_1 + v_2$$

Before $m \quad b \quad m$ Solve for $v_1 \neq v_2 \neq \text{get}$

$$v_1 = \frac{v}{2}(1 - e), v_2 = \frac{v}{2}(1 + e)$$

$$\omega_1 = \frac{v - v_1}{b} = \frac{v}{2b}(1 + e)$$

$$\omega_2 = \frac{v_2}{b} = \frac{v}{2b}(1 + e) = \omega_1$$

After $v_1 \quad v_2$

A-14

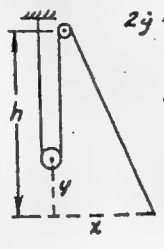
$$\sqrt{x^2 + h^2} + 2(h - y) = 3h, 2y = \sqrt{x^2 + h^2} - h$$

$$2\dot{y} = \frac{x\dot{x}}{\sqrt{x^2 + h^2}}; 2\ddot{y} = \frac{\dot{x}^2}{\sqrt{x^2 + h^2}} - \frac{(x\ddot{x})}{\sqrt{(x^2 + h^2)^3}}$$

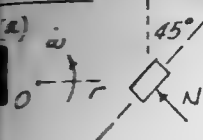
$$\text{so } \ddot{y} = \frac{h^2 \dot{x}^2}{2\sqrt{(x^2 + h^2)^3}} = \frac{h^2 v^2}{2\sqrt{(x^2 + h^2)^3}}$$

$$\Sigma F_y = ma_y; T - mg = ma_y$$

$$T = mg \left(1 + \frac{h^2 v^2}{2g\sqrt{(x^2 + h^2)^3}} \right)$$

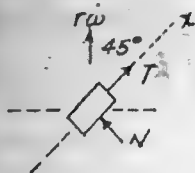


A-15



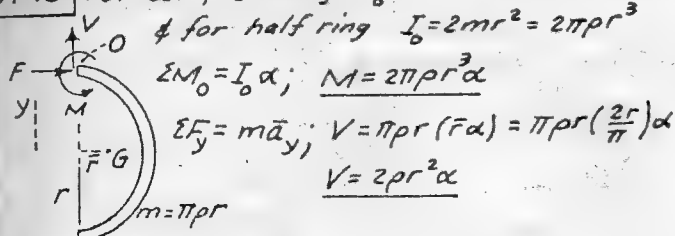
Let B on disk be coincident with A
 $\underline{a}_A = \underline{a}_B + \underline{a}_{rel}$
 \underline{a}_A in dir. of force N
 $\underline{a}_B = r\omega$ From diag.
 $\underline{a}_{rel} = r\omega/\sqrt{2}$

(b)



$$\Sigma F_x = ma_x; \quad T = mr\omega/\sqrt{2}$$

(c) For $T=0$, $\Sigma F_x = ma_x$; $0 = m(\frac{r\omega}{\sqrt{2}} - \frac{r\omega^2}{\sqrt{2}})$
 $\omega \neq 0$
 $\dot{\omega} = \omega^2$, $\omega = \sqrt{\dot{\omega}}$

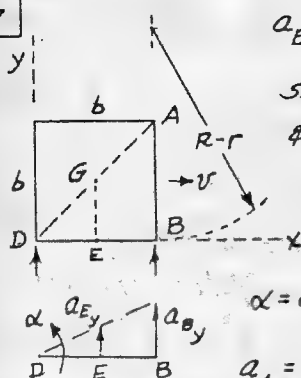
A-16 For complete ring $I_o = (2m)r^2 + (2m)r^2 = 4mr^2$ 

$$\Sigma M_o = I_o \alpha; \quad M = 2\pi r^3 \alpha$$

$$\Sigma F_y = m\ddot{a}_y; \quad V = \pi r (\bar{F}\alpha) = \pi r (\frac{2r}{\pi}) \alpha$$

$$V = 2pr^2 \alpha$$

A-17



$$\underline{a}_{By} = \frac{v^2}{R-r}, \quad \underline{a}_{Ay} = \frac{1}{2} \underline{a}_{By}$$

$$\text{Since } \underline{v}_{By} = 0, \quad \omega = 0$$

$$\& (a_{G/E})_n = (a_{G/E})_y = 0$$

$$\text{Thus } \underline{a}_{Gy} = \underline{a}_{Ey} = \frac{1}{2} \frac{v^2}{R-r}$$

$$\Sigma F_x = 0 \text{ so } \underline{a}_{Gx} = 0$$

$$\alpha = \underline{a}_{By} / \overline{DB} = \frac{v^2/b}{R-r}$$

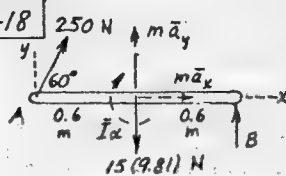
$$\underline{a}_A = \underline{a}_G + (\underline{a}_A/G)_t$$

$$\underline{a}_A/G = (\underline{a}_A/G)_t = \overline{AG} \alpha_{AG} = \frac{b}{\sqrt{2}} \frac{v^2/b}{R-r} = \frac{1}{\sqrt{2}} \frac{v^2}{R-r}$$

$$\underline{a}_A = \frac{v^2}{R-r} \left(-\frac{1}{\sqrt{2}} \underline{i} + \left[\frac{1}{\sqrt{2}} \underline{i} + \frac{1}{2} \underline{j} \right] \right)$$

$$= \frac{v^2}{R-r} \left(-\frac{1}{2} \underline{i} + \underline{j} \right)$$

A-18



$$\Sigma M_B = \bar{I}\alpha + m\ddot{a}_y d$$

$$250 \frac{\sqrt{3}}{2} 1.2 - 15(9.81)(0.6)$$

$$= \frac{1}{12} 15(1.2)^2 \frac{\ddot{a}_y}{0.6} + 15 \ddot{a}_y (0.6)$$

$$\ddot{a}_y = 14.29 \text{ m/s}^2, \quad \alpha = \frac{14.29}{0.6} = 23.8 \text{ rad/s}^2$$

$$\Sigma F_y = m\ddot{a}_y; \quad B - 15(9.81) + 250 \frac{\sqrt{3}}{2}$$

$$= 15(14.29)$$

$$B = 145.0 \text{ N}$$

A-19

Let A = cross-sectional area of tubes

 ρ = density of water

For vertical section

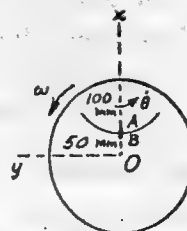
$$p_o A = \rho g A h, \quad p_o = \rho g h \text{ (negative pressure)}$$

For horizontal section

$$\Sigma F_n = m\ddot{a}_n; \quad p_o A = \rho A r \frac{1}{2} \omega^2, \quad p_o = \frac{1}{2} \rho r^2 \omega^2$$

$$\text{Thus } \rho g h = \frac{1}{2} \rho r^2 \omega^2, \quad \omega = \sqrt{2gh/r}$$

A-20

Let B = point on disk coincident with A at $\theta = 0$.

$$\underline{a}_A = \underline{a}_B + 2\omega \times \underline{r}_{rel} + \underline{a}_{rel}$$

$$\underline{a}_B = -50(10)^2 \underline{i} + 50(25) \underline{j} \text{ mm/s}^2$$

$$2\omega \times \underline{r}_{rel} = 2(10 \underline{k}) \times (100)(3)(-\underline{j})$$

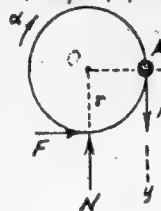
$$= 6000 \underline{i} \text{ mm/s}^2$$

$$\underline{a}_{rel} = 100(3^2) \underline{i} - 100(5) \underline{j} \text{ mm/s}^2$$

$$\underline{a}_A = 1900 \underline{i} + 750 \underline{j} \text{ mm/s}^2$$

A-21

Hoop & pipe A



$$\Sigma M_A = \Sigma \bar{M} = 0; \quad Nr - Fr = 0, \quad F = N$$

regardless of f

$$\Sigma F_x = ma_x; \quad F = ma_A$$

$$\Sigma F_y = ma_y; \quad mg - N = ma_A$$

(a) If $f > 1$, F may equal N, & hoop will not slip; $\underline{a}_x = r\alpha$, $\underline{a}_y = r\alpha$

$$\text{Thus, } mg - mr\alpha = mr\alpha, \quad \alpha = \frac{g}{2r}$$

$$\& F = mg/2 = N$$

(b) If $f < 1$, F cannot equal N > 0

so $F = N = 0$ & hoop slips with $\underline{a}_y = g$

$$\text{so } \alpha = \frac{g}{r}$$

A-22 Use the equation of work-energy for a small displacement dx upward.

Drum turns through angle $2 dx / 0.2$ & pinion turns through angle $\frac{0.3}{0.1} \frac{2 dx}{0.2} = 30 dx$ rad for x in metres.

$$dU = dT + dV; T = \sum \frac{1}{2} m v^2 + \sum \frac{1}{2} \bar{I} \omega^2$$

$$dT = \sum m v dv + \sum \bar{I} \omega d\omega$$

$$= \sum m \ddot{x} dx + \sum \bar{I} \ddot{\theta} d\theta$$

$$= \sum m \ddot{x} dx + \bar{I} \frac{2 \ddot{x}}{r} \frac{2 dx}{r}$$

$$= \left[450 + 100(0.15)^2 \frac{4}{(0.2)^2} \right] \ddot{x} dx$$

$$= 675 \ddot{x} dx$$

$$dV = \sum mg dx = 450(9.81) dx$$

$$dU = M d\theta = 240(30 dx) = 7200 dx \text{ J}$$

$$\text{So } 7200 dx = 675 \ddot{x} dx + 450(9.81) dx$$

$$\ddot{x} = \frac{2780}{675} = 4.13 \text{ m/s}^2 \text{ v}$$

A-23 During impact $\Delta H_0 = 0$

$$0.06(300)r = (5 + 0.06)vr, v = 3.56 \text{ m/s}$$

$$f = \frac{p}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}, k = 4\pi^2 m f^2 = 4\pi^2 (3.06)(4^2)$$

$$= 3200 \text{ N/m}$$

$$V_{e \max} = T_{\max}; \frac{1}{2} k A^2 = \frac{1}{2} m v^2$$

$$\frac{1}{2} 3200 A^2 = \frac{1}{2} 5.06 (3.56)^2, A^2 = 0.0200 \text{ m}^2$$

$$A = 0.1415 \text{ m}$$

$$\text{or } A = 141.5 \text{ mm}$$

$$f = 4 \text{ Hz}$$

For damped motion

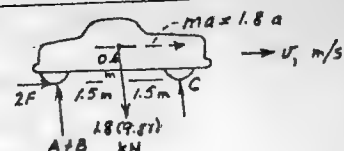
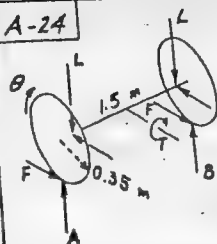
$$\ln \frac{x_0}{x_n} = n b T, n = \text{No. of cycles} = 10$$

$$b T = \frac{c}{2m} \frac{2\pi}{p} = \frac{\pi c}{m} \sqrt{\frac{m}{k}} = \frac{\pi c}{\sqrt{mk}}$$

$$\text{Thus } \ln \frac{1}{0.6} = 10 \frac{\pi c}{\sqrt{5.06(3200)}}, 0.511 = \frac{31.42 c}{127.2}$$

$$c = 2.07 \text{ N.s/m}$$

A-24



For 80 km/h, speed of propeller shaft is $3.7 \frac{v}{r} = 3.7 \frac{80/3.6}{0.35} = 235 \text{ rad/s}$

$$\text{Power } P = \omega T, T = \frac{75}{235} = 0.319 \text{ kN.m}$$

$$\text{For rear wheels, } dU = 0, 0.319(3.7 d\theta) = 2F(0.35) d\theta$$

$$F = 1.688 \text{ kN}$$

$$\text{For entire car, } \sum F = ma; 2(1.688) = 1.8 a, a = 1.875 \text{ m/s}^2$$

$$\sum M_C = mad; 3(A+B) - 1.8(9.81)(1.5) = 2(1.688)(0.6)$$

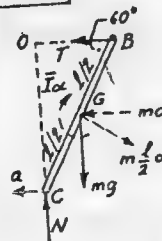
$$A+B = 9.50 \text{ kN} \text{ --- (1)}$$

$$\text{For rear-wheel assembly, } \sum M_{\text{propeller}} = 0$$

$$(B-A) \frac{1.5}{2} = 0.319, B-A = 0.426 \text{ kN} \text{ --- (2) Solve (1) \& (2) get}$$

$$B = 4.96 \text{ kN}, A = 4.54 \text{ kN}$$

A-25



$$a_B = a_C + (a_{B/C})_t, \bar{a} = a_C + (a_{C/t})_t$$

$$a_C = a, (a_{C/t})_t = \frac{l}{2} \alpha$$

$$\text{so } a_B = \frac{l}{2} \alpha \cos \theta$$

$$a = l \alpha \sqrt{3}/2$$

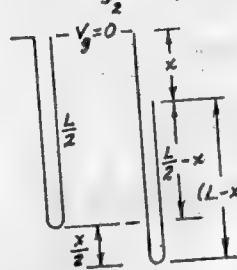
$$\bar{I} \alpha = \frac{1}{12} m l^2 \frac{2a}{l \sqrt{3}} = \frac{m a l}{6 \sqrt{3}}; m \frac{l}{2} \alpha = m a / \sqrt{3}$$

$$\sum M_O = \bar{I} \alpha + m \bar{a} d; mg \frac{l}{2} \frac{1}{2} = \frac{m a l}{6 \sqrt{3}} + m a \frac{l \sqrt{3}}{2} \frac{1}{2} - \frac{m a}{\sqrt{3}} \left(\frac{l}{2} - \frac{l}{2} \right)$$

$$\text{Solve \& get } a = \frac{3\sqrt{3}}{8} g$$

A-26

$$V_g = -pg \frac{L}{2} \frac{L}{4} - 2pg \frac{x}{2} \left(\frac{L}{2} + \frac{x}{4} \right) - pg \left(\frac{L}{2} - x \right) \left(x + \frac{1}{2} \left[\frac{L}{2} - x \right] \right)$$



$$V_g = -\frac{pg}{4} (L^2 + 2Lx - x^2)$$

$$V_g = -pg \frac{L^2}{4}; \Delta V_g = -\frac{pg}{4} (2Lx - x^2)$$

$$\Delta T = \frac{1}{2} \rho \frac{L-x}{2} \dot{x}^2$$

$$\Delta T + \Delta V_g = 0;$$

$$\frac{1}{2} \rho \frac{L-x}{2} \dot{x}^2 - \frac{pg}{4} (2Lx - x^2) = 0$$

$$\text{so } \dot{x} = \sqrt{\frac{g x (2L-x)}{L-x}}$$

① ② $\rho = \text{mass per unit length}$

$$\text{As } x \rightarrow L, \dot{x} \rightarrow \infty$$

For $x = L, \Delta V_g = -pg \frac{L^2}{4} \neq pg \frac{L^2}{4}$ represents the kinetic energy concentrated in the last element of falling rope. Impact occurs as the last element is brought to rest & the energy $pg \frac{L^2}{4}$ is lost during impact.

A-27

Let ρ = mass of rope per unit length.Assume $\bar{x} = L/2$ For complete rope ($x=L, T=0$)

$$\Sigma F_y = 0; T \sin \theta = \rho g L$$

$$\Sigma F_x = m a_x; T \cos \theta = \rho L \frac{L}{2} \omega^2$$

For section of length x ,

$$\Sigma F_y = 0; T \sin \theta + \rho g x = \rho g L, T \sin \theta = \rho g (L-x)$$

$$\Sigma F_x = m a_x; \rho L \frac{L}{2} \omega^2 - T \cos \theta = \rho x \frac{x}{2} \omega^2, T \cos \theta = \frac{\rho \omega^2}{2} (L^2 - x^2)$$

$$\tan \theta = \frac{dy}{dx} = \frac{\rho g (L-x)}{\frac{\rho \omega^2}{2} (L^2 - x^2)} = \frac{2g}{\omega^2} \frac{1}{L+x}$$

$$\int_0^y dy = \frac{2g}{\omega^2} \int_0^x \frac{dx}{L+x}, y = \frac{2g}{\omega^2} \ln \left(1 + \frac{x}{L} \right)$$

A-28

(a) Accel. in dir. of force is $x_0 \omega^2 - \ddot{x}$

$$\text{so } \Sigma F_x = m a_x; k x_0 = m (x_0 \omega^2 - \ddot{x})$$

$$\ddot{x} = - \left(\frac{k}{m} - \omega^2 \right) x_0 \text{ where } \frac{k}{m} > \omega^2$$

(b) For general position $\ddot{x} + p^2 x = 0$

$$\text{where } p^2 = \frac{k}{m} - \omega^2$$

$$F \rightarrow \square \rightarrow 2|x|\omega \quad x = A \sin pt + B \cos pt$$

$$x = x_0 \text{ \& } \dot{x} = 0 \text{ when } t=0$$

$$\text{gives } B = x_0 \text{ \& } A = 0$$

$$\text{so } x = x_0 \cos pt, \dot{x} = -x_0 p \sin pt, |\dot{x}|_{x=0} = x_0 p$$

$$\text{Thus accel is } 2|\dot{x}|\omega = 2x_0 \omega \sqrt{\frac{k}{m} - \omega^2}$$

$$F = ma; F = 2m x_0 \omega \sqrt{\frac{k}{m} - \omega^2}$$

A-29

$$\theta = \tan^{-1} \frac{250}{150}, \sin \theta = 0.857, \cos \theta = 0.514$$

$$u = v_A \sin \theta - v_B \cos \theta = \dot{r}$$

$$= 0.1 \frac{3}{2} (0.857 - 0.1(0.514)) = 0.0772 \text{ m/s}$$

$$\dot{\theta} = \frac{1}{r} (v_A \cos \theta + v_B \sin \theta)$$

$$= \frac{1}{0.292} [0.15(0.514) + 0.1(0.857)]$$

$$= 0.559 \text{ rad/s CCW}$$

$$a_A = a_B + a_{A/B}, a_B = 0$$

$$(a_{A/B})_r = \ddot{r} - r \dot{\theta}^2 = \ddot{u} - 0.292(0.559)^2 = \ddot{u} - 0.0910 \text{ m/s}^2$$

$$(a_{A/B})_\theta = r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 0.292 \ddot{\theta} + 2(0.0772)(0.559) = 0.292 \ddot{\theta} + 0.0863 \text{ m/s}^2$$

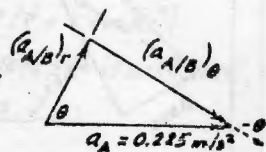
$$a_A = 0.1 \left(\frac{3}{2} \right)^2 = 0.225 \text{ m/s}^2 \rightarrow$$

$$(a_{A/B})_r = 0.225(0.514) = \ddot{u} - 0.0910$$

$$\ddot{u} = 0.207 \text{ m/s}^2$$

$$(a_{A/B})_\theta = -0.225(0.857) = 0.292 \ddot{\theta} + 0.0863$$

$$\ddot{\theta} = -\ddot{\theta} = 0.958 \text{ rad/s}^2$$



A-30

Let M = point on slotted link coincident with A at instant shown.



$$v_M = v_P \text{ \& } a_M = a_P$$

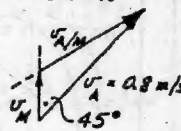
$$v_A = v_M + v_{A/M}$$

$$v_A = R \dot{\theta} = 0.1(8) = 0.8 \text{ m/s}$$

Solution of triangle

gives

$$v_P = v_M = 0.8 \frac{\sqrt{3}-1}{\sqrt{6}}, v_P = 0.239 \text{ m/s up}$$



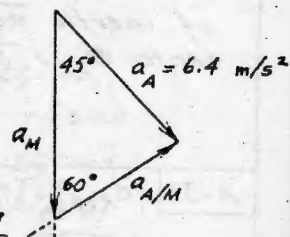
$$a_A = a_M + a_{A/M}$$

$$a_A = R \ddot{\theta} = 0.1(8^2) = 6.4 \text{ m/s}^2$$

 $a_{A/M}$ is along slot a_M is vertical

Solution of triangle gives

$$a_M = a_P = 7.14 \text{ m/s}^2 \text{ down}$$



A-31

$v_B = 2\dot{x}/\cos 30^\circ = 2(0.25)\frac{2}{\sqrt{3}} = 0.577 \frac{m}{s}$
 $a_{A n_2} = \frac{v_A^2}{AO} = \frac{(0.289)^2}{2} = 0.0417 \frac{m}{s^2}$
 $v_{A t_1} = \dot{x} \tan 30^\circ = 0.25 \frac{1}{\sqrt{3}} = 0.1443 \frac{m}{s}$
 $a_{A n_1} = -(\ddot{x} - v_{A t_1}^2/x) = 0 + (0.1443)^2/2 = 0.01042 \frac{m}{s^2}$

$a_{A n_1} = 0.01042 \frac{m}{s^2}$
 From vector solution
 $a_A = 0.0434 \frac{m}{s^2}$
 $a_{B t_1} = 2a_{A t_1}$; $a_{B n_1} = 2a_{A n_1}$
 so $a_B = 2a_A = 0.0867 \frac{m}{s^2}$
 in the same direction
 as a_A .

A-32 Let I = moment of inertia of complete sphere about a diametral axis.

By symmetry $I_{xx} = I_{yy} = I_{zz} = \frac{1}{2}I$ for the half-sphere.

Since x, y, z axes are principal axes through O , it follows that the ellipsoid of inertia about O is a sphere. Hence, moments of inertia about all axes through O are identical and equal $\frac{2}{5}mr^2$. Similarly for a hemispherical shell where moment of inertia about any axis through its center O is $\frac{2}{3}mr^2$.

A-33 $\{a_{R\theta\phi}\} = [T_\theta][T_\phi]\{a_{xyz}\}$; $a_x = -v^2/r$, $a_y = 0$, $a_z = 0$

$\{a_{R\theta\phi}\} = \begin{bmatrix} \cos\phi \cos\theta & \cos\phi \sin\theta & \sin\phi \\ -\sin\theta & \cos\theta & 0 \\ -\sin\phi \cos\theta & -\sin\phi \sin\theta & \cos\phi \end{bmatrix} \begin{Bmatrix} -v^2/r \\ 0 \\ 0 \end{Bmatrix}$

Expand & get

$a_R = -\frac{v^2}{r} \cos\phi \cos\theta$
 $a_\theta = \frac{v^2}{r} \sin\theta$
 $a_\phi = \frac{v^2}{r} \sin\phi \cos\theta$

Ans.

A-34

$\underline{a}_A = \underline{a}_C + \underline{\Omega} \times \underline{r} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$
 where $\underline{\Omega} = \omega \underline{k}$ = angular velocity of reference axes
 $\underline{a}_C = -R\Omega^2 \underline{j}$, $\underline{\Omega} \times \underline{r} = 0$
 $\underline{\Omega} \times (\underline{\Omega} \times \underline{r}) = \Omega \underline{k} \times (\Omega \underline{k} \times \underline{r}) = -r\Omega^2 \underline{i}$
 $2\underline{\Omega} \times \underline{v}_{rel} = 2\underline{\Omega} \underline{k} \times R\Omega \underline{k} = 0$
 $\underline{a}_{rel} = -\frac{(R\Omega)^2}{r} \underline{i}$
 Thus $\underline{a}_A = -R\Omega^2 \underline{j} - r\Omega^2 \underline{i} - \frac{(R\Omega)^2}{r} \underline{i} = -\Omega^2 \left(\frac{r^2 + R^2}{r} \underline{i} + R \underline{j} \right)$

A-35 $\Delta V_g + \Delta T = 0$; $-mgh + \frac{1}{2}mv^2 = 0$, $v^2 = 2gh$

$\Sigma F_r = ma_r$; $N_1 = m \frac{(v \cos \gamma)^2}{b} = \frac{2mgh \cos^3 \gamma}{b}$
 $\Sigma F_n = 0$; $N_2 - mg \cos \gamma = 0$
 $N = mg \cos \gamma \sqrt{1 + \frac{4h^2 \cos^2 \gamma}{b^2}}$
 But $\tan \gamma = \frac{h}{2\pi b}$, $\cos^2 \gamma = \frac{1}{1 + (\frac{h}{2\pi b})^2}$
 substitute, simplify, & get
 $N = \frac{mg}{1 + (\frac{h}{2\pi b})^2} \sqrt{1 + \frac{h^2}{b^2} (4 + \frac{1}{4\pi^2})}$

A-36 $\omega_x = v_o/r = 0.9/0.2 = 4.5 \text{ rad/s}$; $\omega_y = \omega_z = 0$

$H_x = I_{xx} \omega_x$, $H_y = -I_{xy} \omega_x$
 $H_z = -I_{xz} \omega_x$
 $I_{xx} = \frac{1}{2}mr^2 + 2m'd^2 = \frac{1}{2}(2)(0.2)^2 + 2(4)(0.15)^2 = 0.660 \text{ kg} \cdot \text{m}^2$
 $I_{xy} = 0 + 2(4)(0.15)(0.075) = 0.0900 \text{ kg} \cdot \text{m}^2$
 $H_x = 0.660(4.5) \underline{i} - 0.0900(4.5) \underline{j}$
 $H_o = 2.97 \underline{i} - 0.405 \underline{j} \text{ kg} \cdot \text{m}^2/\text{s}$

A-37

$I_{xx} = \rho c \frac{\pi}{2} (-b) = -\frac{\rho b c^2}{2}$, $\rho = \text{unit mass}$
 $I_{yy} = \rho c a \left(\frac{\pi}{2}\right) = \frac{\rho a c^2}{2}$
 $I_{zz} = \frac{1}{3} \rho a^3 + \rho b \left(\frac{1}{12} b^2 + a^2 + \left[\frac{b}{2}\right]^2\right) + \rho c (b^2 + a^2)$
 $= \rho \left[\frac{a^3}{3} + \frac{b^3}{3} + b a^2 + c b^2 + c a^2\right]$

Substitute $a = 0.125 \text{ m}$, $b = 0.075 \text{ m}$, $c = 0.100 \text{ m}$, $\rho = 1.2 \text{ kg/m}$ & get

$I_{xx} = -0.450 (10^{-3}) \text{ kg} \cdot \text{m}^2$
 $I_{yy} = 0.750 (10^{-3}) \text{ kg} \cdot \text{m}^2$
 $I_{zz} = 4.91 (10^{-3}) \text{ kg} \cdot \text{m}^2$

$M_x = -I_{xz} \dot{\omega}_z = 0.450 (10^{-3}) (10^3) = 0.450 \text{ N} \cdot \text{m}$
 $M_z = I_{zz} \dot{\omega}_z = 4.91 (10^{-3}) (10^3) = 4.91 \text{ N} \cdot \text{m}$
 $M = \sqrt{M_x^2 + M_z^2} = \sqrt{0.450^2 + 4.91^2} = 4.93 \text{ N} \cdot \text{m}$

A-38 Using the Lagrangian approach,

$T = \frac{1}{2} m (\dot{x}^2 + x^2 \omega^2) + \frac{1}{2} I \omega^2$, $V = \frac{1}{2} k x^2$, $L = T - V$
 $\frac{\partial L}{\partial \theta} = 0$, $\frac{\partial L}{\partial \dot{\theta}} = \frac{\partial L}{\partial \omega} = m x^2 \omega + I \omega$
 $\frac{d}{dt} (m x^2 \omega + I \omega) - 0 = 0 \dots (1)$

$\frac{\partial L}{\partial x} = m x \omega^2 - k x$, $\frac{\partial L}{\partial \dot{x}} = m \dot{x}$
 $m \ddot{x} - m x \omega^2 + k x = 0 \dots (2)$

From (1) $(m x^2 + I) \omega = \text{const } C$; $(m x_0^2 + I) \omega_0 = C$

$(m x^2 + I) \omega = (m x_0^2 + I) \omega_0$ [conservation of angular momentum]

Substitute into (2) & get

$\ddot{x} + \left\{ \frac{k}{m} - \frac{[m x_0^2 + I]^2 \omega_0^2}{m x^2 + I} \right\} x = 0$

A-39

$z^2 + r^2 (1 + \cos \theta)^2 = R^2 = L^2 - r^2 \sin^2 \theta$
 $z^2 + 2 r^2 (1 + \cos \theta) = L^2$, $2 z \dot{z} + 2 r^2 (-\dot{\theta} \sin \theta) = 0$
 $\dot{z} = \frac{r^2 \dot{\theta} \sin \theta}{z} = \frac{r^2 \dot{\theta} \sin \theta}{\sqrt{L^2 - 2 r^2 (1 + \cos \theta)}}$

$\mathbf{V}_A = \mathbf{V}_B + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$
 For $\theta = \pi/2$, $\mathbf{V}_A = 2 \mathbf{k} = \frac{r^2 \dot{\theta}}{\sqrt{L^2 - 2 r^2}} \mathbf{k}$

$\mathbf{V}_B = -r \dot{\theta} \mathbf{j}$, $\mathbf{r}_{A/B} = -r \mathbf{i} - r \mathbf{j} + \sqrt{L^2 - 2 r^2} \mathbf{k}$
 $\mathbf{V}_A = \frac{r^2 \dot{\theta}}{\sqrt{L^2 - 2 r^2}} \mathbf{k} = -r \dot{\theta} \mathbf{j} + \left[\frac{r}{\omega_x} \quad \frac{r}{\omega_y} \quad \frac{r}{\omega_z} \right]$

Expand, equate like coefficients of $\mathbf{i}, \mathbf{j}, \mathbf{k}$ & get

$\dot{\theta} = -\omega_y \frac{\sqrt{L^2 - 2 r^2}}{r}$, $\omega_x = \omega_y = \frac{r \dot{\theta}}{\sqrt{L^2 - 2 r^2}}$

Also $\boldsymbol{\omega} \cdot \mathbf{r}_{A/B} = 0$ so $-r \omega_x - r \omega_y + \sqrt{L^2 - 2 r^2} \omega_z = 0$

Combine & get

$\omega_x = \frac{r \dot{\theta} (r^2 - L^2)}{L^2 \sqrt{L^2 - 2 r^2}}$, $\omega_y = \frac{r \dot{\theta}}{L^2 \sqrt{L^2 - 2 r^2}}$, $\omega_z = -\frac{r^2 \dot{\theta}}{L^2}$

A-40

$I = I_{zz} = m r^2$, $I_0 = I_{xx} = I_{yy} = \frac{1}{2} m r^2$
 For precession with zero moment
 $\dot{\psi} = \frac{I \dot{\phi}}{(I_0 - I) \cos \theta} = \frac{\dot{\phi}}{\left(\frac{I_0}{I} - 1\right) \cos \theta}$, Eq. 174
 $\dot{\psi} = \frac{60}{\left(\frac{1}{2} - 1\right) \cos 15^\circ} = -124.2 \text{ rev/min}$
 or $\dot{\psi} = -13.01 \text{ rad/s}$
 (+ $\dot{\psi}$ is in +Z-dir.)

$\omega_x = 0$, $\omega_y = \dot{\psi} \sin \theta$, $\omega_z = \dot{\phi} + \dot{\psi} \cos \theta$
 $\mathbf{H} = \frac{1}{2} m r^2 (0) \mathbf{i} + \frac{1}{2} m r^2 (\dot{\psi} \sin \theta) \mathbf{j} + m r^2 (\dot{\phi} + \dot{\psi} \cos \theta) \mathbf{k}$
 $\boldsymbol{\omega} = (0) \mathbf{i} + \dot{\psi} \sin \theta \mathbf{j} + (\dot{\phi} + \dot{\psi} \cos \theta) \mathbf{k}$
 $T_{\text{rot}} = \frac{1}{2} \boldsymbol{\omega} \cdot \mathbf{H} = \frac{1}{2} m r^2 [(\dot{\psi} \sin \theta)^2 + 2(\dot{\phi} + \dot{\psi} \cos \theta)^2]$
 $= \frac{1}{2} (8) (0.300)^2 [(13.01)^2 (0.2588)^2 + 2(6.28 - 13.01 \times 0.9659)^2]$
 $= 0.18 [11.34 + 78.96] = 16.25 \text{ J}$

A-41

$\Sigma \mathbf{F} = m \mathbf{a}$; $m g \cos \theta = m (r \ddot{\theta} + r \omega^2 \cos \theta \sin \theta)$
 Also $\dot{\theta} d\theta = \dot{\theta} d\theta$
 So $g \cos \theta - r \omega^2 \cos \theta \sin \theta = r \frac{d(\frac{1}{2} \dot{\theta}^2)}{d\theta}$
 $\int_0^{\dot{\theta}} d(\frac{1}{2} \dot{\theta}^2) = \int_0^{\theta} (g \cos \theta - r \omega^2 \cos \theta \sin \theta) d\theta$
 $\frac{1}{2} \dot{\theta}^2 = \frac{g}{r} \sin \theta - \frac{\omega^2}{2} \sin^2 \theta$
 Now let $\dot{\theta} = 0$ for $\theta = \pi/2$ & get
 $\omega^2 = \frac{2g}{r \sin \pi/2} = \frac{2g}{r}$
 So $\omega = \sqrt{2g/r}$

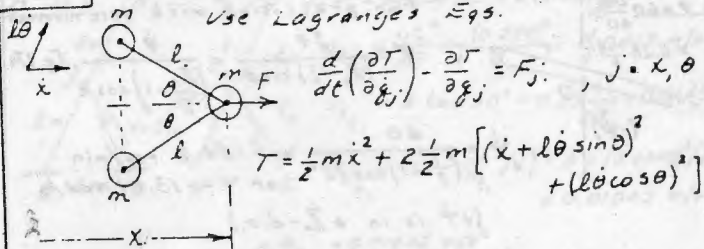
Coriolis term is a vector in the y-direction

$2 \boldsymbol{\omega} \times \mathbf{v} = 2 \omega r \dot{\theta} \sin \theta \mathbf{j}$
 Thus $m g \sin \theta - N_1 = m (-r \ddot{\theta} - r \omega^2 \cos^2 \theta)$
 $N_1 = m (g \sin \theta + 2g \sin \theta - r \omega^2 \sin^2 \theta + r \omega^2 \cos^2 \theta)$
 $= m (3g \sin \theta + r \omega^2 \cos 2\theta)$
 For $\omega = \sqrt{2g/r}$, N_1 & N_2 become
 $N_1 = m g (3 \sin \theta + 2 \cos 2\theta)$
 $N_2 = 2m \sqrt{\frac{2g}{r}} r \sin \theta \sqrt{\frac{2g}{r} \sin \theta - \frac{2g}{r} \sin^2 \theta}$
 $N_2 = 4m g (\sin^{3/2} \theta) \sqrt{1 - \sin \theta}$

A-42

Two degrees of freedom x, θ

Use Lagrange's Eqs.



For x ; $Q_x \delta x = F \delta x$; $Q_x = F$; $\frac{\partial T}{\partial x} = 0$

$$\frac{\partial T}{\partial \dot{x}} = m \dot{x} + 2m(\dot{x} + l \dot{\theta} \sin \theta)$$

So $m \ddot{x} + 2m \ddot{x} + 2ml(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta) - 0 = F$

& for $\dot{\theta} = 0$, $3m \ddot{x} + 2ml \ddot{\theta} \sin \theta = F$ ----- (1)

For θ ; $Q_\theta \delta \theta = 0$, $Q_\theta = 0$;

$$\frac{\partial T}{\partial \theta} = 2m[(\dot{x} + l \dot{\theta} \sin \theta) l \dot{\theta} \cos \theta - l^2 \dot{\theta}^2 \cos \theta \sin \theta]$$

$= 0$ for $\dot{\theta} = 0$

$$\frac{\partial T}{\partial \dot{\theta}} = 2m[(\dot{x} + l \dot{\theta} \sin \theta) l \sin \theta + l^2 \dot{\theta} \cos^2 \theta]$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\theta}} \right) = 2m[l \ddot{x} \sin \theta + l \dot{x} \dot{\theta} \cos \theta + l^2 \ddot{\theta} \sin^2 \theta + 2l^2 \dot{\theta}^2 \sin \theta \cos \theta + l^2 \ddot{\theta} \cos^2 \theta - 2l^2 \dot{\theta}^2 \cos \theta \sin \theta]$$

& for $\dot{\theta} = 0$, $\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\theta}} \right) = 2m[l \ddot{x} \sin \theta + l^2 \ddot{\theta}]$

Thus for $\dot{\theta} = 0$, $2m[l \ddot{x} \sin \theta + l^2 \ddot{\theta}] - 0 = 0$ ---- (2)

Combine (1) & (2) & get $m \ddot{x} (3 - 2 \sin^2 \theta) = F$

or $\ddot{x} = \frac{F/m}{3 - 2 \sin^2 \theta}$